INSIGHT
Year 12 *Trial Exam Paper*

2013

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.

Question 1 (4 marks)

a. Find all solutions to the equation $z^4 + 2z^2 = 8,\ z \in C$.

2 marks

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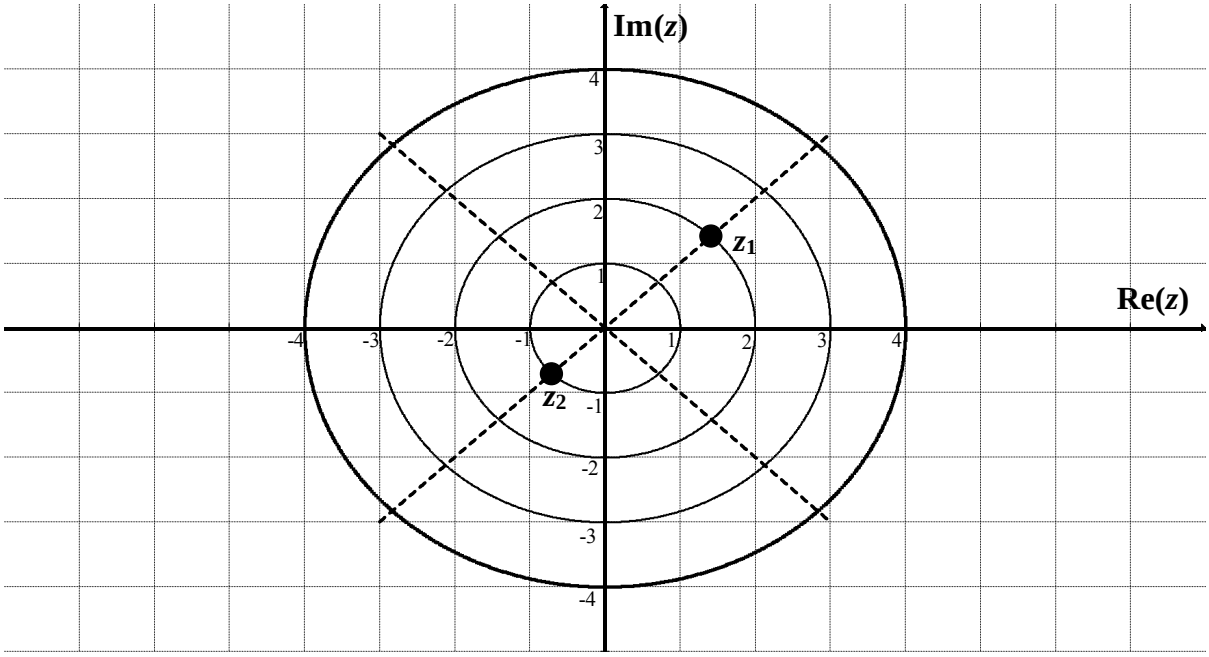
Question 1 – continued

b. Consider z_1 and z_2 as shown on the Argand diagram below. On the same diagram plot the following points:

$u = \dfrac{z_1}{z_2^{\,2}}$ and $w = i^2\left(\overline{z}_1\right)^2$



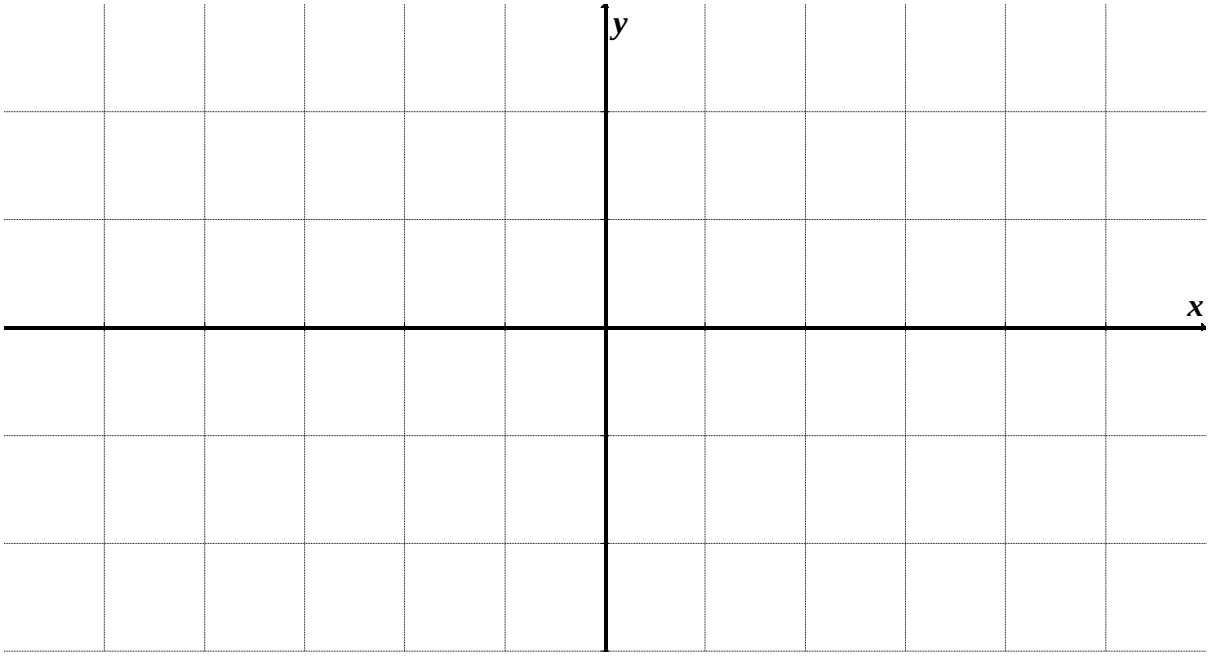
2 marks

End of Question 1
TURN OVER

Question 2 (6 marks)

a. Given that $f(x) = \dfrac{x^2+2}{x^2+4} = 1 - \dfrac{2}{x^2+4}$, sketch the graph of $y = f(x)$ on the axes below.
Label any asymptotes with their equations and any axes intercepts as coordinates.

3 marks



Question 2 – continued

b. Find the exact area enclosed by the graph of $f(x) = \dfrac{x^2 + 2}{x^2 + 4}$, the x-axis, the y-axis and the line $x = 2$.

3 marks

End of Question 2
TURN OVER

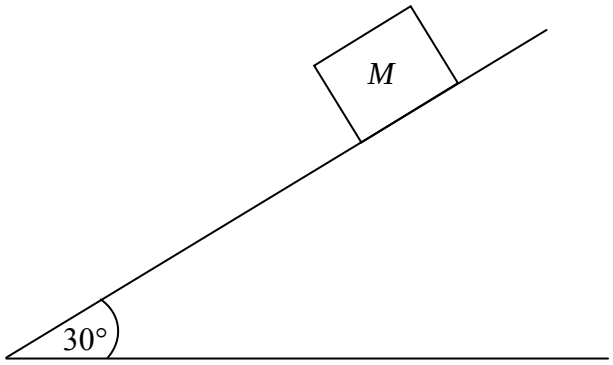
Question 3 (3 marks)

Evaluate $\int_0^{e-1} \frac{\sin(\pi \log_e(x+1))}{x+1}\, dx$.

End of Question 3

Question 4 (5 marks)

A mass of M kg slides from rest down a rough plane, which makes an angle of 30º with the horizontal ground. The coefficient of friction is μ.



a. On the diagram above, mark all the forces acting on the mass, M.

1 mark

b. Find the acceleration of the mass in terms of g and μ.

2 marks

c. If the mass reaches a speed of 3 m/s after sliding for 3 metres, find μ in the form $\mu = \frac{g - c}{\sqrt{c}g}$ where c is an integer.

2 marks

Question 5 (5 marks)

A particle moves in a straight line with an acceleration of a m/s^2, which is given by $a = x + 1$.
At time t seconds, its displacement is x metres from a fixed point and its velocity is v m/s.
When $t = 0$, $x = 0$ and $v = 1$.

a. Show that $v = x + 1$.

3 marks

Question 5 – continued

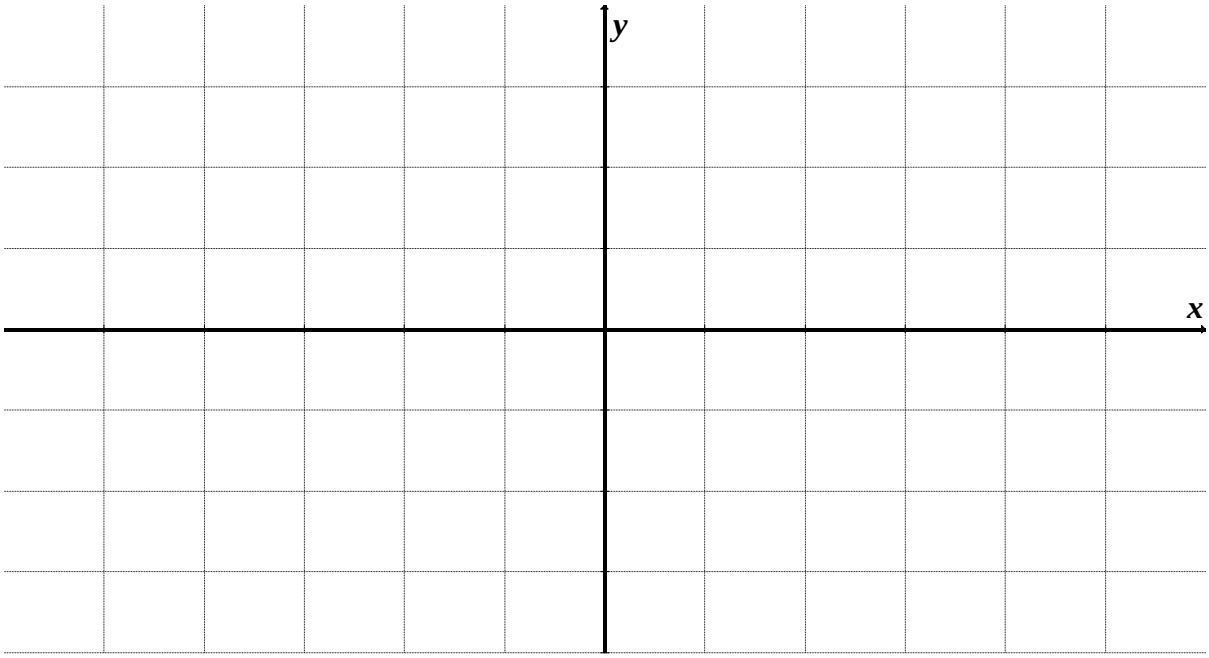
b. Hence, express x as a function of t.

2 marks

End of Question 5
TURN OVER

Question 6 (2 marks)

On the axes below, sketch the graph of the relation $y^2 = 4 - 4(x-1)^2$.
Show all axes intercepts.



End of Question 6

Question 7 (6 marks)

The motion of a particle is described by the following parametric equations:

$x - y = 0.5\sec(t)$

$y = 2 + \tan^2(t)$

a. Show that the Cartesian equation is given by the relation $y = 4x^2 - 8xy + 4y^2 + 1$.

2 marks

b. Show that when time $t = \frac{\pi}{3}$ the coordinate of the particle is (6, 5).

2 marks

c. Hence, find the gradient of the curve at the point (6, 5).

2 marks

Question 8 (4 marks)

Three points, A, B and C, are given by $A(2, 2, 1)$, $B(3, 0, 4)$ and $C(5, -4, 10)$.

a. Show that A, B and C are collinear.

2 marks

b. The respective unit vectors in the direction of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are $\underset{\sim}{a} = \frac{1}{3}(2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k})$ and $\underset{\sim}{b} = \frac{1}{5}(3\underset{\sim}{i} + 4\underset{\sim}{k})$.

Find a vector that bisects the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

2 marks

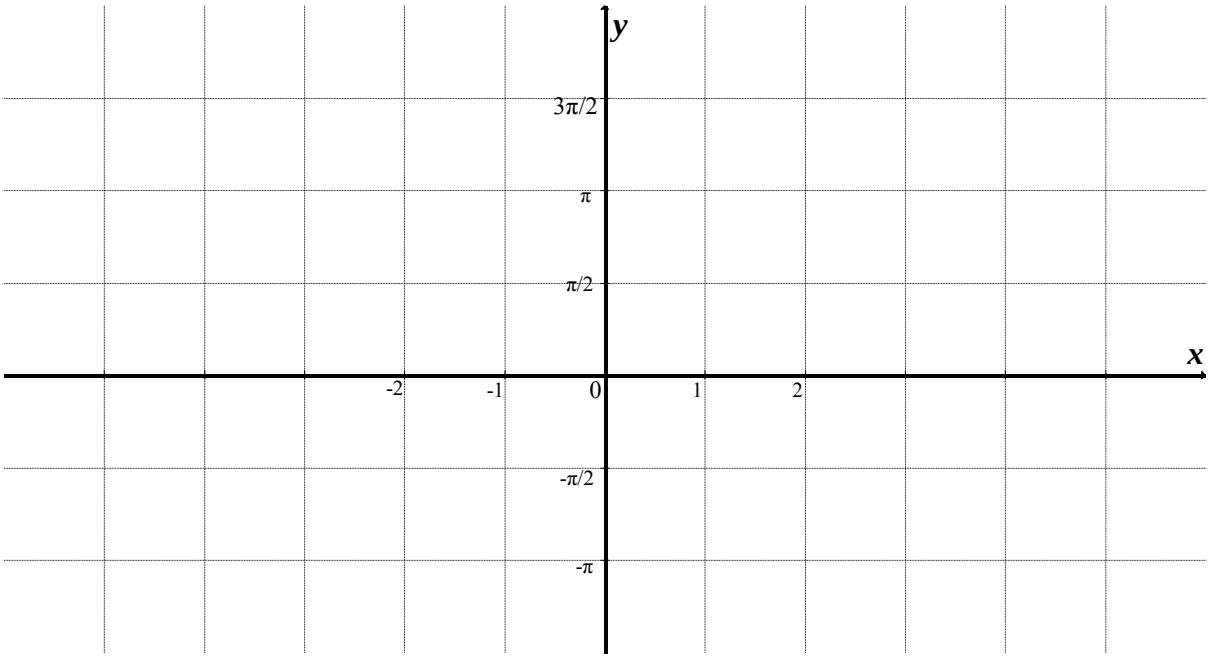
End of Question 8
TURN OVER

Question 9 (5 marks)

a. On the set of axes below, sketch the graph of the curve with equation $f(x) = \frac{\pi}{2} + \sin^{-1}(x)$.

Clearly show the coordinates of the end points.

2 marks



b. Find the volume generated when the area between the curve $y = f(x)$ and the y-axis is rotated about the y-axis to form a solid of revolution.

3 marks

END OF QUESTION AND ANSWER BOOK