Specialist Mathematics

2012

Trial Examination 1

Instructions

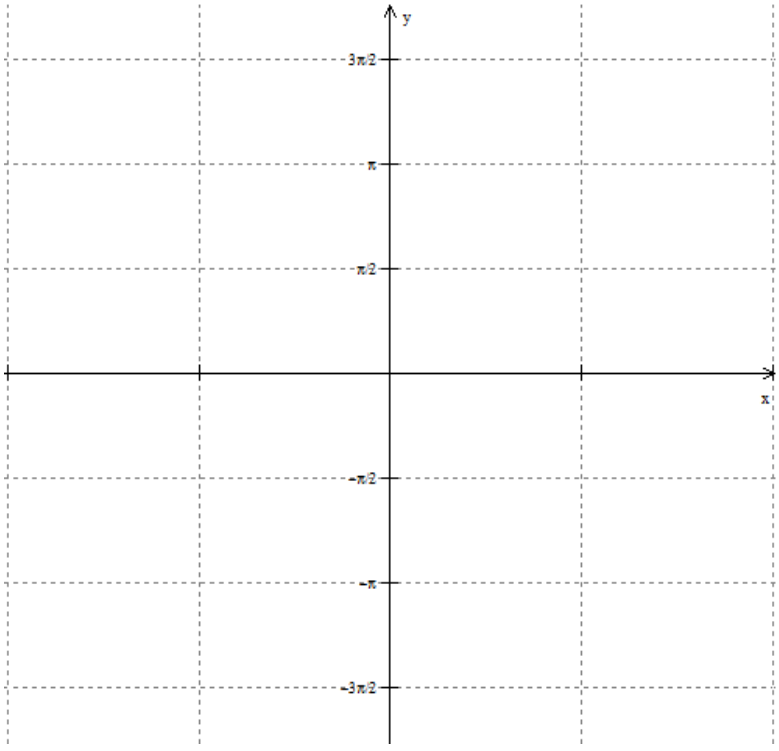
Answer **all** questions. Do **not** use calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working or explanation **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Let $y = \tan^{-1}(x) + \tan^{-1}(2x) + \tan^{-1}(3x)$.

a. Find the exact value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$. 2 marks

b. Sketch the graph of $y = \tan^{-1}(x) + \tan^{-1}(2x) + \tan^{-1}(3x)$. Clearly draw and label the asymptotes. 2 marks



Question 2

Given the relation $(x+2)^2 + 2\left(y-\sqrt{2}\right)^2 = 4$.

a. Find the maximal domain and range of the given relation. 2 marks

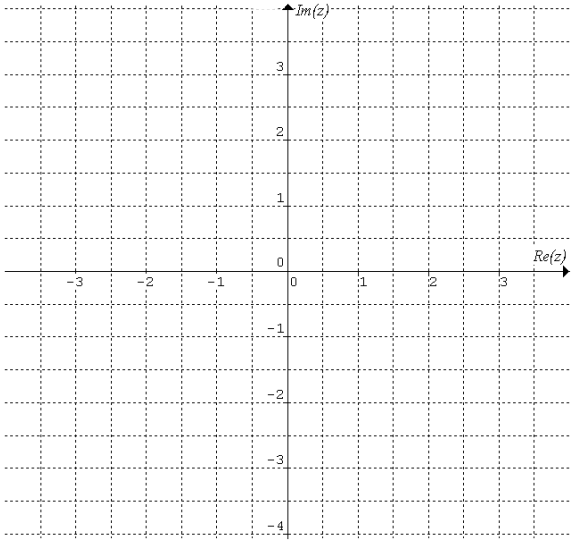
b. The relation is dilated from the *x*-axis by a factor of $\sqrt{2}$. Find the equation of the relation after the dilation. 2 marks

c. Find the exact area of the region enclosed by the relation in part **b**. 1 mark

Question 3

Let $z \in \{z : |z - 2i| \leq 1\}$.

a. Shade clearly the region in the complex plane for $\{z : |z - 2i| \leq 1\}$. 2 marks



b. Find the maximum value of $Arg(z)$. 2 marks

Question 4

Solve $z^5 = \bar{z}$ for z. 3 marks

Question 5

Given 3-dimensional position vector $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$.

a. Find a 3-dimensional vector perpendicular to $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$. 2 marks

b. Hence or otherwise, find a 3-dimensional *unit* vector perpendicular to $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$. 1 mark

c. Hence or otherwise, find the exact coordinates of a point which is 1 unit from $\tilde{r} = 2\tilde{i} - \tilde{j} + 3\tilde{k}$. 1 mark

Question 6

The position of a particle is given by $\tilde{r} = (\cos t)\tilde{i} - (2\sin t)\tilde{j} + 3\tilde{k}$.

a. State the position of the centre of the motion. 1 mark

b. Show that the particle's acceleration always points towards the centre of the motion. 3 marks

Question 7

Given $f(x)=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$ where $-1<x<1$.

a. Find the exact value of $f'\left(\dfrac{\sqrt{2}}{2}\right)$. 2 marks

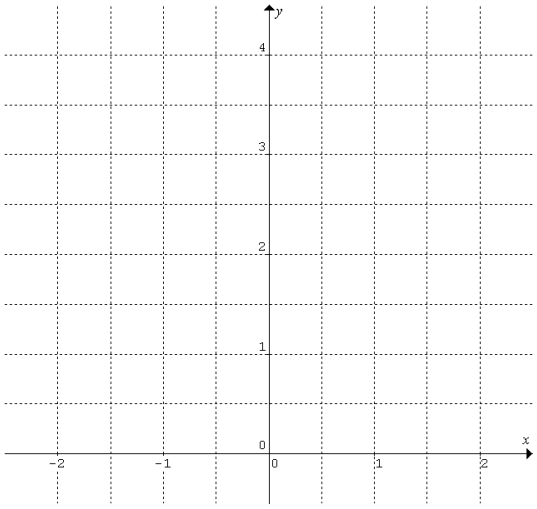
b. Find the exact area of the region bounded by the x-axis, $y=f(x)$, $x=-1$ and $x=1$. 2 marks

Question 8

Given $\frac{dy}{dx} = xy$ where $-2 \le x \le 2$ and $0 \le y \le 4$.

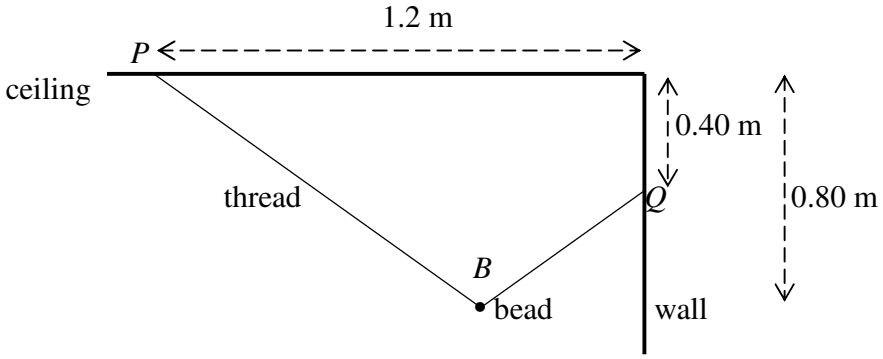
a. Construct a slope field for the differential equation using 1 unit interval for both *x* and *y*. Draw each tangent line about 0.5 unit long.

3 marks



b. Sketch the solution curve to the differential equation $\frac{dy}{dx} = xy$ through $(0,1)$. 1 mark

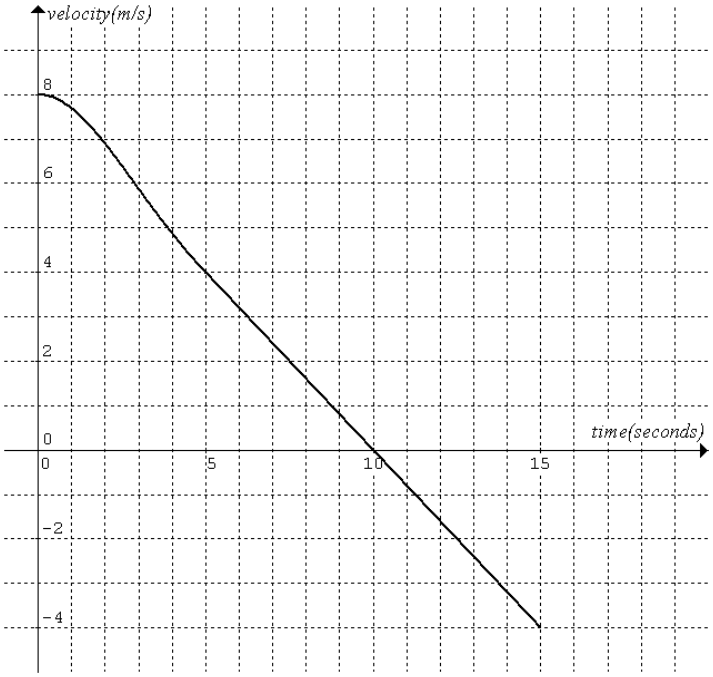
Question 9



A 0.010 kg bead is free to slide along a weightless and frictionless thread fixed to the ceiling and wall as shown above. The bead is in equilibrium when it is 0.80 m below the ceiling.
Points *P*, *B* and *Q* are on the same vertical plane.
Calculate the tension (2 significant figures) in the thread. 3 marks

Question 10

The velocity-time graph of a 0.50 kg particle for the first 15 seconds is shown below. Time is measured in seconds and distance in metres.



The equation of the graph is $v(t) = \begin{cases} \dfrac{8}{1+0.04t^2} & 0 \le t < 5 \\ -0.8t + 8 & 5 \le t \le 15 \end{cases}$

a. Find the magnitude of the change in momentum of the particle in the first 15 seconds. 1 mark

b. Find the magnitude of the resultant force on the particle at $t = 5$. 1 mark

c. Find the exact distance travelled by the particle in the first 15 seconds. 3 marks

End of Exam 1