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Specialist Mathematics

2012

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 Complex number z is 4 units from complex number $1-5i$ and 3 units from complex number $4-i$, a possible value of $|z|$ is

A. $\sqrt{2}$

B. $\sqrt{3}$

C. 2

D. 3

E. 5

Question 2 Given $z = -icis(2)$, $Arg(z)$ is

A. $\frac{\pi}{2}-2$

B. $\frac{\pi}{2}+2$

C. $2-\frac{\pi}{2}$

D. $-2-\frac{\pi}{2}$

E. $2-\pi$

Question 3 Given a, b, $c \in R^+$, and $z \in C$, the maximum number of solutions to the equation $az^2 + b|z|^2 - c = 0$ is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4 Given $Arg[(1+i)z] \in \left(0, \frac{\pi}{2}\right)$, the maximal possible range of values of $Arg(z)$ is

A. $(-\pi, \pi)$

B. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C. $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

D. $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

E. $\left(0, \frac{\pi}{6}\right)$

Question 5 The lengths of the longest and the shortest chords through the centre of an ellipse are 6 and 2 units. A possible equation of the ellipse is

A. $\frac{(x+2)^2}{4} + \frac{(y+6)^2}{36} = 1$

B. $\frac{(x+2)^2}{2} + \frac{(y+6)^2}{6} = 1$

C. $\frac{(x-2)^2}{2} + \frac{(y-6)^2}{6} = 1$

D. $\frac{(x+2)^2}{4} + \frac{(y-2)^2}{9} = 1$

E. $(x-2)^2 + \frac{(y+2)^2}{9} = 1$

Question 6 The graph of $y = \frac{1}{a+bx+ax^2}$, where $a \in R \setminus \{0\}$, has exactly two asymptotes when

A. $-1 < \frac{b}{a} < 1$

B. $-1 \le \frac{b}{a} \le 1$

C. $-2 < \frac{a}{b} < 2$

D. $-2 \le \frac{a}{b} \le 2$

E. $\frac{a}{b} = -\frac{1}{2}$

Question 7 A hyperbola has asymptotes $y = 2 \pm \left(\frac{1}{2}x + 1\right)$. An equation of the hyperbola is

A. $(x+2)^2 - 4(y-2)^2 + k = 0$, $k \in R \setminus \{0\}$

B. $(x-2)^2 - 4(y+2)^2 - k = 0$, $k \in R \setminus \{0\}$

C. $(x+1)^2 - 2(y-1)^2 + k = 0$, $k \in R \setminus \{0\}$

D. $(x-1)^2 - 2(y+1)^2 - k = 0$, $k \in R \setminus \{0\}$

E. $4(x+1)^2 - 2(y-1)^2 + k = 0$, $k \in R \setminus \{0\}$

Question 8 Given $\sec(a) + \text{cosec}(b) = 0$, $\pi < a < \frac{3\pi}{2}$ and $0 < b < \frac{\pi}{2}$, then

A. $\frac{a}{2} + b = \pi$

B. $a + \frac{b}{2} = \frac{3\pi}{2}$

C. $\frac{a}{2} - b = \frac{\pi}{2}$

D. $a - \frac{b}{2} = \frac{5\pi}{4}$

E. $a + b = \frac{3\pi}{2}$

Question 9 The domain of the *inverse* of $f : \left(1, \frac{5}{3}\right] \to R$, $f(x) = \sec\frac{\pi}{2}(x+1) + a\pi$ is

A. $(-\infty, \ a\pi - 2]$

B. $(-\infty, \ a + 2]$

C. $[a\pi - 2, \ a\pi - 1)$

D. $[a\pi - 1, \ \infty)$

E. $[a\pi + 1, \ \infty)$

Question 10 The number of solutions to the equation $\tan^{-1}(x-a+b)-\tan^{-1}(x-a)=\pi$, where $a,\ b\in R$, is

A. 0

B. 1

C. 2

D. 3

E. infinitely many

Question 11 $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are unit vectors such that $\underset{\sim}{a}-\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{0}$, the value of $\underset{\sim}{a}.\underset{\sim}{b}-\underset{\sim}{b}.\underset{\sim}{c}+\underset{\sim}{c}.\underset{\sim}{a}$ is

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{3}{2}$

D. $-\frac{3}{2}$

E. $\frac{\sqrt{3}}{2}$

Question 12 Vectors $\underset{\sim}{k}+a\underset{\sim}{i}$, $\underset{\sim}{i}+b\underset{\sim}{j}$ and $\underset{\sim}{j}+c\underset{\sim}{k}$ are linearly dependent if abc equals

A. 3

B. -2

C. 2

D. -1

E. 1

Question 13 A vector makes *equal* angles with orthogonal unit vectors $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$. The magnitude of each angle is closest to

A. 135°

B. 125°

C. 115°

D. 75°

E. 65°

Question 14 Given $f'(x)=\dfrac{1}{\cos^4 x}$ and $f(0)=1.2$, the average value of $f(x)$ in the interval $[0,1]$ is closest to

A. 1

B. 2

C. 3

D. 4

E. 5

Question 15 The graph of $y=3x^5+5x^4-10x^3-30x^2+5x-10$ has n inflection point(s). The value of n is

A. 0

B. 1

C. 2

D. 3

E. not determinable

Question 16 Consider $y=\tan^2 x$, $0\le x\le\dfrac{3}{2}$. The area of the region enclosed by the y-axis, the line $y=4$ and the curve $y=\tan^2 x$ is closest to

A. 3

B. 3.5

C. 4

D. 4.5

E. 5

Question 17 The differential equation $\left(\frac{dy}{dx}\right)^2 + k\frac{dy}{dx} + 1 = 0$ has exactly one solution, where $k > 0$ and $y = 1$ when $x = 1$. A solution to the differential equation is

A. $x + y = 2$

B. $2x - y = 1$

C. $y = e^{x-1}$

D. $y = e^{\frac{x-1}{k}}$

E. $y = 1 + \log_e x$

Question 18 A particle moves in a straight line with an acceleration of 2 m s^{-2} east. At $t = 0$ it is 5 m east of a reference point and has a velocity of 10 m s^{-1} west.
The total distance (m) travelled when it is 16 m east of the reference point is closest to

A. 40

B. 50

C. 60

D. 70

E. 80

Question 19 The velocity of a particle at position $x \geq 0$ is given by $v = 2 - e^x$ m s^{-1}, and $x = 0$ initially. The distance (m) travelled by the particle when its acceleration is zero is closest to

A. 0.9

B. 0.8

C. 0.7

D. 0.6

E. 0.5

Question 20 A particle of mass 1.3 kg slides down from rest a frictionless plane inclined at $30°$ to the horizontal. Its momentum (kg m s^{-1}) after sliding 1.3 m down the plane is closest to

A. 4.0

B. 4.5

C. 5.5

D. 5.7

E. 5.8

Question 21 A particle of mass 2.04 kg slides at constant velocity of 2.0 m s^{-1} down a plane inclined at $30°$ to the horizontal. The reaction force of the plane on the particle is closest to

A. 10 N normal to the plane

B. 10 N up the plane

C. 17 N normal to the plane

D. 20 N upward

E. 20 N up the plane

Question 22 A 500 kg crate is dragged along a horizontal floor by a horizontal 10 kg chain. The tensions in the chain at its two ends are 1000 N and 1010 N. Ignore the force of gravity on the chain.
The acceleration (m s^{-2}) of the crate is closest to

A. 2.02

B. 2.01

C. 2

D. 1.98

E. 1

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 Consider the relation $x^2 + 4y^2 = 4$. The relation undergoes two transformations in the order shown below:

(1) Translate upwards by $\frac{1}{2}$ of a unit. (2) Dilate vertically by a factor of 2.

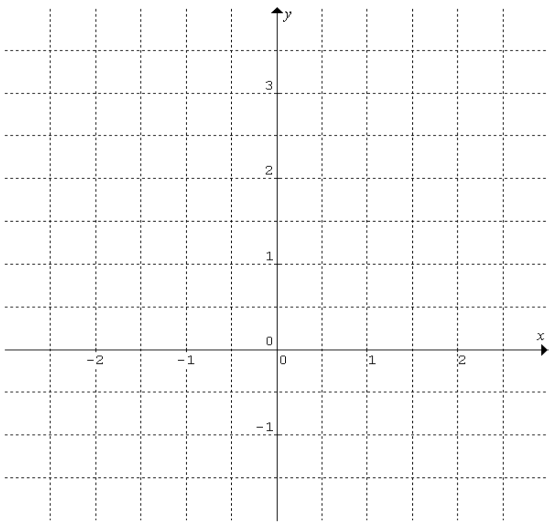
a. Show that the equation of the relation after the two transformations is $x^2 + (y-1)^2 = 4$. 1 mark

b. Find the x-intercepts of $x^2 + (y-1)^2 = 4$. 1 mark

Let the section of $x^2 + (y-1)^2 = 4$ below the x-axis be $g(x)$.

c i. Find the rule of $f(x) = |g(x)|$. 2 marks

c ii. Sketch the graph of $x^2 + (y-1)^2 = 4$ for $y \geq 0$ and the graph of $y = f(x)$ in the following grid. 2 marks



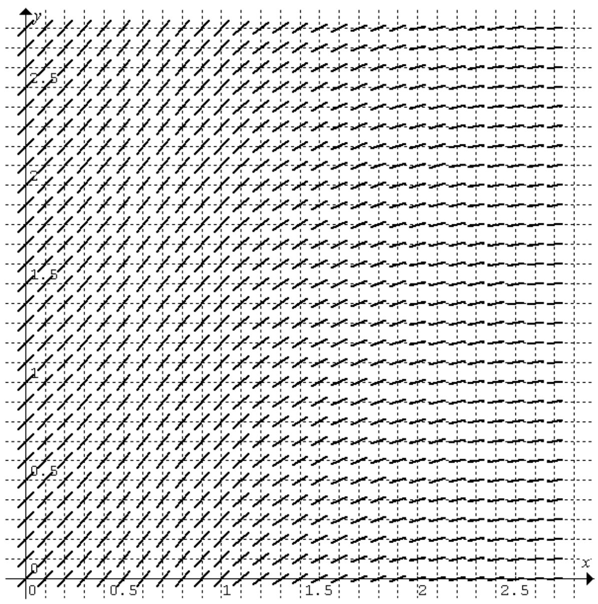
d. The region enclosed by the two curves in part **c ii** is rotated about the y-axis. Find the exact volume of the solid of revolution.

4 marks

Question 2 Given differential equation $\frac{dy}{dx} = \frac{1+x^2}{1+x^4}$ and $y = 0.5$ when $x = 0.5$.

a. The slope field of $\frac{dy}{dx} = \frac{1+x^2}{1+x^4}$ is shown below.

Sketch the solution curve on the slope field and find the approximate value of y when $x = 2$. 2 marks



b. Use Euler's method (step size of 0.5) to find the approximate value of y when $x = 2$, starting from $(0.5, 0.5)$.

2 marks

c. Use your CAS to find the value (rounded to 1 decimal place) of y when $x = 2$. 1 mark

d i. Write down the resulting expression when the numerator and denominator of $\dfrac{1+x^2}{1+x^4}$ is divided by x^2.

1 mark

d ii. $x^2 + \dfrac{1}{x^2}$ can be expressed in the form $\left(x + \dfrac{b}{x}\right)^2 + c$ where $c > 0$. Find b and c. 1 mark

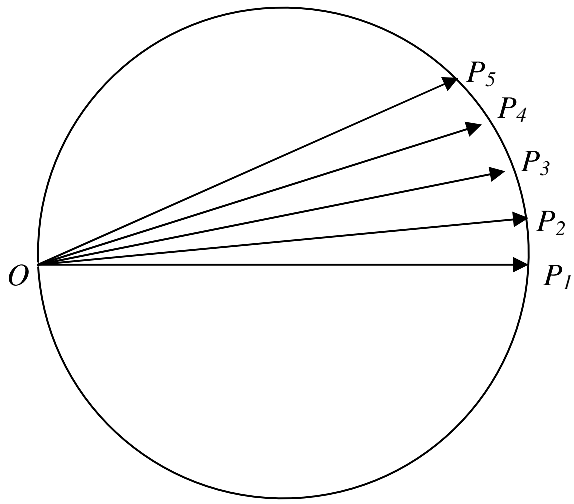
d iii. By calculus use an appropriate substitution method to find the antiderivative of $\dfrac{1+x^2}{1+x^4}$. 3 marks

d iv. Hence find the value (rounded to 1 decimal place) of y of differential equation $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+x^4}$ when $x = 2$, given $y = 0.5$ when $x = 0.5$.

2 marks

Question 3 In the following diagram, P_1, P_2, P_3, P_4, P_5, …… are points on the circumference of the circle. These points divide the circumference into arcs of equal length. OP_1 is a diameter of the circle. The diameter is a unit in length.

(Circle geometry knowledge required: (1) Angles at the circumference subtended by the same arc (or equal arcs) are equal. (2) Angle at the circumference subtended by a diameter is a right angle)



Let $\overrightarrow{OP_1} = \tilde{a}$, $\overrightarrow{OP_2} = \tilde{b}$, $\overrightarrow{OP_3} = \tilde{c}$, $\overrightarrow{OP_4} = \tilde{d}$, $\overrightarrow{OP_5} = \tilde{e}$, ……, and the angle between $\tilde{a}$ and $\tilde{b}$ is α.

a. Use vector methods to show $\left|\overrightarrow{P_1P_2}\right|^2 = |\tilde{a}|^2 + |\tilde{b}|^2 - 2|\tilde{a}||\tilde{b}|\cos\alpha$ 2 marks

b. Use part **a.** and circle geometry to find $\left|\overrightarrow{P_2P_3}\right|^2$ in terms of $\tilde{b}$, $\tilde{c}$ and α. 1 mark

c. Hence show $\cos\alpha = \dfrac{|\tilde{a}| + |\tilde{c}|}{2|\tilde{b}|}$. 2 marks

d. Explain why $\tilde{b}$ and $\tilde{c}$ are vector resolutes of $\tilde{a}$ in the directions of $\overrightarrow{OP_2}$ and $\overrightarrow{OP_3}$ respectively.

2 marks

e. Find $\left|\tilde{b}\right|$ and $\left|\tilde{c}\right|$ in terms of α. 2 marks

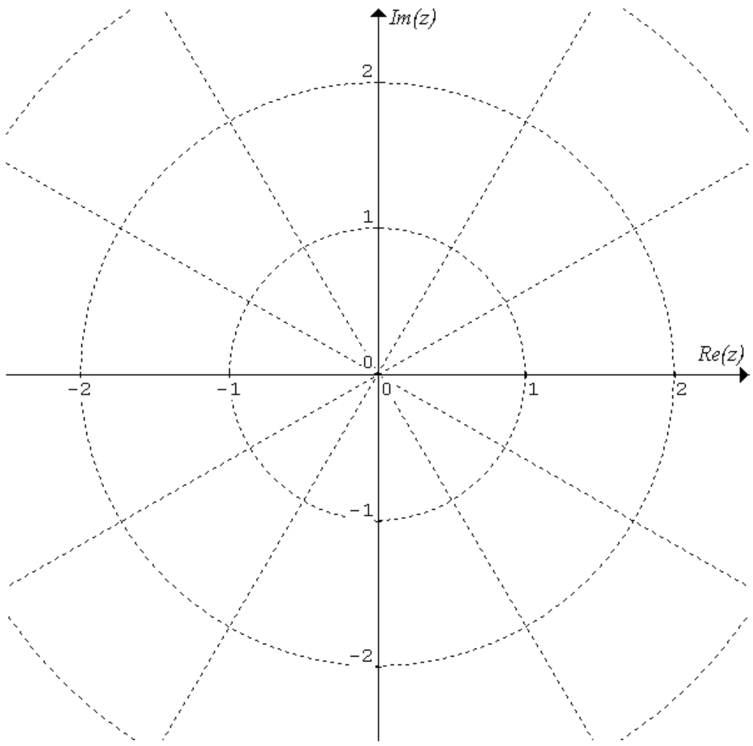
f. Use the results in parts **c.** and **e.** to show $1+\cos 2\alpha = 2\cos^2 \alpha$. 1 mark

g. *Use similar method as shown in pervious parts* to show $\cos\alpha + \cos 3\alpha = 2\cos\alpha\cos 2\alpha$. 3 marks

Question 4 Given $P_1(z) = z^9 + 1$ and $P_2(z) = z^6 - z^3 + 1$, $z \in C$.

a. Express $\dfrac{z^9 + 1}{z^6 - z^3 + 1}$ in the form of a polynomial function of z by means of a long division. 2 marks

b. Plot accurately the roots of $P_1(z) = 0$ in the grid below. 2 marks



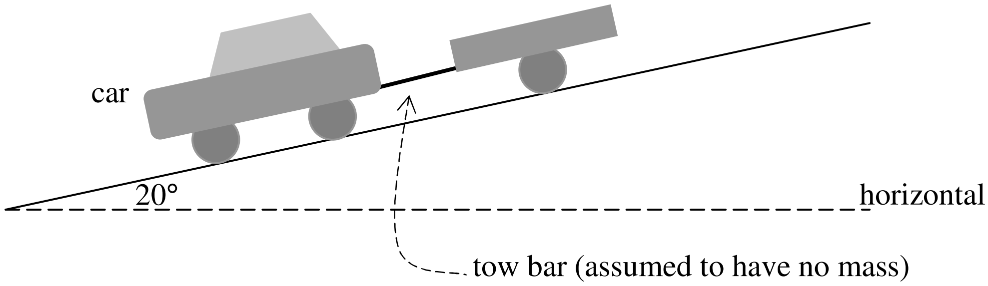
c. State the difference in the Arguments of two *adjacent* roots of $P_1(z) = 0$. 1 mark

d i. Write down the roots of $P_2(z)=0$ in polar form. 2 marks

d ii. Hence find the roots of $P_2(iz)=0$ in polar form. 3 marks

e. Show that the product of the roots of $P_2(iz)=0$ is -1. 2 marks

Question 5



The car with the trailer *reverses* up the slope with an acceleration $a = \begin{cases} 0.8\cos^2\left(\dfrac{\pi t}{4}\right) & 0 \le t \le 2 \\ 0 & t > 2 \end{cases}$ in m s^{-2} at time t (s). As a result the tow bar is in compression. The slope inclined at an angle of $20°$ with the horizontal.

Weight of the car $\left|\vec{W}_c\right| = 15000$ N, weight of the trailer $\left|\vec{W}_t\right| = 5000$ N, resistive force on the car $\left|\vec{R}_c\right| = 300$ N, resistive force on the trailer $\left|\vec{R}_t\right| = 200$ N. Resistive forces are assumed to be constant.

Let $\vec{F}_d$, $\vec{N}_c$, $\vec{N}_t$ and T be the driving force of the car, the normal reaction force on the car, the normal reaction force on the trailer and the compression (force) in the tow bar respectively, all are measured in newtons.

a. Use arrows to show all forces acting on the car and the trailer. Label them as $\vec{W}_c$, $\vec{W}_t$, $\vec{R}_c$, $\vec{R}_t$, $\vec{F}_d$, $\vec{N}_c$, $\vec{N}_t$ or T. 2 marks

b. Determine the magnitude of driving force $\vec{F}_d$ at $t = 0$. 3 marks

c. Determine the compression (force) in the tow bar at $t = 0$. 2 marks

The car starts from rest and accelerates up the slope.

d. Determine the exact speed of the trailer after the first 3 seconds. 3 marks

e. Calculate the distance (in metres) travelled in the time interval from $t = 3$ to $t = 8$. 1 mark

End of Exam 2

