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Unit 3 and 4 Specialist Mathematics: Exam 1

Practice Exam Solutions

Stop!

Don’t look at these solutions until you have attempted the exam.

Found a mistake?

Check the Engage Education website for updated solutions, and then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$3xy^2 + 4y = 12 - 2x$

$3y^2 + 6xy\frac{dy}{dx} + 4\frac{dy}{dx} = -2$	implicit differentiation using the chain rule [1]
$\frac{dy}{dx}(6xy + 4) = -2 - 3y^2$	factorise out $\frac{dy}{dx}$
$\frac{dy}{dx} = -\frac{3y^2+2}{6xy+4}$	make $\frac{dy}{dx}$ the argument [1]

Question 1b

$\frac{dy}{dx} = -\frac{3y^2+2}{6y^2+4}$	substitute in $x = y$ or $y = x$
$\frac{dy}{dx} = -\frac{3y^2+2}{2(3y^2+2)}$	simplify the terms
$\frac{dy}{dx} = -\frac{1}{2}$	simplify [1]

Question 2

$y = \int x\sqrt{x^2+9}\,dx + c$	fundamental theorem of calculus
$y = \frac{1}{2}\int 2x\sqrt{x^2+9}\,dx + c$	add a $2x$ factor because of the x^2

Use substitution $u = x^2 + 9$, $\frac{du}{dx} = 2x$ [1]

$y = \frac{1}{2}\int \frac{du}{dx}\sqrt{u}\,dx + c$	substitute in known terms
$y = \frac{1}{2}\int \sqrt{u}\,du + c$	chain rule
$y = \frac{1}{2} * \frac{2}{3} * u^{\frac{3}{2}} + c$	integrate [1]
$y = \frac{1}{3}(x^2+9)^{\frac{3}{2}} + c$	replace u by $x^2 + 9$

Now $y = 8$ when $x = 0$, so:

$8 = \frac{1}{3}(3^2)^{\frac{3}{2}}+c$	substitute in known point
$c = -1$	[1]
Hence, $y = \frac{1}{3}(x^2+9)^{\frac{3}{2}} - 1$	[1]

Question 3a

$$y = \int -x + \frac{1}{x-2}\,dx$$

$y = -\frac{x^2}{2} + \log_e(x-2) + c$	[1] for integration, [1] for including c

Question 3b
Find the x-axis intercept:

Let $y = 0$, then $0 = -x + \frac{1}{x-2}$

$0 = x^2 - 2x - 1$

$x = \dfrac{2 \pm \sqrt{4+4}}{2}$

$x = 1 \pm \sqrt{2}$ [1]

Let A be the area:

$A = -\int_{1+\sqrt{2}}^{3} \left(-x + \frac{1}{x-2}\right) dx$ [1]

$A = -\left[-\frac{x^2}{2} + \log_e|x-2|\right]_{1+\sqrt{2}}^{3}$ using answer from 3a

$A = \dfrac{9}{2} - \log_e 1 + \left(-\dfrac{\left(1+\sqrt{2}\right)^2}{2} + \log_e\left|\sqrt{2}-1\right|\right)$

$A = \dfrac{9 - 1 - 2\sqrt{2} - 2}{2} + \log_e\left(1 - \sqrt{2}\right)$

$A = 3 - \sqrt{2} + \log_e\left(1 - \sqrt{2}\right)$ [2]

Question 4a
$\boldsymbol{i}$ is the motion in the x plane, $\boldsymbol{j}$ is the motion in the y plane [1 for this fact, or its use, doesn't need to be stated] Hence:

$y = \sin^2 t$

$x = \cos 2t = 1 - 2\sin^2 t$ [1]

$\therefore x = 1 - 2y$

$y = \frac{1-x}{2}$ [1]

Question 4b
A straight line between (1, 0) and (-1, 1). [1 mark for each endpoints (total 2), 1 mark for shape]

Question 5
$z^2 + 2z + 5 = 0$ [1]

$z = \frac{-2 \pm \sqrt{4-20}}{2}$

$z = \frac{-2 \pm \sqrt{16}i}{2}$ [1]

$z = -1 \pm 2i$ [1]

Question 6
$y = sin(mx)$

$\frac{dy}{dx} = mcos(mx)$

$\frac{d^2y}{dx^2} = -m^2 \sin(mx)$ [1]

$sin(mx) = 3m^2 \sin(mx)$ [1]

$1 = 3m^2$, and also $m = 0$ [1]

$\frac{1}{3} = m^2$

$m = 0, \pm\frac{1}{\sqrt{3}}$ [1]

Question 7a

$-1 \le \frac{x}{2} + 1 \le 1$ [1]

$-2 \le \frac{x}{2} \le 0$

$-4 \le x \le 0$

Therefore the domain is $[-4, 0]$. [1]

Alternatively, you could build up the expression (rather than reducing it to x), as is used to find the range:

$0 \le \cos^{-1} a \le \pi,$ where $a = \frac{x}{2} + 1$ [1]

$0 \le \frac{1}{\pi}\cos^{-1} a \le 1$

$-2 \le \frac{1}{\pi}\cos^{-1} a - 2 \le -1$

Therefore the range is $[-2, -1]$. [1]

Question 7b

$\frac{d}{dx}\left(\frac{1}{\pi}\text{arccos}\left(\frac{x}{2} + 1\right) - 2\right)$

$= \frac{1}{\pi}\frac{d}{dx}\left(\text{arccos}\left(\frac{x}{2} + 1\right)\right)$ take out constant, $\frac{d}{dx}$ of 2 is 0

Use chain rule where u=$\left(\frac{x}{2} + 1\right)$: [1]

$= \frac{\frac{d}{dx}\left(\frac{x}{2}+1\right)}{\pi\sqrt{1-\left(\frac{x}{2}+1\right)^2}}$ [1]

$= \frac{1}{2\pi\sqrt{1-\left(\frac{x}{2}+1\right)^2}}$ [1]

$= \frac{1}{\pi\sqrt{-x(x+4)}}$ [2]

Hence, $a = 1$, $b = -1$, and $c = 4$

Question 8a

$v(t) = 0$

$\dfrac{5(1-2t)}{1-2t} = 0$

$t = \frac{1}{2}$ [1]

Question 8b

$v(t) = \frac{5(1-2t)}{1+2t} = \frac{10}{1+2t} - 5$ [1]

From part a, the particle is moving forwards for $0 \leq t \leq \frac{1}{2}$ and backwards for $\frac{1}{2} \leq t \leq 1$.

Therefore,

$d = \int_0^{\frac{1}{2}} \left(\frac{10}{1+2t} - 5\right) dt - \int_{\frac{1}{2}}^1 \left(\frac{10}{1+2t} - 5\right) dt$ [1]

$= [5\log_e(1+2t) - 5t]_0^{1/2} - [5\log_e(1+2t) - 5t]_{1/2}^1$

$= 5\log_e 2 - \frac{5}{2} - (5\log_e 3 - 5) + 5\log_e 2 - \frac{5}{2}$ [1]

$= 10\log_e 2 - 5\log_e 3$

$= 5\log_e 4 - 5\log_e 3$

$= 5\log_e \frac{4}{3}$ [1]