INSIGHT
Year 12 *Trial Exam Paper*

2012

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.

Question 1

a. Show that for $0 < x < 1$, $\dfrac{d}{dx}\left(\arcsin(2x-1)\right) = \dfrac{1}{\sqrt{x-x^2}}$.

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2 marks

b. Hence, find the exact value of $\displaystyle\int_{\frac{1}{2}}^{\frac{3}{4}} \dfrac{6}{\sqrt{x-x^2}}\,dx$

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2 marks

END OF QUESTION 1

Question 2

a. Solve the following equation over C.

$z^2 - 2iz + 5 = 0$

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2 marks

b. Let $z_1 = \sqrt{3} - i$.

Express z_1 in polar form, $rcis\,\theta$ where $\theta = Arg(z_1)$.

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1 mark

c. On the argand diagram below, plot and clearly label

    i. z_1

    ii. $z_2 = |\overline{z_1}| i$

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                                                 2 marks

END OF QUESTION 2

Question 3

A particle moves in a straight line with an acceleration of a m/s^2, as given by

$$a = \frac{v^2 + v}{1 + \log_e(v+1)} \ , \quad v > 0.$$

At time t seconds, its displacement is x metres from a fixed point and its velocity is v m/s. What is the displacement of the particle as it moves from its position where $v = (e - 1)$ m/s to its position where $v = (e^2 - 1)$ m/s?

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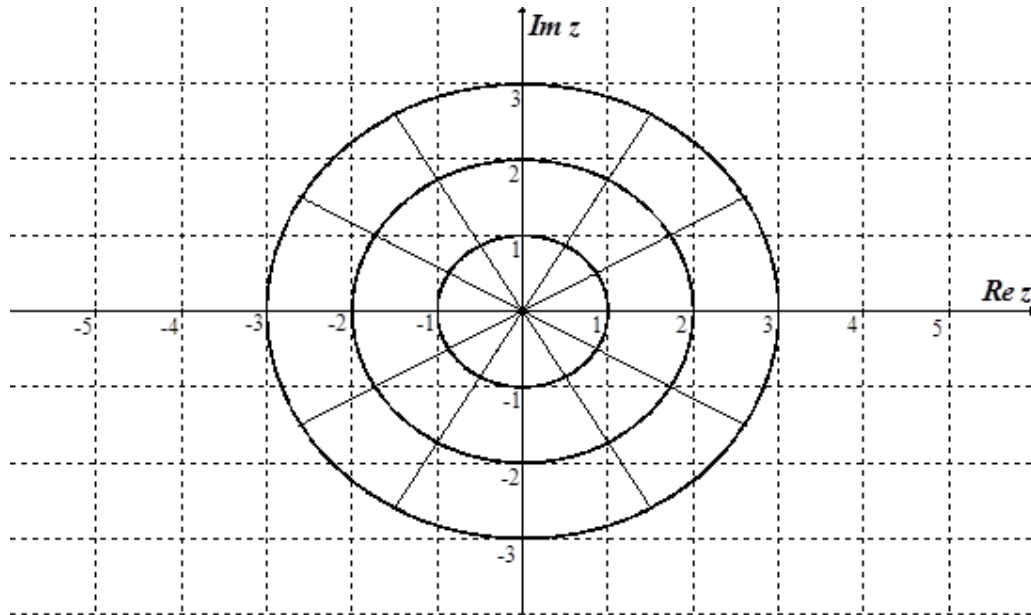
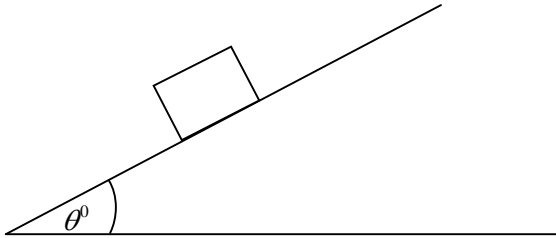3 marks

Question 4

A container of mass 400 kg rests on the rough surface of an inclined tray truck. The tray is inclined at an angle of $\theta °$ to the horizontal.
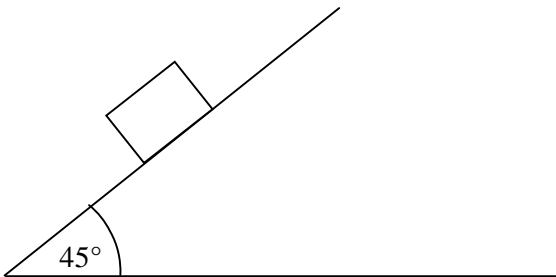
a.   On the diagram below, clearly label the three forces, including the normal force, N, and the friction force, F, acting on the container. All forces are in Newtons.

<div align="right">1 mark</div>

When the tray is raised to an angle of $45°$ to the horizontal, the container accelerates down the tray at $\dfrac{g\sqrt{2}}{20}$ m/s².

b.   What is the coefficient of friction between the container and the surface of the tray?

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<div align="right">3 marks</div>

<div align="right">**END OF QUESTION 4**</div>

Question 5

a. Use a compound angle formula to show that $\cos\left(\dfrac{\pi}{12}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$.

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<div align="right">1 mark</div>

b. Hence, evaluate $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{2}} 4\sin x \cos^3 x\,.dx$

Express the answer in the form $\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)^n$, where n is an integer.

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<div align="right">2 marks</div>

<div align="right">**END OF QUESTION 5**
TURN OVER</div>

Question 6

a. Sketch the graph of the curve with equation $y = \cos^{-1}(x) - \dfrac{\pi}{2}$ on the set of axes below.

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<div align="right">1 mark</div>

b. Find the volume generated when the region enclosed by the curve with equation
$y = \cos^{-1}(x) - \dfrac{\pi}{2}$, the *y*-axis and the lines $y = 0$ and $y = \dfrac{\pi}{2}$ is rotated about the *y*-axis to
form a solid of revolution.

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4 marks

END OF QUESTION 6
TURN OVER
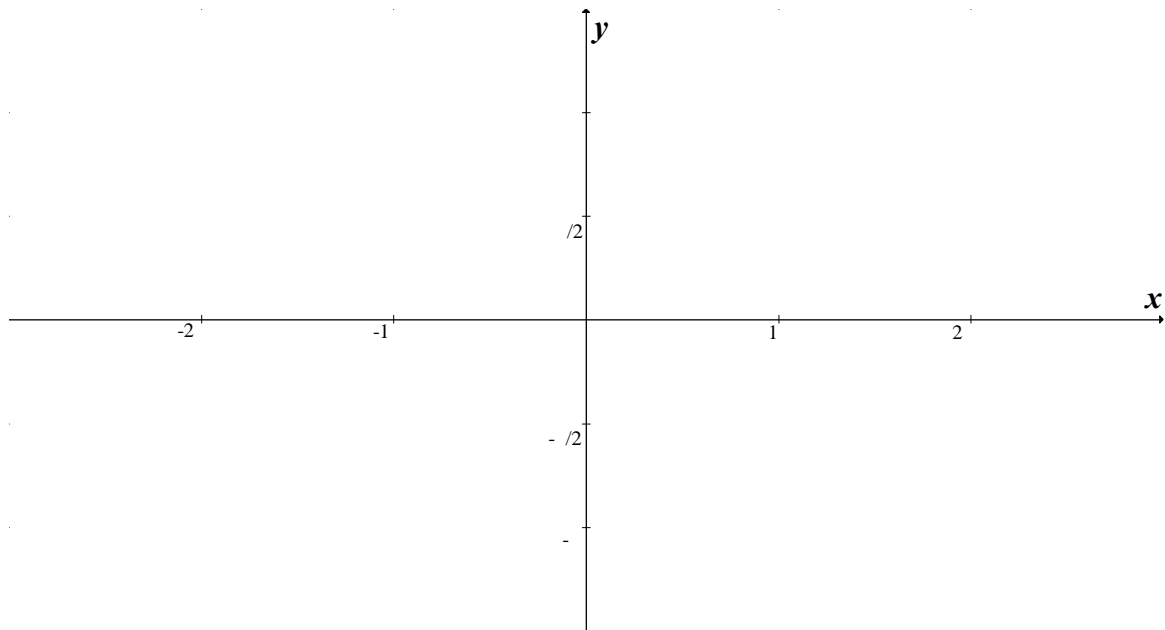
Question 7

For the relation $\log_e(xy) = x^2 y^2$, show that $\dfrac{dy}{dx} = \dfrac{-y}{x}$.

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3 marks

END OF QUESTION 7

Question 8

The position vector of a moving particle, $\underset{\sim}{r}(t)$ metres, at any time, t seconds, is given by

$$\underset{\sim}{r}(t) = 2\tan(t)\underset{\sim}{i} + \sec^2(t)\underset{\sim}{j}, \quad t \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right).$$

a. Determine the Cartesian equation for the path of the particle. State the domain and range.

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3 marks

b. Find the minimum speed of the particle.

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3 marks

END OF QUESTION 8

Question 9

Three points, A, B and O, are given by $A(2,1,2)$, $B(2,2,0)$ and $O(0,0,0)$.

a. Find the vector $\overrightarrow{AB}$ expressed in the form $x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

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1 mark

b. A point, C, on vector $\overrightarrow{AB}$ is closest to O. Find the coordinates of point C.

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3 marks

Question 10

On the axes supplied, sketch the graph of $f:[0, 2\pi] \to R, f(x) = \cot\left(\dfrac{x}{2}\right) - 1$, clearly indicating the location of any asymptotes and intercepts with the axes.

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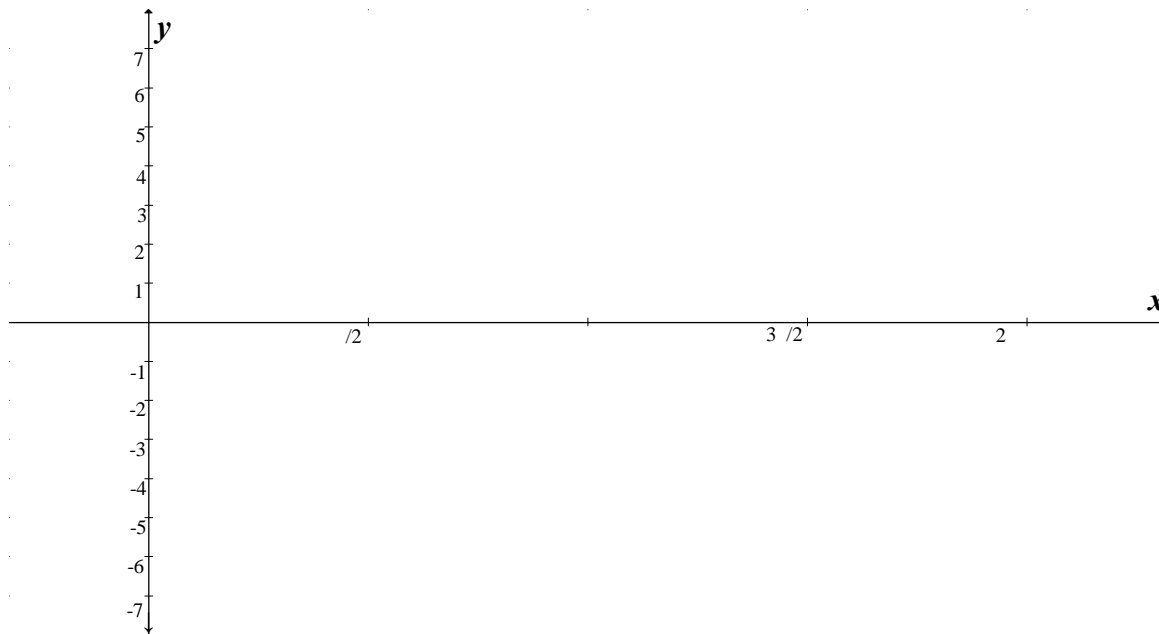
3 marks

END OF QUESTION AND ANSWER BOOK