Neap:

Trial Examination 2012

VCE Specialist Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 22 pages. Formula sheet of miscellaneous formulas.

Answer sheet for multiple-choice questions.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page and in the space provided on the answer sheet for multiple-choice questions.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2012 VCE Specialist Mathematics Units 3 & 4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The equation of the oblique asymptote of the graph of $y = \frac{2x^2 - x + 3}{4 - x}$ is

A. $x = 4$

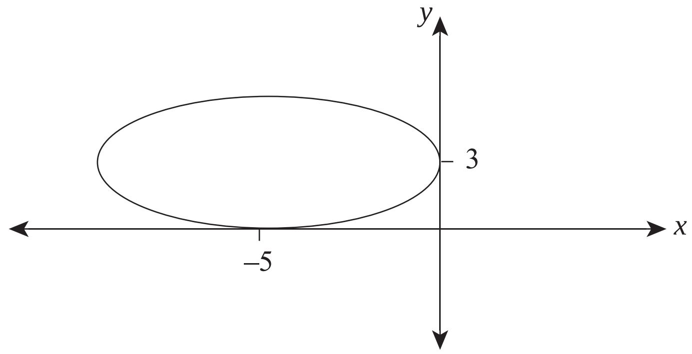
B. $y = 3$

C. $y = -2$

D. $y = -2x - 7$

E. $y = -2x$

Question 2



A possible equation of the ellipse shown above is

A. $25(x-5)^2 + 9(y+3)^2 = 225$

B. $9(x-5)^2 + 25(y+3)^2 = 225$

C. $9(x+5)^2 + 25(y-3)^2 = 225$

D. $25(x+5)^2 + 9(y-3)^2 = 225$

E. $9(x+5)^2 + 25(y+3)^2 = 225$

Question 3

An hyperbola is defined by the rule $\frac{(x-2)^2}{9} - \frac{(y+3)^2}{16} = 1$.

The domain of the hyperbola is

A. $x \in R$

B. $x \in R \setminus (-2, 2)$

C. $x \in R \setminus (-3, 3)$

D. $x \in R \setminus (-5, 1)$

E. $x \in R \setminus (-1, 5)$

Question 4

The function with rule $f(x) = a\sin^{-1}(x) + c$ has range

A. $\left[-\frac{a\pi}{2}, \frac{a\pi}{2}\right]$

B. $\left[-\frac{a\pi + 2c}{2}, \frac{a\pi + 2c}{2}\right]$

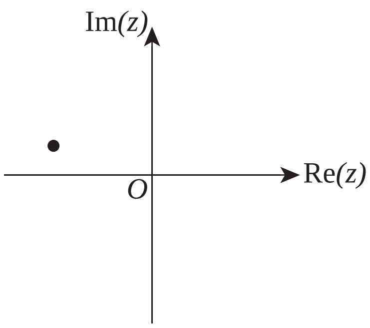
C. $\left[-\frac{a\pi - 2c}{2}, \frac{a\pi + 2c}{2}\right]$

D. $\left[-\frac{a\pi - 2c}{2}, \frac{a\pi - 2c}{2}\right]$

E. $\left[-\frac{a\pi + 2c}{2}, \frac{a\pi - 2c}{2}\right]$

Question 5

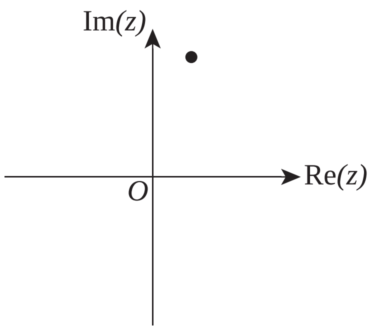
A certain complex number z is represented by the point on the Argand diagram below.



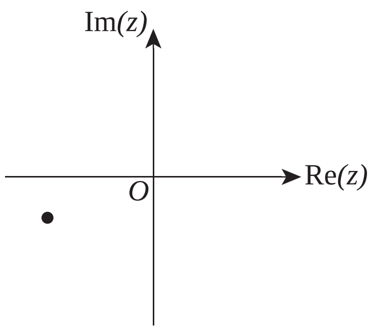
All axes below have the **same scale** as those in the diagram above.

The complex number $\frac{\bar{z}}{i}$ is best represented by

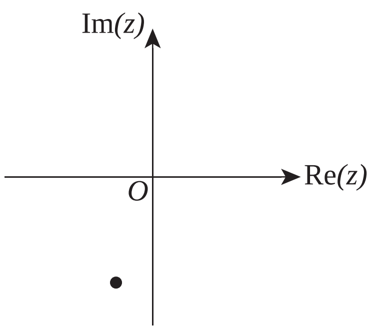
A.



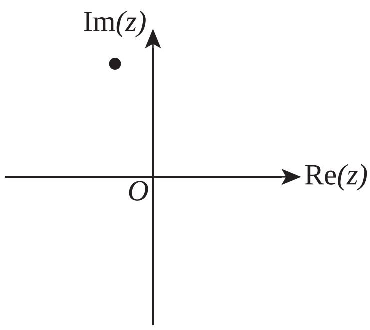
B.



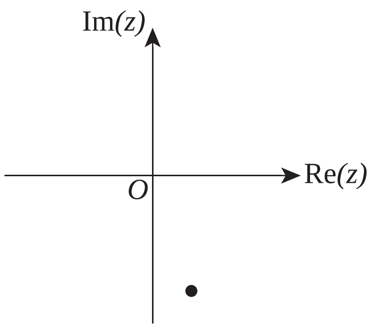
C.



D.



E.



Question 6

If $z^5 = ai$, where a is a positive real constant, the number of distinct solutions for which $\text{Arg}(z) > 0$ is

A. 1
B. 2
C. 3
D. 4
E. 5

Question 7

Two of the solutions to the equation $z^3 - 5z^2 + 8z = k$, where $k \in R$, are 3 and $1 - i$.

The value of k is

A. –3
B. 3
C. –6
D. 6
E. 9

Question 8

The relation $|z - ai| - |z + a| = 0$, where $a \in R\backslash\{0\}$, when graphed on the Argand plane would be

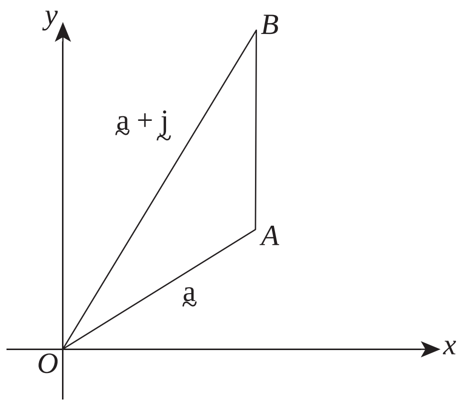
A. a circle centred at $(-a, a)$ with a radius of a.
B. a circle centred at $(a, -a)$ with a radius of a.
C. a circle centred at $(-a, a)$ with a radius of a^2.
D. a straight line passing through the origin with a gradient of 1.
E. a straight line passing through the origin with a gradient of –1.

Question 9

The sum of the solutions to the equation $\dfrac{\cos(2x) - \sec(2x)}{\cos(x)} = 0$, $x \in [0, 2\pi]$, is

A. $\dfrac{\pi}{2}$
B. π
C. 2π
D. 3π
E. 5π

Question 10



The point A is such that its position vector is $\overrightarrow{OA} = \underset{\sim}{a} = \frac{1}{\sqrt{2}}(\underset{\sim}{i} + \underset{\sim}{j})$.

The point B has position vector $\overrightarrow{OB} = \underset{\sim}{a} + \underset{\sim}{j}$, as shown in the diagram above.

The angle BOA, in radians, and the area of triangle BOA, in square units, respectively equal

A. $\frac{\pi}{6}$ and $\sqrt{2}$

B. $\frac{\pi}{6}$ and $\frac{\sqrt{2}}{2}$

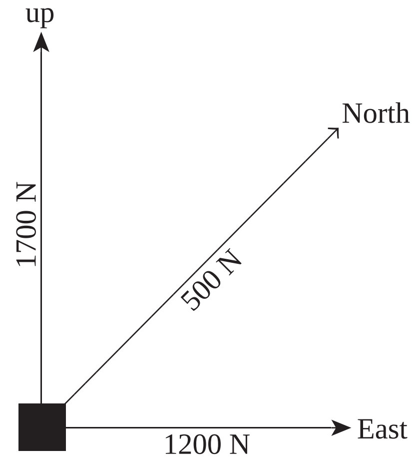
C. $\frac{\pi}{8}$ and $\frac{1}{2}$

D. $\frac{\pi}{8}$ and $\frac{\sqrt{2}}{4}$

E. $\frac{\pi}{8}$ and $\sqrt{2}$

Question 11

Three mutually perpendicular forces of magnitudes 1200 newtons East, 500 newtons North and 1700 newtons vertically upwards are pulling on a box, as shown in the diagram below.



The direction that the resultant force makes with the horizontal, correct to the nearest degree, equals

A. 35°

B. 37°

C. 53°

D. 55°

E. 74°

Question 12

The angle between the vectors $\underset{\sim}{a}$ and $(\underset{\sim}{a} - \underset{\sim}{b})$, where $\underset{\sim}{a} = 4\underset{\sim}{i} + 2\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, equals

A. $\arccos\left(\frac{2}{3}\right)$

B. $\arccos\left(\frac{\sqrt{2}}{3}\right)$

C. $-\arccos\left(\frac{\sqrt{2}}{3}\right)$

D. $\arccos\left(\frac{2\sqrt{2}}{3}\right)$

E. $-\arccos\left(\frac{2\sqrt{2}}{3}\right)$

Question 13

A curve has parametric equations given by $x = \tan(\theta)$ and $y = \tan(2\theta)$.

The equation of the normal to the curve at the point where $\theta = \frac{\pi}{6}$ is

A. $y = 6x - \sqrt{3}$

B. $4y = 6x + 3\sqrt{3}$

C. $x + 6y = 3\sqrt{3}$

D. $3x + 18y = 17\sqrt{3}$

E. $3x + 18y = 19\sqrt{3}$

Question 14

Using a suitable substitution, the definite integral $\int_0^{\frac{\pi}{4}} \frac{\cos(2x)}{\sqrt{1 + 2\sin(2x)}} dx$ can be simplified to

A. $\frac{1}{4}\int_0^2 \frac{1}{\sqrt{1+u}} du$

B. $\frac{1}{2}\int_0^2 \frac{1}{\sqrt{1+u}} du$

C. $\int_0^{\frac{\pi}{4}} \frac{1}{\sqrt{1+u}} du$

D. $\int_0^1 \frac{1}{\sqrt{1+2u}} du$

E. $\frac{1}{4}\int_0^2 \frac{1}{\sqrt{1+2u}} du$

Question 15

A girl of mass 65 kg is standing on the floor of a lift of mass 835 kg. The lift is descending with a constant acceleration and the tension in the lift cable is 8550 N.

The reaction force, in N, of the floor on the girl is closest to

A. 617.5

B. 630.5

C. 637

D. 656.5

E. 669.5

Question 16

At time $t = 0$ seconds an object passes through a fixed origin, *O*, with a velocity of $4\underset{\sim}{i} - \underset{\sim}{j}$ m/s. The object is subject to a constant force which results in an acceleration of $2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ m/s^2.

The distance of the object from *O*, in metres, at $t = 4$ is

A. $12\underset{\sim}{i} + 3\underset{\sim}{j} - 8\underset{\sim}{k}$

B. $32\underset{\sim}{i} + 4\underset{\sim}{j} - 16\underset{\sim}{k}$

C. $4\sqrt{89}$

D. 36

E. $\sqrt{217}$

Question 17

The volume of the regular tetrahedron is changing at a rate of 6 cm^3/s, with the length measurements changing uniformly so that the shape remains a tetrahedron.

The height of a regular tetrahedron with side length *L* equals $\frac{\sqrt{6}}{3}L$, and the area of one of the faces is $\sqrt{3}$ cm^2.

How fast, in cm/s, is the side length changing?

A. $3\sqrt{2}$

B. $5\sqrt{3}$

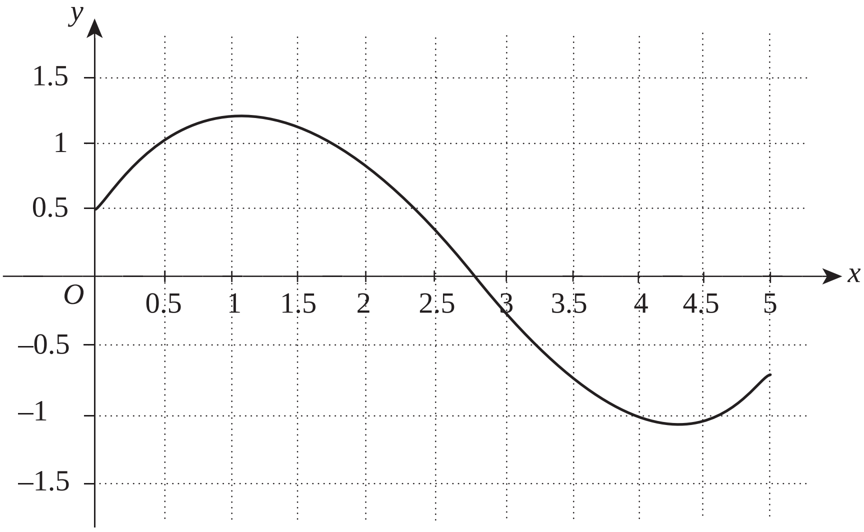
C. $8\sqrt{2}$

D. $6\sqrt{2}$

E. $2\sqrt{3}$

Question 18

The graph of $y = f'(x)$ is shown below. It is known that $f(4) = 1$.



Euler's method is used to estimate the value of $f(3.8)$.

Using a step size of 0.2, the approximate value of $f(3.8)$ is

A. 1.2, which is an overestimate.
B. 1.2, which is an underestimate.
C. 0.80, which is an overestimate.
D. 0.80, which is an underestimate.
E. 1.1, which is an overestimate.

Question 19

Consider a particle which is moving along the *x*-axis at time *t*.

Let the continuous function $v(t)$ represent the velocity of the particle. For some part of the interval [0,20], $|v(t)| \neq v(t)$.

The value of the definite integral $\int_0^{20} v(t)dt$ represents

A. the total distance travelled by the particle over [0, 20].
B. the position of the particle at $t = 20$.
C. the change in velocity of the particle over [0, 20].
D. the average rate of change of the particle over [0, 20].
E. the displacement of the particle over [0, 20].

Question 20

The acceleration of an object is inversely proportional to its velocity at any given time.

The object is travelling at 1 m/s when its acceleration is 3 m/s^2.

Given that the object moved with an initial velocity of 2 m/s to the left, its velocity at any time is given by

A. $v = \sqrt{4 + 6t}$

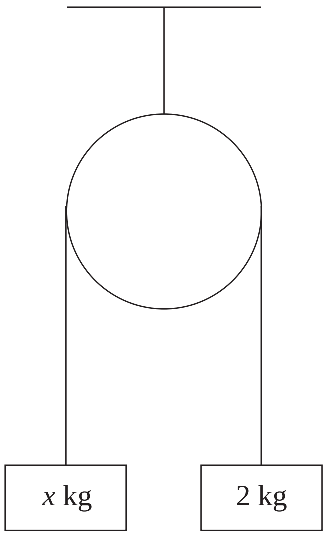
B. $v = -\sqrt{6t + 4}$

C. $v = \sqrt{6t - 4}$

D. $v = -\sqrt{4 - 6t}$

E. $v = -2\sqrt{3t + 1}$

Question 21



A light inelastic string passes over a smooth pulley. A mass of 2 kg is attached to one end of the string and a mass of x kg is attached to the other end of the string.

When the system is released from rest the 2 kg mass falls 3 metres in $2\sqrt{3}$ seconds.

The value of x is given by

A. $\frac{2(2g - 1)}{g + 1}$

B. $\frac{2(2g + 1)}{2g - 1}$

C. $\frac{2(g - 1)}{g + 1}$

D. $\frac{2(2g - 1)}{2g + 1}$

E. $\frac{2g - 1}{2g + 1}$

Question 22

A stationary submarine is in difficulty in very deep water. It projects an emergency beacon of mass 25 kg vertically upwards through the ocean water. When the beacon has risen *h* metres, its velocity is *V* m/s. The beacon is subject to a retarding force through the ocean water of *R* newtons, where $R = 25kV^2, k > 0$.

The equation of motion which correctly describes the movement of the beacon is

A. $V\frac{dV}{dh} + kV^2 + g = 0$

B. $V\frac{dV}{dh} + kV^2 - g = 0$

C. $V\frac{dV}{dh} - kV^2 - g = 0$

D. $V\frac{dV}{dh} + 25kV^2 = 0$

E. $V\frac{dV}{dh} - 25kV^2 = 0$

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SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Consider the cubic polynomial, $p(z)$ over C, where $p(z) = z^3 - 2(1 - \sqrt{3}i)z^2 - 4(1 + \sqrt{3}i)z + 8$.

a. **i.** Show that $z = 2$ is a root of the equation $p(z) = 0$.

1 mark

ii. Dividing $p(z)$ by $z - 2$ gives $q(z)$, a quadratic polynomial, where $q(z) = z^2 + bz + c$ with $b \in C$ and $c \in R$.

Write down the values of b and c.

1 mark

iii. Solving $z^2 + bz + c = 0$ is equivalent to solving the equation $(z + h)^2 = k$, with $h \in C$ and $k \in R$.

Find the values of h and k and hence show that the roots of the equation $p(z) = 0$ are given by $z_1 = 2$, $z_2 = 1 - \sqrt{3}i$, $z_3 = -1 - \sqrt{3}i$.

3 marks

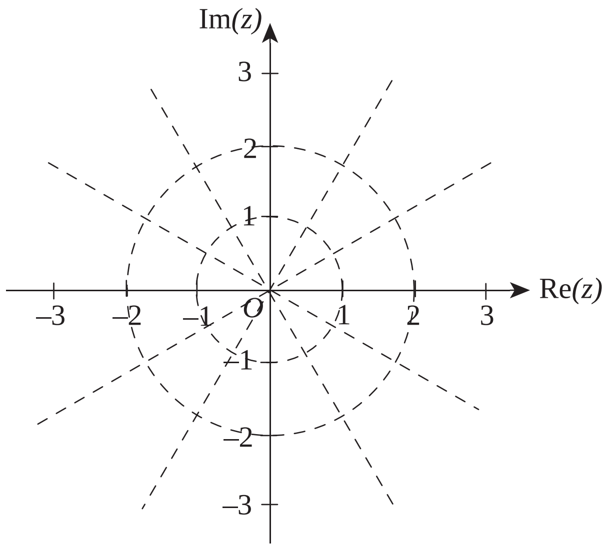
iv. What is the geometrical relationship between these three roots?

1 mark

b. **i.** Express z_3 in polar form.

1 mark

ii. Plot the three roots on the Argand diagram below. Label the real root *P*, and the complex roots *M* and *N* where *M* has the larger principal argument.



2 marks

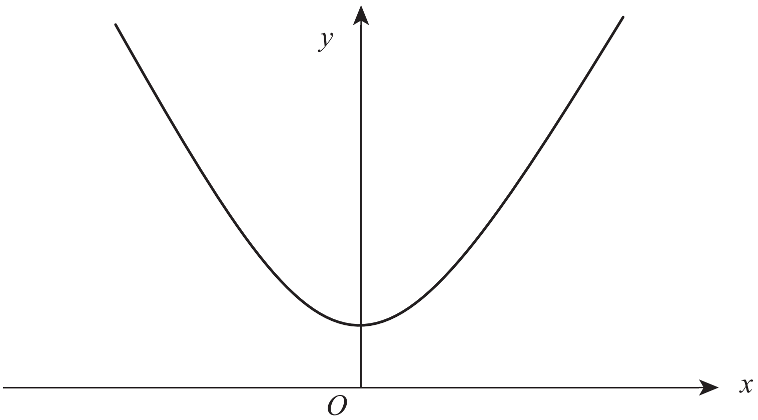
Let $\underset{\sim}{i}$ be a unit vector in the direction of the real axis and let $\underset{\sim}{j}$ be a unit vector in the direction of the imaginary axis.

c. By expressing the vectors $\overrightarrow{NP}$ and $\overrightarrow{OM}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$, and finding an appropriate scalar product, show that *ONMP* is a rhombus.

3 marks
Total 12 marks

Question 2

A graph of the hyperbola with equation $\frac{(y+1)^2}{9} - \frac{x^2}{4} = 1$ for $y > 0$ is shown on the axes below.



a. Write down the coordinates of the *y*-intercept.

1 mark

b. Show that the equations of the asymptotes are $3x - 2y = 2$ and $3x + 2y = -2$, and sketch these on the graph above for $y \geq 0$.

3 marks

The region in the first quadrant bounded by the hyperbola, the line $y = 10$, the asymptote and the coordinate axes are rotated about the *y*-axis to form a volume of revolution in the shape of a bowl which will be filled with water. Values on the coordinate axes are in centimetres.

c. Write down a definite integral in terms of *y* which will find the volume of water that the bowl will hold, and hence find the this volume correct to the nearest ml.

3 marks

d. The bowl is to be made from a particular type of glass which has a density of 2.5 g per cubic centimetre.

Write down an expression using a series of definite integrals that could be used to determine the mass of the bowl (you are **not** required to find this mass).

2 marks

Total 9 marks

Question 3

The volume, V m^3, of wine in a holding tank is given by $V = \pi(6h^3 + 216h)$, where h m is the depth of wine and $0 \le h \le 12$. The holding tank is initially empty.

The wine is poured into the holding tank at a constant rate of 6000 m^3/hour.

a. **i.** Find the time taken for the holding tank to be filled.

ii. At what rate is the depth of the wine in the holding tank rising when the depth of the holding tank is 6 m?

2 + 3 = 5 marks

b. The wine is stored in the holding tank to mature. When the wine is ready for bottling it is slowly removed over a period of time. The wine is removed via a tap which is 1 metre from the bottom of the holding tank. The wine is removed at a rate given by $\frac{dV}{dt} = -\frac{36\pi}{\sqrt{h}}$ m^3/hour, where t is the number of hours after the wine starts to be removed.

i. Show that *h* and *t* satisfy the differential equation: $\frac{dh}{dt} = -\frac{2}{\sqrt{h}(h^2 + 12)}$.

ii. Find the time, correct to the nearest hour, taken to remove as much of the wine as possible via the tap which is 1 metre above the bottom of the holding tank.

2 + 2 = 4 marks
Total 9 marks

Question 4

Melanie is a keen adventure tourist who has travelled to Switzerland to engage in some skydiving activities.

Melanie is in a plane which is circling at an altitude of 3500 metres. She has all her parachute equipment attached and is ready to leap out of the plane. Her total mass including the parachute is 120 kg.

During free fall, when the parachute is not engaged, the total resistive force on Melanie is $0.3v^2 + 80$ newtons, where v is her velocity in m/s.

a. Show that the terminal velocity Melanie would reach is approximately 60.4 m/s, correct to one decimal place.

2 marks

b. Show that the differential equation $\frac{dv}{dt} = \frac{137}{15} - \frac{v^2}{400}$ applies until the terminal velocity is reached.

3 marks

c. Melanie started to feel panic when her velocity first exceeded 150 km/h.

i. Determine a definite integral expression which gives the time at which Melanie started to feel panic.

ii. Calculate, correct to the nearest tenth of a second, when this occurred.

2 + 1 = 3 marks

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d. Assuming terminal velocity to be 60.4 m/s, correct to one decimal place, use part **b** to show that Melanie travels a distance of $200\log_e\left(\frac{68\ 500}{97}\right)$ metres before reaching this velocity.

2 marks

Suppose Melanie engages her parachute at the time when she reaches her terminal velocity, i.e. when her velocity is 60.4 m/s. After her parachute opens her velocity satisfies the differential equation

$$\frac{dv}{dt} = -9.8 - 0.6v.$$

e. Find an expression which gives Melanie's approximate velocity as a function of the time after which the parachute is opened.

3 marks

f. How long, correct to the nearest second, will it take Melanie to reach the ground from the time the parachute is opened?

2 marks

g. Suppose now that a person who is skydiving has the differential equation $\frac{dv}{dt} = -kv - g$ as a suitable model for acceleration from the time after their parachute is opened. Assume that such a skydiver engages the parachute at the time they are descending at 50 m/s. For the person to land safely they must not hit the ground at a speed which exceeds 5.5 m/s.

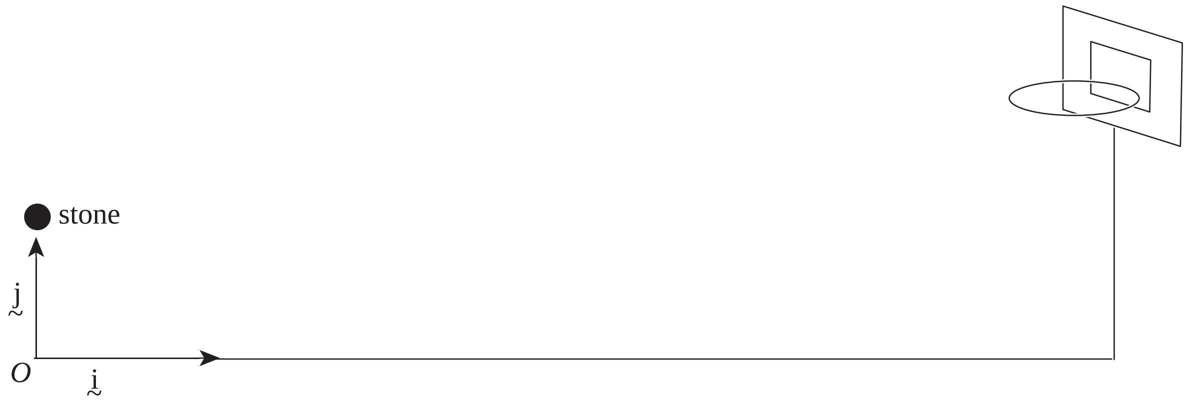
Find a suitable range of values for k.

3 marks

Total 18 marks

Question 5

Basil is a keen basketball player. One day he is locked out of the house and whilst he is waiting for someone to arrive with the key, he occupies himself by throwing stones at his basketball ring, trying to shoots goals. The basketball ring has a diameter of 50 cm and is at a height of 3 m above the ground. He is projecting the stones from a height of 2 m at an angle of 45° to the horizontal, and at the point of release the stone is a horizontal distance of 25 m from the centre of the basketball ring.



The path of a projectile projected from (0, 0) can be modelled by the equation $y = \frac{-g\sec^2(\theta)}{2v^2}x^2 + \tan(\theta)x$,

where θ is the angle of projection measured from the horizontal, v m/s is the initial speed of the stone, x is the horizontal distance from the origin O, and y is the vertical distance from the origin O.

For this modelling, the stone will be considered to be a particle; that is, the size of the stone is not considered in the calculations. A goal is considered scored if the path of the particle passes within the circumference of the basketball ring.

Use a Cartesian axes where the origin O at Basil's feet is at $(0, 0)$, the stone is released from $(0, 2)$, the middle of the basketball ring is at $(25, 3)$ and $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors parallel to the x-axis and y-axis respectively.

a. Show that if the stone is thrown with an initial speed of 16 m/s it will pass through the basketball ring.

3 marks

b. Show that the initial velocity of the stone is $\underset{\sim}{v} = 8\sqrt{2}\underset{\sim}{i} + 8\sqrt{2}\underset{\sim}{j}$.

1 mark

c. The acceleration of the stone can be modelled by the equation $\underset{\sim}{a} = -g\underset{\sim}{j}$.

i. Show by integration, that the position vector of the basketball is given by

$\underset{\sim}{r} = 8\sqrt{2}t\underset{\sim}{i} + \left(2 + 8\sqrt{2}t - \frac{g}{2}t^2\right)\underset{\sim}{j}$, where t is the time in seconds after the basketball is projected.

4 marks

ii. Hence or otherwise determine the speed, correct to one decimal place, of the stone as it enters the basketball ring.

2 marks
Total 10 marks

END OF QUESTION AND ANSWER BOOKLET