Year 2013

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education 2013

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 32 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graphs of $\frac{(x+a)^2}{a^2} + \frac{(y+a)^2}{b^2} = 1$ and $\frac{(y+a)^2}{b^2} - \frac{(x+a)^2}{a^2} = 1$, where a and b are non-zero real constants,

A. do not intersect.

B. touch at the two points $(-2a, -a)$ and $(0, -a)$.

C. touch at the two points $(-a, b-a)$ and $(-a, -(a+b))$.

D. intersect at the three points.

E. intersect at four points.

Question 2

The graph of $y = \frac{1}{ax^2 + b(1-a)x - b^2}$ where a and b are non-zero real constants, has asymptotes at

A. $x = -b$ and $x = \frac{b}{a}$ only.

B. $x = b$ and $x = -\frac{b}{a}$ only.

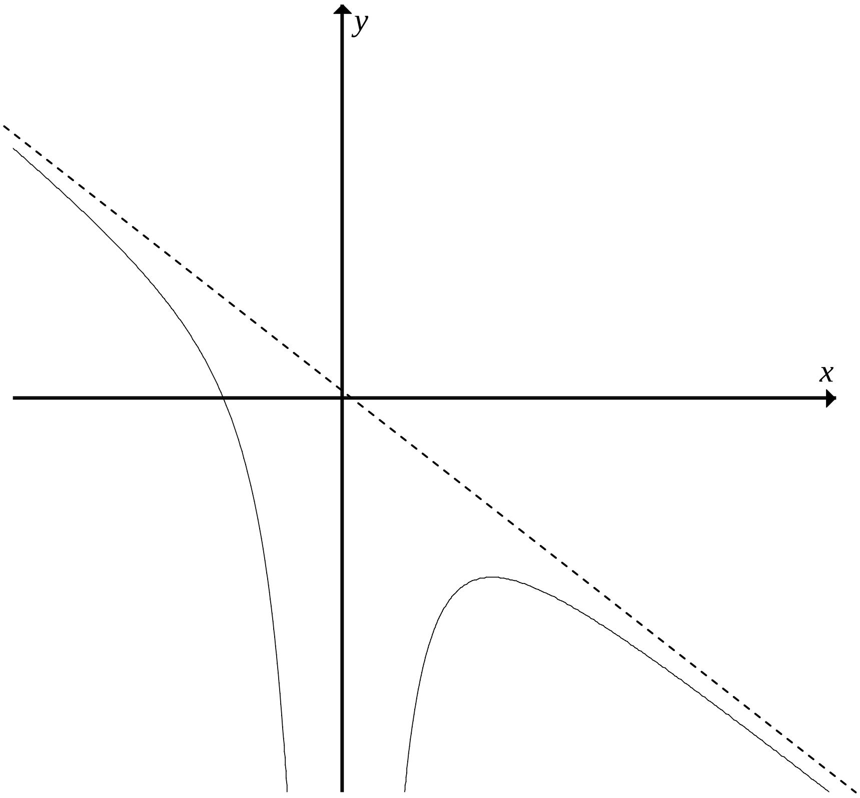
C. $x = -b$, $x = \frac{b}{a}$ and $y = 0$.

D. $x = b$, $x = -\frac{b}{a}$ and $y = 1$.

E. $x = b$, $x = -\frac{b}{a}$ and $y = 0$.

Question 3

The graph of $y = \dfrac{ax^n + b}{x^2}$ is shown below.



Then

A. $n = 3$, $a > 0$, $b < 0$.

B. $n = 3$, $a > 0$, $b > 0$.

C. $n = 3$, $a < 0$, $b < 0$.

D. $n = 3$, $a < 0$, $b > 0$.

E. $n = 2$, $a < 0$, $b > 0$.

Question 4

The graph of $y = a\sin^{-1}(bx - c) + d$ has both the domain and range equal to $[0, \pi]$. Then

A. $a = c = 1$, $b = d = \dfrac{\pi}{2}$

B. $a = c = 1$, $b = \dfrac{2}{\pi}$, $d = \dfrac{\pi}{2}$

C. $a = c = 1$, $b = d = \dfrac{2}{\pi}$

D. $a = c = -1$, $b = \dfrac{2}{\pi}$, $d = \dfrac{\pi}{2}$

E. $a = c = -1$, $b = d = \dfrac{\pi}{2}$

Question 5

If $a + bi = r\,\text{cis}(\theta)$ where *a* and *b* are positive real constants, then $-b - ai$ is equal to

A. $-r\,\text{cis}(\theta)$

B. $-r\,\text{cis}\left(\dfrac{\pi}{2} - \theta\right)$

C. $r\,\text{cis}(-\theta)$

D. $r\,\text{cis}(\pi - \theta)$

E. $r\,\text{cis}\left(\theta - \dfrac{3\pi}{2}\right)$

Question 6

The polynomial $P(z)$ has real coefficients. Two of the roots of the equation $P(z) = 0$ are $z = (a + b)i$ and $z = (a - b)i$, where *a* and *b* are non-zero real constants, and $|a| \neq |b|$. Then the minimum degree of the polynomial $P(z)$ is

A. 2

B. 3

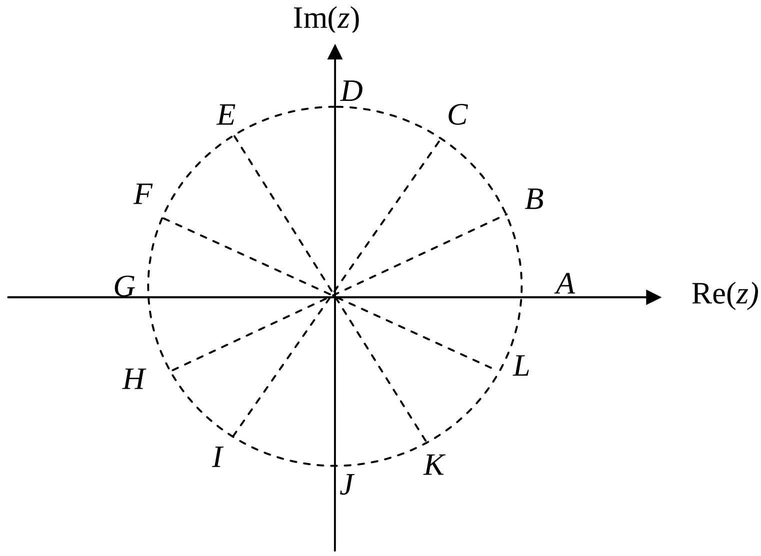
C. 4

D. 5

E. 6

Question 7

The diagram shows a circle of radius a on an argand diagram.



The roots of the equation $z^3 - ia^3 = 0$ where a is a positive real number, are given by

A. D only.

B. G ,C and K.

C. A, E and I.

D. B, F and J.

E. D, H and L.

Question 8

A cricket ball is hit from ground level at an angle of 30^0 to the horizontal, with a speed of 19.6 ms^{-1}. Then

A. it takes one second to reach a maximum height of $2g\sqrt{3}$ metres.

B. it takes two seconds to reach a maximum height of 4.9 metres.

C. it takes two seconds to reach a maximum height of $2g\sqrt{3}$ metres.

D. it takes one second to reach a maximum height of 4.9 metres and hits the ground 49 metres from the point of impact.

E. it reaches a maximum height of 4.9 metres and hits the ground after two seconds $2g\sqrt{3}$ metres from the point of impact.

Question 9

A boy is standing on top of a two metre high fence. He jumps off the fence and when his hand is one metre above the ground, he throws a ball vertically upwards, with a speed of $4.9\ \text{ms}^{-1}$. The time taken in seconds, for the ball to hit the ground is closest to

A. 1.79

B. 1.17

C. 1.15

D. 0.50

E. 0.17

Question 10

If b is a non-zero real number, and $z = x + yi$, then the set of points in the argand plane defined by $\{z : |z-b|^2 + |z+bi|^2 = 2b^2\}\ z \in C$ represents a circle with

A. centre $(b,-b)$ and radius $\sqrt{2}\,b$

B. centre $(-b,b)$ and radius $\sqrt{2}\,b$

C. centre $(b,-b)$ and radius $\dfrac{\sqrt{2}\,b}{2}$

D. centre $\left(-\dfrac{b}{2},\dfrac{b}{2}\right)$ and radius $\dfrac{\sqrt{2}\,b}{2}$

E. centre $\left(\dfrac{b}{2},-\dfrac{b}{2}\right)$ and radius $\dfrac{\sqrt{2}\,b}{2}$

Question 11

A particle is acted upon by two forces. One has a magnitude of $\sqrt{2}\,b$ newtons and acts in the direction $S\,45^0\,E$, the other has a magnitude of $\sqrt{2}\,c$ newtons and acts in the direction $N\,45^0\,E$, where *b* and *c* are non-zero real positive constants. The magnitude of the resultant force is equal to

A. $\sqrt{2}(b+c)$

B. $\dfrac{\sqrt{2}}{2}(b+c)$

C. $2\sqrt{2\left(b^2+c^2\right)}$

D. $\sqrt{2\left(b^2+c^2\right)}$

E. $2\sqrt{b^2+c^2}$

Question 12

A car of mass m kg is travelling on a level roadway. The engine exerts a constant propulsive force of F newtons and the total resistance to the motion of the car is kv newtons, where k is positive constant and v is its speed in m/s. The car moves from rest, and travels a distance of D metres until it obtains a speed of V m/s, in a time of T seconds.

Five students stated some relationships between the constants, m, V, k, F , D and T.

Alan stated that $mV = (F - kV)T$

Ben stated that $2mD = (F - kV)T^2$

Colin stated that $\frac{1}{2}mV^2 = (F - kV)D$

David stated that $D = \int_0^V \frac{mv}{F - kv}\,dv$

Edward stated that $T = \int_0^V \frac{m}{F - kv}\,dv$

Then

A. Alan, Ben and Colin are all correct.

B. Alan and Colin are both correct.

C. Only Colin is correct.

D. David and Edward are both correct.

E. Only Edward is correct.

Question 13

A girl of mass 50 kg is standing in a lift. The reaction of the lift floor on the girl is equal to 60 kg-wt. Then the lift is moving

A. with constant speed.

B. down with an acceleration equal to 1.96 ms^{-2}.

C. up with an acceleration equal to 1.96 ms^{-2}.

D. down with an acceleration equal to 0.2 ms^{-2}.

E. up with an acceleration equal to 0.2 ms^{-2}.

Question 14

The gradient of the normal to a curve at any point $P(x, y)$ is twice the square root of the gradient joining *P* and the origin. The coordinates on the curve satisfy the differential equation

A. $\frac{dy}{dx}+\sqrt{\frac{x}{4y}}=0$

B. $\frac{dy}{dx}-\sqrt{\frac{x}{4y}}=0$

C. $\frac{dy}{dx}-2\sqrt{\frac{y}{x}}=0$

D. $\frac{dy}{dx}+2\sqrt{\frac{y}{x}}=0$

E. $\frac{dy}{dx}+\frac{4y^2}{x^2}=0$

Question 15

Two vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are such that $|\underset{\sim}{u}|=|\underset{\sim}{v}|=L$. The angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is θ. Then $|\underset{\sim}{u}-\underset{\sim}{v}|$ is equal to

A. 0

B. $2L$

C. $\sqrt{2}L$

D. $L\sin(\theta)$

E. $2L\sin\left(\frac{\theta}{2}\right)$

Question 16

Given the vectors $\underset{\sim}{a}=\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$, $\underset{\sim}{b}=2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}$ and $\underset{\sim}{c}=\frac{1}{3}\left(\underset{\sim}{i}-5\underset{\sim}{j}+4\underset{\sim}{k}\right)$

A. Then the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependant.

B. The vector $\underset{\sim}{c}$ is the scalar resolute of the vector $\underset{\sim}{a}$ parallel to the vector $\underset{\sim}{b}$.

C. The vector $\underset{\sim}{c}$ is the vector resolute of the vector $\underset{\sim}{a}$ parallel to the vector $\underset{\sim}{b}$.

D. The vector $\underset{\sim}{c}$ is the vector resolute of the vector $\underset{\sim}{a}$ perpendicular to the vector $\underset{\sim}{b}$.

E. The vector $\underset{\sim}{c}$ is parallel to the vector $\underset{\sim}{a}+\underset{\sim}{b}$.

Question 17

A tank initially holds 50 litres of a solution, in which 4 kilograms of salt has been dissolved. Another solution containing 4 kilograms of salt per litre is poured into the tank at a rate of 3 litres per minute. The well-stirred mixture leaves the tank at a rate of 2 litres per minute. If Q kilograms is the amount of salt in the tank a time t minutes, and c a constant, then

A. $Q=12(50+t)+\dfrac{c}{50+t}$

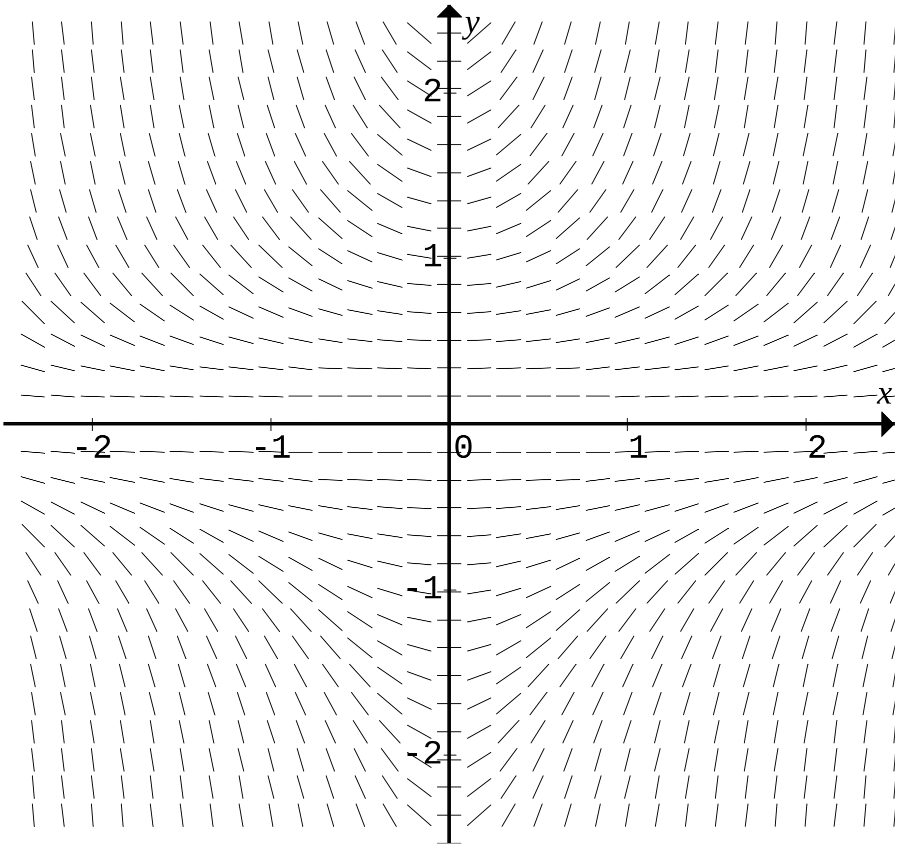
B. $Q=12(50+t)-\dfrac{c}{50+t}$

C. $Q=4(50+t)+\dfrac{c}{(50+t)^2}$

D. $Q=4(50+t)-\dfrac{c}{(50+t)^2}$

E. $Q=4(50-t)-\dfrac{c}{(50-t)^2}$

Question 18



The differential equation which best represents the above direction field is

A. $\dfrac{dy}{dx} = y^2 x$

B. $\dfrac{dy}{dx} = y x^2$

C. $\dfrac{dy}{dx} = y^2 x^2$

D. $\dfrac{dy}{dx} = \dfrac{x^2}{y}$

E. $\dfrac{dy}{dx} = \dfrac{y^2}{x^2}$

Question 19

A moving particle has a velocity vector at a time *t*, given by $e^t \cos(t)\underset{\sim}{i} + e^t \sin(t)\underset{\sim}{j}$, for $t \ge 0$. Initially the particle is at the origin. The position vector is given by

A. $e^t \sin(t)\underset{\sim}{i} - e^t \cos(t)\underset{\sim}{j}$.

B. $e^t \sin(t)\underset{\sim}{i} - \left(e^t \cos(t) - 1\right)\underset{\sim}{j}$.

C. $e^t \left(\cos(t) - \sin(t) - 1\right)\underset{\sim}{i} + e^t \left(\cos(t) + \sin(t) - 1\right)\underset{\sim}{j}$.

D. $\frac{1}{2}\left[e^t \left(\cos(t) + \sin(t)\right) - 1\right]\underset{\sim}{i} + \frac{1}{2}\left[e^t \left(\sin(t) - \cos(t)\right) + 1\right]\underset{\sim}{j}$.

E. $\frac{1}{2}e^t \left(\cos(t) + \sin(t)\right)\underset{\sim}{i} + \frac{1}{2}e^t \left(\sin(t) - \cos(t)\right)\underset{\sim}{j}$

Question 20

When Euler's method, with a step size of $\frac{1}{5}$, is used to solve the differential equation $\frac{dy}{dx} = \tan(x)$ with $x_0 = 0$ and $y_0 = 0$, the value of y_3 would be given as

A. $-\log_e\left(\cos(0.4)\right)$

B. $0.2\left(\tan(0.2) + \tan(0.4)\right)$

C. $0.2\tan(0.6)$

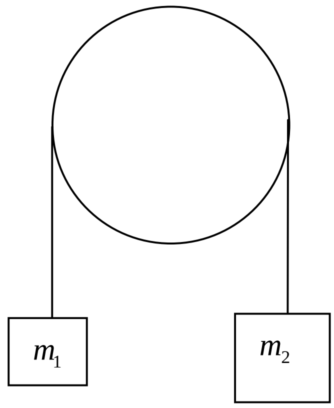
D. $0.2\tan(0.4)$

E. $0.2\tan(0.3)$

Question 21

A light inextensible string passes over a smooth pulley, with particles of masses m_1 and m_2 kg, attached to each end of the string as shown in the diagram.

Which of the following is **false?**



A. If $m_2 = 2m_1$ the mass m_2 moves downwards with an acceleration $\frac{g}{2}$ ms^{-2}.

B. If $m_1 = m_2 = m$ the tension in the string is equal to mg newtons.

C. If $m_1 = m_2$ both masses remain at rest.

D. If $m_2 > m_1$ the mass m_2 moves downwards with an acceleration $\frac{(m_2 - m_1)g}{m_1 + m_2}$ ms^{-2}.

E. If $m_1 \neq m_2$ the tension in the string is equal to $\frac{2m_1 m_2}{m_1 + m_2}$ kg-wt.

Question 22

A particle of mass m kg is acted upon by a variable force, so that its velocity v m/s when the particle is x m from the origin is given by $v = e^{cx}$, where c is a non-zero real constant. The force acting on the particle when $x = \frac{1}{c}$, in newtons, is

A. mc^2

B. mec

C. me

D. me^2

E. mce^2

END OF SECTION 1

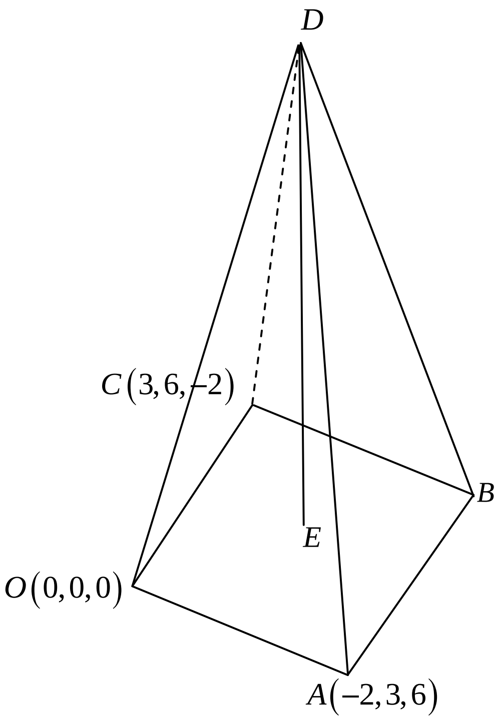
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 11 marks

OABCD is a right pyramid, where *O* is the origin. The coordinates of the points are $A(-2,3,6)$, $C(3,6,-2)$ and $D(x, y, z)$. The height of the pyramid is the length of *ED*, where *E* is a point on the base of *OABC*, such that *E* is the midpoint of *OB*.



a. Given that *OABC* is a square, find the coordinates of the point *B*.

2 marks

b. Find the coordinates of the point E.

1 mark

c. If the vector $\overrightarrow{ED}$ is perpendicular to $\overrightarrow{OE}$ show that $x+9y+4z=49$.

3 marks

d. The faces of the pyramid, OAD, ABD, BCD and OCD are all similar isosceles triangles, with all sloping edges $OD = AD = BD = CD = \frac{7}{6}\sqrt{274}$. Write down a set of four equations, which can be used to solve for x, y and z.

4 marks

e. Find the values of x, y and z and hence determine the height of the pyramid.

1 mark

Question 2 13 marks

a. Given that $\sin\left(\frac{\pi}{8}\right)=\frac{1}{2}\sqrt{2-\sqrt{2}}$, show that $\cos\left(\frac{\pi}{8}\right)=\frac{1}{2}\sqrt{2+\sqrt{2}}$.

2 marks

b. Use a double angle formula to show that $\tan\left(\frac{\pi}{8}\right)=\sqrt{2}-1$.

2 marks

c. If $\text{Arg}\left(1-\sqrt{2}+i\right)=k\pi$, write down the value of *k*, and explain your result geometrically.

2 marks

Let $u=\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}}$

d. Express *u* in exact polar form. 1 mark

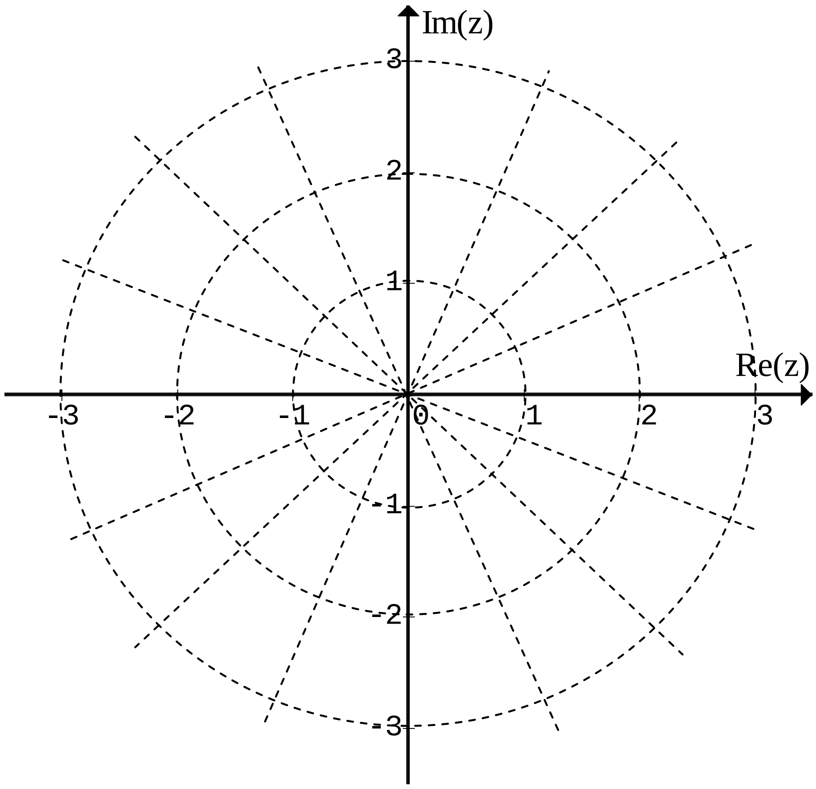
e. Find u^{10} giving your answer in both polar and rectangular form.

2 marks

f.i. On the argand diagram below, shade the region defined by

$\{z : \text{Arg}\left(u^{10}\right) \le z \le \text{Arg}\left(u\right)\} \cap \{z : 1 \le z \le 2\}, z \in C$

2 marks



ii. Find the area corresponding to the region sketched above in **f.i.**

1 mark

g. Find the value(s) of *n*, such that $\text{Im}\left(u^{n}\right) = 0$

1 mark

Question 3 11 marks

A train travelling along a straight line track between two stations has its velocity v ms^{-1} at a time t seconds, given by

$$v(t)=\begin{cases} a\sin\left(\dfrac{\pi t}{100}\right) & 0\le t<50 \\ bt+c & 50\le t\le 120 \\ \dfrac{32}{\pi}\cos^{-1}\left(\dfrac{t-120}{60}\right) & 120<t\le 180 \end{cases}$$

where a, b and c are real constants.

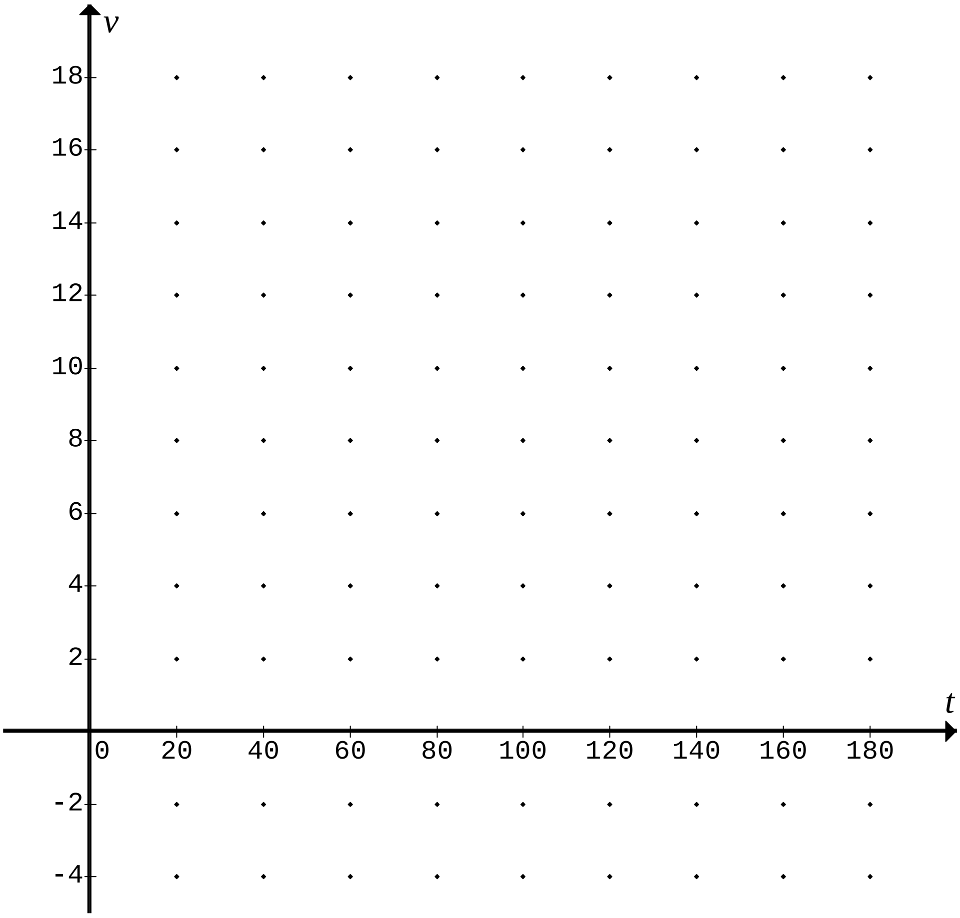
a. Over the time interval $[50,120]$ the train travels a distance of 875 metres. Show that $5950b+70c=875$. 2 marks

b. Write down another two simultaneous equations, which can be used to solve for a, b and c, and hence shown that $a=9$, $b=\dfrac{1}{10}$ and $c=4$

2 marks

c. Sketch the velocity time graph on the axes below.

2 marks



d.i. Write down in terms of definite integrals the total distance between the two stations.

1 mark

ii. Find the total distance between the two stations correct to the nearest metre.

1 mark

e. Find the acceleration of the train after 150 seconds. Given your answer correct to three decimal places.

1 mark

f. The passengers on the train, are most comfortable when the train travelling between the station travels at speeds less than 30 km/hr. For what percentage of the journey are the passengers most comfortable?
Give your answer correct to one decimal place.

2 marks

Question 4 12 marks

A curve is defined by the parametric equations

$$x = \cos(t)\left(4\cos(t) - \sec(t)\right)$$

$$y = \sin(t)\left(4\cos(t) - \sec(t)\right) \quad \text{for} \quad 0 \le t \le \pi$$

a. Show that the curve can be expressed in cartesian form as

$$4x^2 - (x+1)\left(x^2 + y^2\right) = 0$$

2 marks

b.i. Show that $\dfrac{dy}{dx} = \dfrac{6x - 3x^2 - y^2}{2y(x+1)}$.

2 marks

ii. Express the gradient of the curve in terms of t only.

1 mark

iii. Hence find the coordinates of the turning points on the graph of $4x^2-(x+1)(x^2+y^2)=0$, giving your answers correct to three decimal places.

2 marks

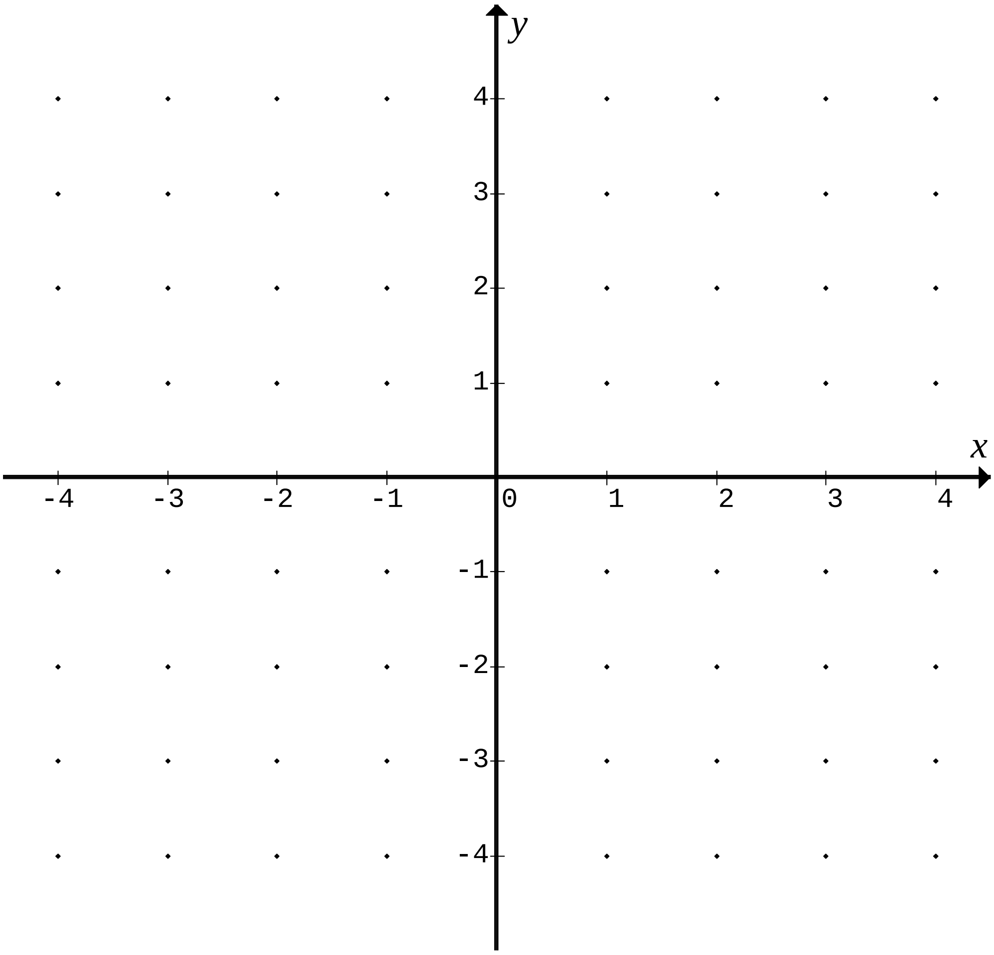
c. Sketch the curve defined by

$x = \cos(t)\left(4\cos(t) - \sec(t)\right)$

$y = \sin(t)\left(4\cos(t) - \sec(t)\right)$ for $0 \le t \le \pi$

labelling any asymptotes with their equations.

2 marks



The region bounded by the curve $4x^2-(x+1)(x^2+y^2)=0$, the x-axis, $x=0$ and $x=3$ is rotated about the x-axis, to form a solid of revolution with volume V.

d.i. Show that $V=\pi\int_0^3 \frac{x^2(3-x)}{x+1}dx$

1 mark

ii. Using a suitable substitution, express V in the form

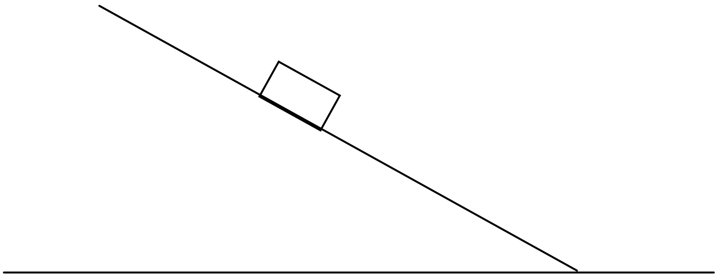
$V=\pi\int_a^b \left(pu^2+qu+r+\frac{s}{u}\right)du$, where a, b, p, q, r and s are integers. 2 marks

Question 5 11 marks

A block of mass *m* kg slides from rest down a plane inclined at an angle of 60^0 to the horizontal. The coefficient of friction between the block and the plane is μ.

a.i. On the diagram below, mark in and label all the forces acting on the block.

1 mark



ii. If a is the acceleration of the block in m/s^2 as it slides down the plane show that $a = \frac{g}{2}\left(\sqrt{3} - \mu\right)$.

2 marks

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The block moves down the plane a distance of D metres and reaches a speed of $\frac{g}{4}$ m/s, in a time of T seconds.

b. If the coefficient of friction is constant and equal to $\mu = \sqrt{3} - 1$,

i. find the value of T.

1 mark

ii. find the value of D.

1 mark

c. For this block and plane, the coefficient of friction between the block and the plane is not constant, but depends on the speed v m/s of the block as it moves down the plane. For this case when $\mu = \sqrt{3} - 1 - \frac{v^2}{5g}$,

i. find the value of T.

3 marks

ii. find the value of D.

3 marks

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\text{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r \qquad -\pi < \text{Arg}(z) \le \pi$$

$$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2) \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r \qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$$

acceleration: $$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x} dx = \log_e(x) + c,\ \text{for}\ \ x > 0$

$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$

$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

										Letter
Figures										
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SIGNATURE ______________________________

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