ENGAGE EDUCATION FOUNDATION

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# Unit 3 and 4 Specialist Mathematics: Exam 1

## Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

| Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|
| 8 | 8 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 11 pages, including a sheet of miscellaneous formulas.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

## Instructions

Answer all questions in the spaces provided.
Unless otherwise specified, an exact answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude $g$ m/s$^2$, where $g = 9.8$.

## Questions

Question 1

Consider the relation $3xy^2 + 4y = 12 - 2x$.

a. Find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$.

2 marks

b. Hence, find the exact value of $\frac{dy}{dx}$ when $y = x$.

1 mark

Question 2

Solve the differential equation $\frac{dy}{dx} = x\sqrt{x^2 + 9}, x \geq 3$ for y, given that $y = 8$ when $x = 0$.

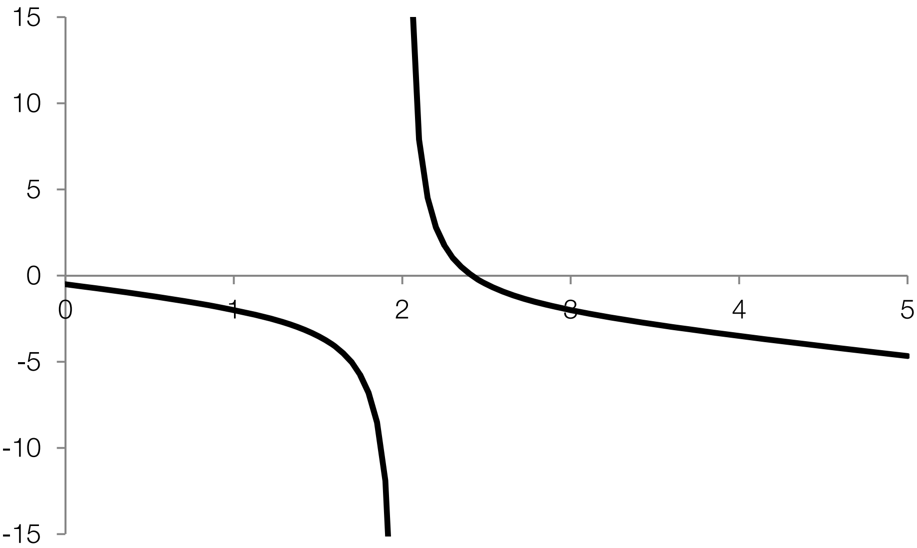
4 marks

Question 3

a. If $\frac{dy}{dx} = -x + \frac{1}{x-2}$, find $y$.

2 marks

b. The graph $y = -x + \frac{1}{x-2}$ is shown below.




Find the exact area bounded by $y$, the x-axis and the line $x = 3$.

4 marks

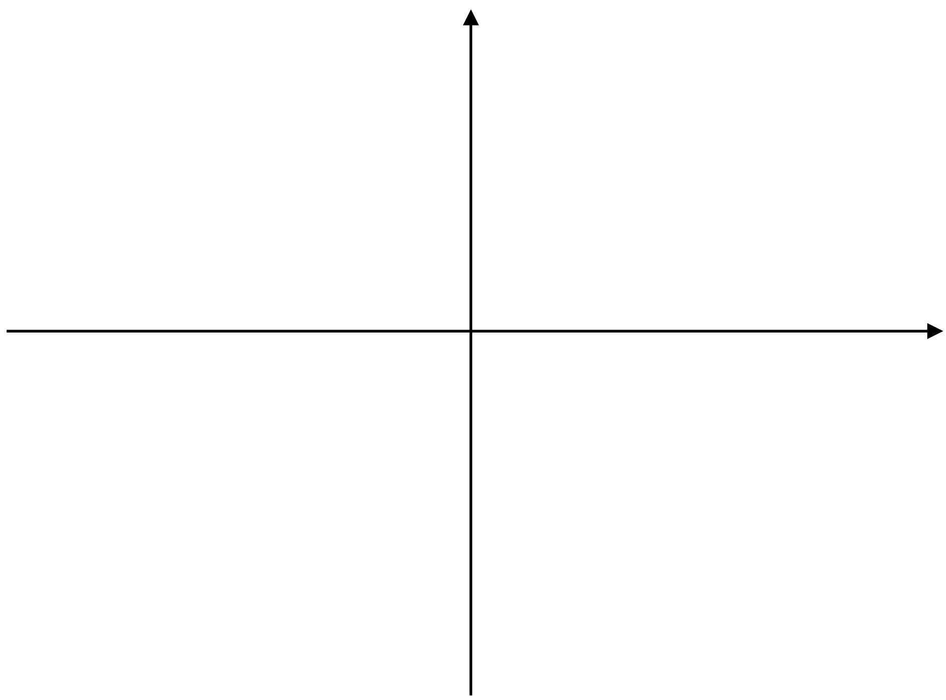
Question 4

The position vector of a moving particle is given by $\boldsymbol{r}(t) = \cos(2t)\,\boldsymbol{i} + \sin^2(t)\,\boldsymbol{j}$, for $t > 0$.

a. Find the Cartesian equation of the path followed by the particle.

3 marks

b. On the axes below, sketch the path of the particle, from $0 > t > \pi$.



3 marks

Question 5
Find the roots of $z^2 + 2z + 5$ over $\mathbb{C}$, expressing your answers in Cartesian form.

3 marks

Question 6
Find all real values of $m$ such that $y = \sin(mx)$ is always a solution of $y = -3\frac{d^2y}{dx^2}$.

4 marks

Question 7

Let $f(x) = \frac{1}{\pi}\arccos\left(\frac{x}{2}+1\right) - 2$.

a. State the implied domain and range of $f$.

4 marks

b. Find $f'(x)$, giving your answer in the form $\frac{a}{\pi\sqrt{bx(x+c)}}$, where $a$, $b$ and $c$ are integers.

5 marks

## Question 8

The velocity of a particle moving in a straight line is given by $v(t) = \frac{5(1-2t)}{1+2t}$, where $t \geq 0$.

a. Find the time when the particle comes to a stop momentarily.

1 mark

b. Find the exact total distance travelled by the particle at $t = 1$.

4 marks

# Formula sheet

## Mensuration

| | |
|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder | $2\pi rh$ |
| volume of a cylinder | $\pi r^2 h$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid | $\frac{1}{3}Ah$ |
| volume of a sphere | $\frac{4}{3}\pi r^3$ |
| area of a triangle | $\frac{1}{2}bc\sin A$ |
| sine rule | $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ |
| cosine rule | $c^2=a^2+b^2-2ab\cos C$ |

## Coordinate geometry

| | | | |
|---|---|---|---|
| ellipse | $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ | hyperbola | $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ |

## Circular (trigonometric) functions

$$\cos^2(x)+\sin^2(x)=1$$

$$1+\tan^2(x)=\sec^2(x) \qquad \cot^2(x)+1=\operatorname{cosec}^2(x)$$

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y) \qquad \sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y) \qquad \cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$$

$$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)} \qquad \tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$$

$$\sin(2x)=2\sin(x)\cos(x) \qquad \tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$$

| function | $\sin^{-1}$ | $\cos^{-1}$ | $\tan^{-1}$ |
|---|---|---|---|
| domain | $[-1,1]$ | $[-1,1]$ | $\mathbb{R}$ |
| range | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ | $[0,\pi]$ | $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ |

## Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}\,z \leq \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

## Calculus

$\dfrac{d}{dx}(x^n) = nx^{n-1}$

$\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\dfrac{d}{dx}(e^{ax}) = ae^{ax}$

$\displaystyle\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$

$\displaystyle\int \frac{1}{x} dx = \log_e|x| + c$

$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\displaystyle\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\displaystyle\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}(\tan(ax)) = \dfrac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}(\sin^{-1}(x)) = \dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}(\cos^{-1}(x)) = -\dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$

$\displaystyle\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

| | |
|---|---|
| product rule | $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ |
| quotient rule | $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{\left(v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right)}{v^2}$ |
| chain rule | $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$ |
| Euler's method | If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = a$, then $y_{n+1} = y_n + hf(x_n)$ |
| acceleration | $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$ |
| constant (uniform) acceleration | $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$, $s = \frac{1}{2}(u+v)t$ |

## Vectors in two and three dimensions

$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

$\dot{\boldsymbol{r}} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$

$|\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\boldsymbol{r}_1.\boldsymbol{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

## Mechanics

| | |
|---|---|
| momentum | $\boldsymbol{p} = m\boldsymbol{v}$ |
| equation of motion | $\boldsymbol{R} = m\boldsymbol{a}$ |
| friction | $F \leq \mu N$ |