TSSM
Creating VCE Success

2011 Trial Examination

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STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 27 pages.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks are **not** deducted for incorrect answers.
If more than 1 answer is completed for any question, no mark will be given.
Take the **acceleration due to gravity**, to have magnitude g m/s^2, where g = 9.8

Question 1

The asymptotes of a hyperbola intersect at the point $(-1,1)$, the hyperbola passes through the point $(-3,1)$ and the gradient of one asymptote is 2. The equation of the hyperbola is

A. $\frac{(x+1)^2}{4}-\frac{(y-1)^2}{16}=1$

B. $\frac{(x+1)^2}{16}-\frac{(y-1)^2}{4}=1$

C. $\frac{(x+1)^2}{4}-(y-1)^2=1$

D. $\frac{(y-1)^2}{16}-\frac{(x+1)^2}{4}=1$

E. $(x+1)^2-\frac{(y-1)^2}{4}=1$

Question 2

Given that the domains of $\sin(x)$ and $\cos(x)$ are restricted to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\left[0,\pi\right]$ respectively, the implied domain of $y=\sin(2\cos^{-1}(2x))$ is

A. $\left[\frac{\sqrt{2}}{2},1\right]$

B. $\left[\frac{\sqrt{2}}{4},\frac{1}{2}\right]$

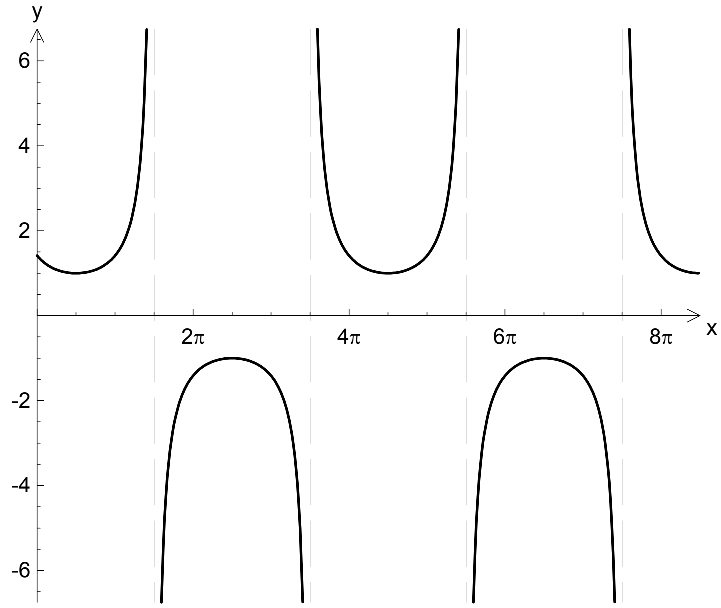
C. $\left[-1,1\right]$

D. $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

E. $\left[-\frac{\sqrt{2}}{4},\frac{\sqrt{2}}{4}\right]$

SECTION 1 - continued

Question 3



The graph of $y = \sec\left(a(b-x)\right)$ is shown above. The values of a and b could be

A. $a = \frac{1}{2}$ and $b = \frac{3\pi}{2}$

B. $a = \frac{1}{2}$ and $b = \frac{\pi}{2}$

C. $a = -\frac{1}{2}$ and $b = \frac{\pi}{2}$

D. $a = \frac{1}{2}$ and $b = -\frac{\pi}{2}$

E. $a = 2$ and $b = \frac{\pi}{2}$

SECTION 1 - continued
TURN OVER

Question 4

If $Arg(z-i)=\frac{\pi}{4}$ and $Arg(z-2)=\frac{\pi}{2}$, then $Arg(z+1)$ is equal to:

A. $\tan^{-1}\left(\frac{3}{2}\right)$

B. $\frac{\pi}{4}$

C. $\tan^{-1}\left(\frac{1}{2}\right)$

D. $-\frac{3\pi}{4}$

E. undefined

Question 5

If the roots of a polynomial $P(z)$ are $1+i,\ 1-i\ and\ 2$, then the roots of the polynomial $Q(z)=P(i\,\overline{z})$ are

A. $1-i,\ -1-i\ and\ -2i$

B. $1+i,\ 1-i\ and\ 2$

C. $1+i,\ 1-i\ and\ 2i$

D. $1+i,\ -1+i\ and\ 2i$

E. $-1-i,\ 1-i\ and\ -2i$

Question 6

The argument of the complex number $w=\frac{2\overline{z}^2(1+i)}{z^2}$, where $Arg(\overline{z})=\frac{\pi}{16}$, is:

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{4}$

E. $2\left(\frac{\pi}{16}\right)^2\times\frac{\pi}{4}$

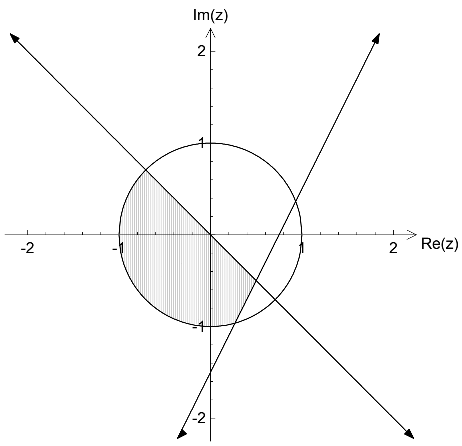
SECTION 1 – continued

Question 7

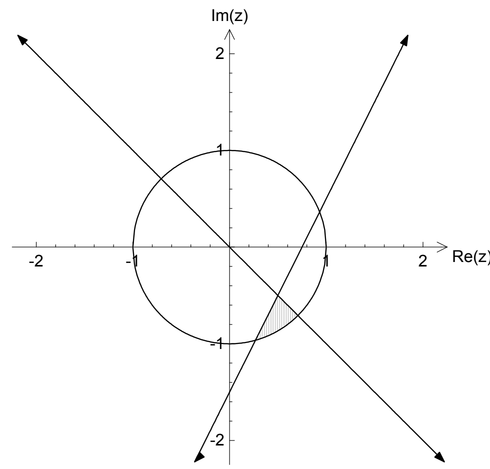
Which one of the following shows the shaded region T of the complex plane specified by

$T = \{z : |z+1| \geq |z-i|,\ z \in C\} \cap \{z : |z-2| \leq |z-i|,\ z \in C\} \cap \{z : |z| \leq 1,\ z \in C\}$?

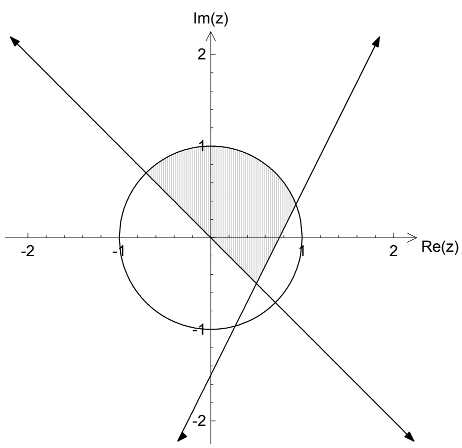
A.



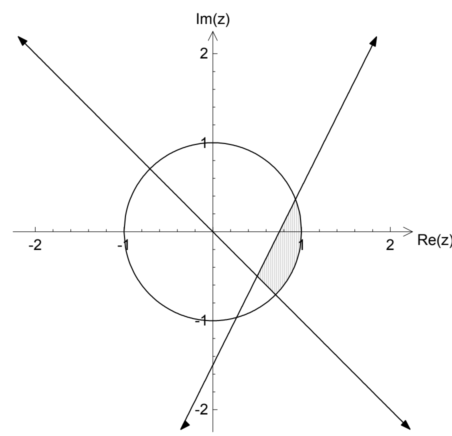
B.



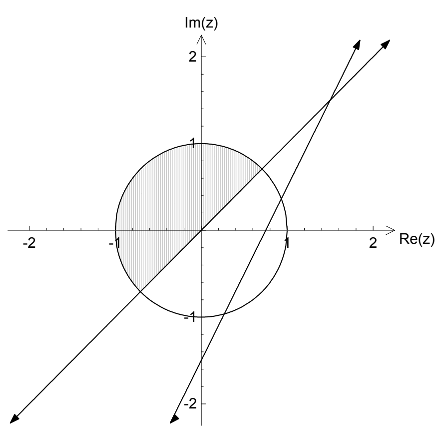
C.



D.



E.



SECTION 1 – continued
TURN OVER

Question 8

To prove that three concurrent non-zero and non- parallel vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are coplanar, it is sufficient to show that:

A. $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent

B. $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly independent

C. Lami's theorem is true for the sum $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}$

D. $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}=0$

E. one vector is the sum of the other two.

Question 9

Consider two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ and their sum $\underset{\sim}{c}=\underset{\sim}{a}+\underset{\sim}{b}$. Which one of the following statements is false?

A. $\underset{\sim}{b}\cdot\underset{\sim}{c}=|\underset{\sim}{b}|^2+\underset{\sim}{a}\cdot\underset{\sim}{b}$

B. $\frac{\underset{\sim}{a}\cdot\underset{\sim}{c}}{|\underset{\sim}{c}|^2}\underset{\sim}{c}+\frac{\underset{\sim}{b}\cdot\underset{\sim}{c}}{|\underset{\sim}{c}|^2}\underset{\sim}{c}=\underset{\sim}{c}$

C. $\underset{\sim}{a}-\frac{\underset{\sim}{a}\cdot\underset{\sim}{c}}{|\underset{\sim}{c}|^2}\underset{\sim}{c}=\left(\underset{\sim}{b}-\frac{\underset{\sim}{b}\cdot\underset{\sim}{c}}{|\underset{\sim}{c}|^2}\underset{\sim}{c}\right)$

D. $\frac{\underset{\sim}{c}\cdot\underset{\sim}{a}}{|\underset{\sim}{c}|^2}\underset{\sim}{c}+\frac{\underset{\sim}{c}\cdot\underset{\sim}{b}}{|\underset{\sim}{c}|^2}\underset{\sim}{c}=\underset{\sim}{c}$

E. $\underset{\sim}{c}\cdot\underset{\sim}{a}=|\underset{\sim}{c}|^2-\underset{\sim}{b}\cdot\underset{\sim}{c}$

Question 10

A and B are defined by the position vectors $\underset{\sim}{a}=\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j}$ and $\underset{\sim}{b}=3\underset{\sim}{k}$. The unit vector that bisects the angle $\angle AOB$ is

A. $\frac{1}{\sqrt{13}}\left(\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j}\right)+\frac{3}{\sqrt{13}}\underset{\sim}{k}$

B. $\frac{1}{2\sqrt{2}}\left(\underset{\sim}{i}+\sqrt{3}\underset{\sim}{j}\right)+\frac{1}{\sqrt{2}}\underset{\sim}{k}$

C. $\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$

D. $\frac{1}{|\underset{\sim}{a}+\underset{\sim}{b}|}(\underset{\sim}{a}+\underset{\sim}{b})$

E. $\frac{1}{2}(\hat{\underset{\sim}{a}}+\hat{\underset{\sim}{b}})$

SECTION 1 – continued

Question 11

Given that $\frac{1-\sin(2\theta)}{\cos(2\theta)} = \tan\left(\frac{\pi}{4}-\theta\right)$ is true, then which one of the following is also true?

A. $\frac{1-\cos(2\theta)}{\sin(2\theta)} = \cot\left(\frac{\pi}{4}-\theta\right)$

B. $\frac{1+\cos(2\theta)}{\sin(2\theta)} = \cot(\theta)$

C. $\frac{1-\sin(\theta)}{\cos(\theta)} = \cot\left(\frac{\theta}{2}\right)$

D. $\frac{\sin(2\theta)}{1-\cos(2\theta)} = \tan\left(\frac{\pi}{4}-\theta\right)$

E. $\frac{1+\cos(2\theta)}{\sin(2\theta)} = \cot\left(\frac{\pi}{4}-\theta\right)$

Question 12

The position vector of a particle at time t, $t \ge 0$, is given by $\underset{\sim}{r}(t) = \tan(t)\underset{\sim}{i} + \sec(t)\underset{\sim}{j}$. The path along which the particle moves is:

A. circular
B. elliptical
C. parabolic
D. straight
E. hyperbolic

Question 13

A particle moves in a straight line with velocity v m/s. At time t seconds, $t \ge 0$, the position is x metres.
If $v^2 = -4x$ and at $t = 0$ $\quad v(0) = 8$, then the particle starts moving with

A. increasing acceleration and increasing velocity

B. decreasing acceleration and decreasing velocity

C. decreasing acceleration and constant velocity

D. constant acceleration and decreasing velocity

E. constant acceleration and increasing velocity

SECTION 1 - continued
TURN OVER

Question 14

Using a suitable substitution $\int_0^1 x^3 e^{x^2}\,dx$ is equal to

A. $\int_0^1 u\,e^u\,du$

B. $\frac{1}{2}\int_0^1 u^{\frac{3}{2}}\,e^u\,du$

C. $\frac{1}{2}\int_0^1 u\,e^{u^2}\,du$

D. $\frac{1}{2}\int_0^1 u\,e^u\,du$

E. $\int_0^1 u^{\frac{3}{2}}\,e^u\,du$

Question 15

The position of a particle at time t, $t \geq 0$, is given by $\underset{\sim}{r}(t) = -\sin(2\pi t)\underset{\sim}{i} + \cos(\pi t)\underset{\sim}{j}$.

The speed of the particle at time $t = \frac{3}{2}$ is:

A. $\pi\sqrt{5}$

B. $-\pi\sqrt{5}$

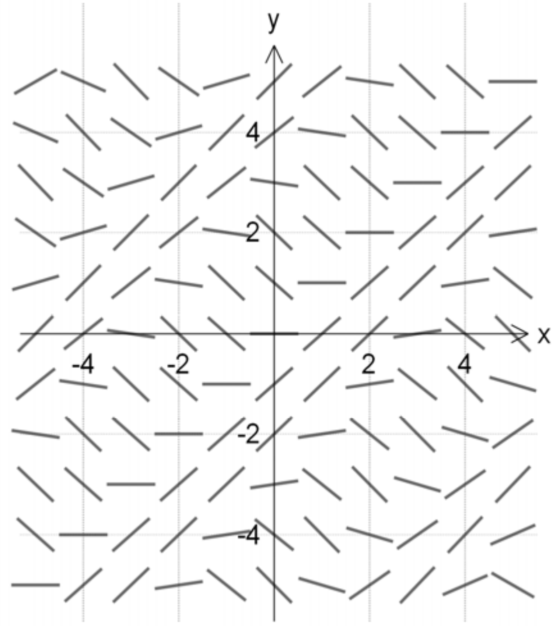
C. $\sqrt{5\pi}$

D. 3π

E. 0

SECTION 1 – continued

Question 16



The family of solutions of a first order differential equation is shown above. The differential equation could be

A. $\dfrac{dy}{dx} = \tan(y - x)$

B. $\dfrac{dy}{dx} = \tan(x - y)$

C. $\dfrac{dy}{dx} = \sin(y - x)$

D. $\dfrac{dy}{dx} = \sin(x - y)$

E. $\dfrac{dy}{dx} = \cos(y - x)$

Question 17
A particle, starting from rest, moves with constant acceleration in a straight line. It covers a distance d in the first second. The distance covered by the particle in the n^{th} second is

A. nd

B. $(2n-1)d$

C. $(2n+1)d$

D. n^2d

E. d

Question 18
An object starts falling from a high altitude and, apart from gravity, it experiences an air resistance force $F_r = -mkv$, where m is the mass of the object , v ms^{-1} is its velocity t seconds after it is dropped and k is a constant. The time after which the acceleration of the object decreases to $\frac{g}{2}$ is

A. $\frac{1}{k}\log_e\left(\frac{4}{3}\right)$

B. k

C. $\frac{\log_e 2}{k}$

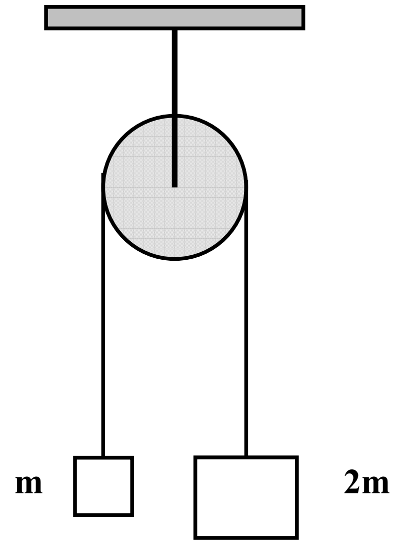
D. undefined

E. $-\frac{\log_e 2}{k}$

SECTION 1 – continued

Question 19

Two blocks of masses *m* and *2m* are connected by a light inextensible string that passes over a smooth fixed pulley as shown in the diagram. The system is released from rest. The tension in the string is



A. $\frac{2}{3}mg$

B. mg

C. $\frac{3}{4}mg$

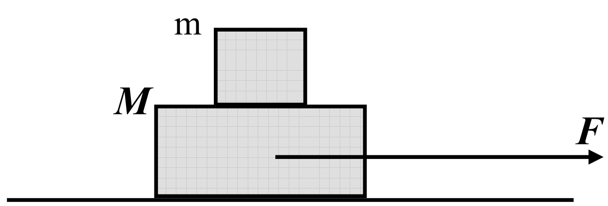
D. $\frac{3}{2}mg$

E. $\frac{4}{3}mg$

Question 20

Two objects of mass M and m respectively, lie on a rough surface as in the diagram below. The coefficient of friction between M and m and between M and the surface is μ. A force F acts on the object of mass M.

The minimum value of the magnitude of force F for which the object m slides on the object M is



A. μmg

B. μMg

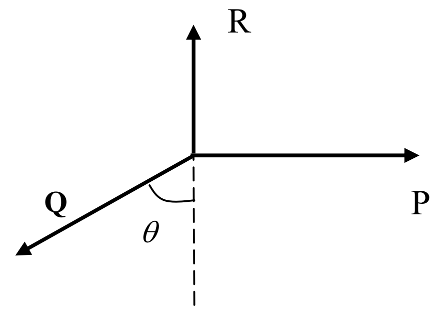
C. $2\mu(M+m)g$

D. $\mu(2M+m)g$

E. $\mu(M+m)g$

SECTION 1 - continued
TURN OVER

Question 21
The diagram shows a particle held in equilibrium by three concurrent coplanar forces $\underset{\sim}{P}, \underset{\sim}{Q}$ *and* $\underset{\sim}{R}$.
Which of the following is **FALSE?**



A. If $\theta = 45$ then $R = P = \sqrt{2}\,Q$

B. $Q^2 = R^2 + P^2$

C. $P = Q\sin(\theta)$ and $R = Q\cos(\theta)$

D. $\tan(\theta) = \dfrac{P}{R}$

E. $\underset{\sim}{P} + \underset{\sim}{Q} + \underset{\sim}{R} = \underset{\sim}{0}$

Question 22
As a space shuttle moves through the space, its mass decreases due to fuel consumption.
At time t the mass of the shuttle is $m\ kg$, its velocity is $\underset{\sim}{v}\ ms^{-1}$, its acceleration is $\underset{\sim}{a}\ ms^{-2}$ and the net force acting on the shuttle is $\underset{\sim}{F}$ *Newton*. Which one of the following statements is true?

A. $\underset{\sim}{F} = m\underset{\sim}{a} + \underset{\sim}{v}\dfrac{dm}{dt}$

B. $\underset{\sim}{F} = m\underset{\sim}{a}$

C. $F = ma + v\dfrac{dm}{dt}$

D. $\underset{\sim}{F} = m\dfrac{d\underset{\sim}{v}}{dt}$

E. $\underset{\sim}{F} = 0$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.
For questions worth more than one mark, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the **acceleration due to gravity**, to have magnitude g m/s^2 , where $g = 9.8$.

Question 1

Let u and w *be* the roots of the equation $z+\frac{1}{z}=-2\sin\theta$ where: $-\frac{\pi}{4}\le\theta\le 0$ and $Arg(u)>0$

a. By solving the equation and bringing the roots to the polar form, show that

$u=cis\left(\frac{\pi}{2}+\theta\right)$ and $w=cis\left(-\frac{\pi}{2}-\theta\right)$

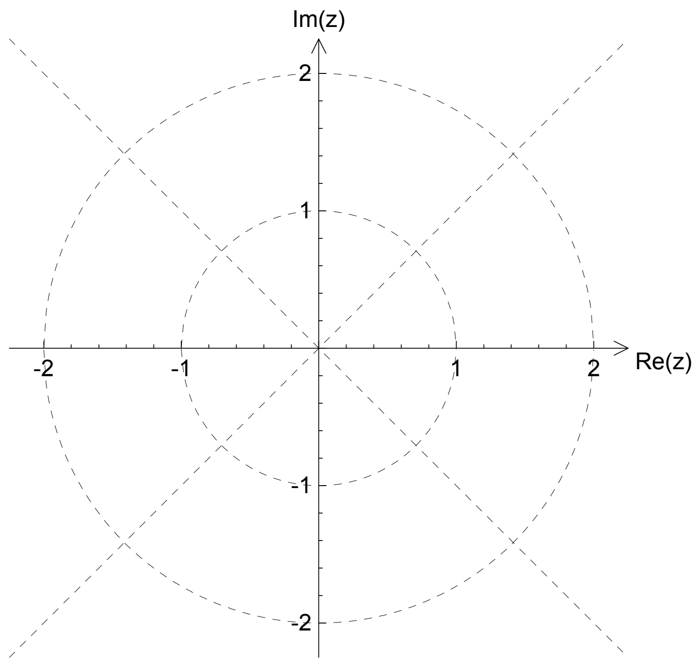
4 marks

SECTION 2 – Question 1- continued
TURN OVER

Let $A=\left\{z: z=u, \quad -\frac{\pi}{4} \leq \theta \leq 0\right\}$, $B=\left\{z: z=w, \quad -\frac{\pi}{4} \leq \theta \leq 0\right\}$ and

$T=\left\{z: z=2u^2, \quad -\frac{\pi}{4} \leq \theta \leq 0\right\}$

b. Sketch the complex regions *A, B* and *T* on the Argand diagram below.



3 marks

SECTION 2 – Question 1- continued

c. Find the minimum and maximum values of $|z_1 - z_2|$ where $z_1 \in A$ and $z_2 \in T$.

2 marks

d.

i. Show that $u^n - w^n = 2i\sin\left(\frac{n\pi}{2} + n\theta\right)$ where n is a natural number.

2 marks

ii. Hence evaluate $u^4 - w^4$ for $\theta = \frac{\pi}{4}$

1 mark

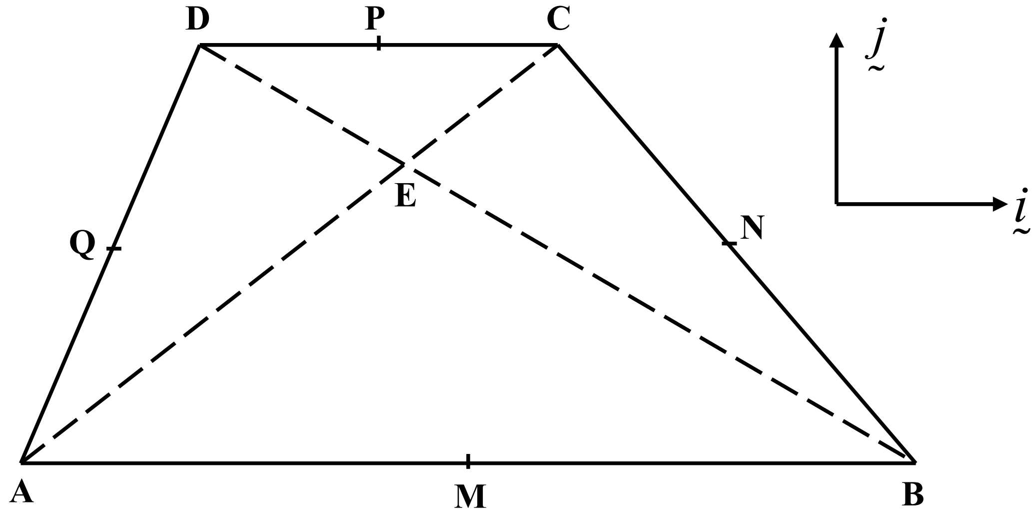
Total 12 marks

SECTION 2 – continued

TURN OVER

Question 2

In the diagram below, *ABCD* is a trapezium. Points *M, N, P* and *Q* are the midpoints of *AB, BC, CD* and *DA* respectively. Point *E* is the point of intersection of the diagonals *AC* and *DB*.



Let $\overrightarrow{AB} = \underset{\sim}{a}$, $\overrightarrow{AD} = \underset{\sim}{b}$ and $\overrightarrow{DC} = \underset{\sim}{c}$

a. Show that the quadrilateral *MNPQ* is a parallelogram.

2 marks

SECTION 2 –Question 2-continued

Let $\underset{\sim}{a} = 8\underset{\sim}{i}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{c} = 3\underset{\sim}{i}$

b.

i. Express the diagonals $\overrightarrow{AC}$ and $\overrightarrow{DB}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$

1 mark

ii. Hence find the acute angle formed by the diagonals, correct to the nearest degree.

2 marks

SECTION 2 – Question 2- continued
TURN OVER

c. Let $\overrightarrow{AE} = m\overrightarrow{AC}$ and $\overrightarrow{DE} = n\overrightarrow{DB}$

i. Show that $m = \frac{8}{11}$ and $n = \frac{3}{11}$.

4 marks

SECTION 2 – Question 2-continued

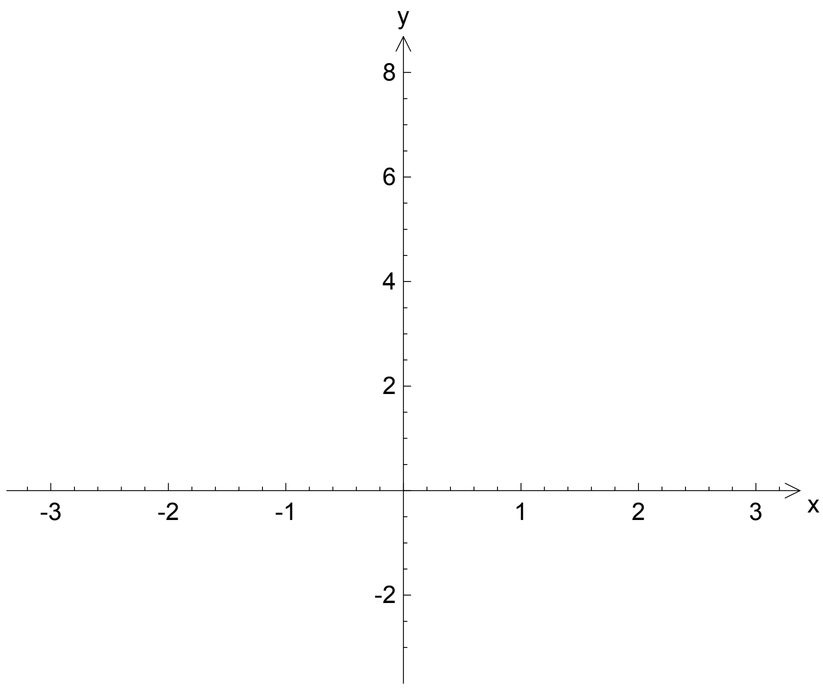
ii. Hence find the area of the triangle *ABE*.

2 marks
Total 11 marks

Question 3

The equation of a hyperbola is described by the equation $(y+3)^2 - 9x^2 = 9 \quad , \quad y \geq 0$

a. Sketch the graph of the hyperbola on the axes below and label all its important features.



4 marks

b. Let $f(x)$ be the gradient to the hyperbola, as a function of x only.

i. Show that $f(x) = \dfrac{3x}{\sqrt{x^2+1}}$

2 marks

SECTION 2 –Question 3- continued

ii. The range of $f(x)$ can be written as an interval $(a, \quad b)$. Find the values for a and b.

1 mark

c. The hyperbola is rotated about the y-axis to form a solid of revolution.

i. Express as a definite integral the volume V of this hyperboloid in terms of the depth h.

2 marks

ii. Use calculus to show that $V(h) = \pi\left(\frac{h^3}{27} + \frac{h^2}{3}\right)$

1 mark

d. Imagine the hyperboloid is a container with very thin walls. Water is continuously added , so at time t seconds the volume of water in the container is $V\ cm^3$, the depth of water is $h\ cm$ and the area of the surface of water is $A\,cm^2$. Write A as the rate of change of other variables involved.

1 mark
Total 11 marks

SECTION 2 – continued
TURN OVER

Question 4

A rocket is launched horizontally with an initial speed u from a height h above the ground. Define the positive x-axis as horizontal in the direction of the original flight, the positive y-axis as downward and the origin as the launching point.
The air resistance at time t seconds after launching is $\underset{\sim}{F} = -mk\underset{\sim}{v}$, where m is the mass of the rocket, in kg, $\underset{\sim}{v}$ is the velocity, in ms^{-1}, at time t seconds and k is a constant. The acceleration at any time t is $\underset{\sim}{a}$. $\underset{\sim}{a} = a_x \underset{\sim}{i} + a_y \underset{\sim}{j}$ and $\underset{\sim}{v} = v_x \underset{\sim}{i} + v_y \underset{\sim}{j}$. Assume that the rocket is in flight.

a. Show that $\frac{dv_x}{dt} = -k v_x$ and $\frac{dv_y}{dt} = g - k v_y$

2 marks

b. Use calculus or calculator to find v_x and v_y at any time t.

4 marks

SECTION 2 –Question 4- continued

c. Use calculus to show that the *y*-coordinate of the rocket at any time *t* is given by

$$y(t) = \frac{g}{k}t + \frac{g}{k^2}\left(e^{-kt} - 1\right)$$

2 marks

d.

i. Assume that $h = 490\,m$ and $k = 0$ (no air resistance) . Find the time when the rocket hits the ground.

1 mark

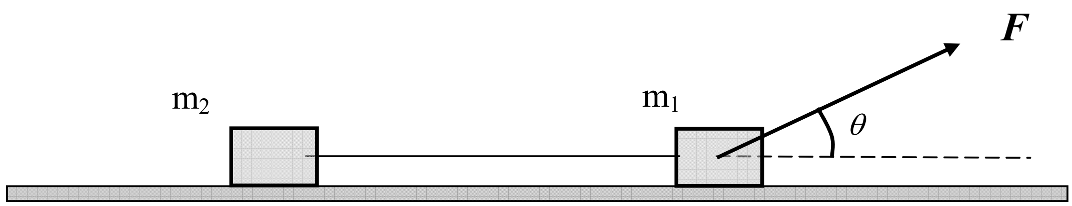
ii. Assume that $h = 490\,m$ and $k = 0.01g$. Find the time when the rocket hits the ground correct to one decimal place.

1 mark
Total 10 marks

SECTION 2 – continued
TURN OVER

Question 5

An object of mass $m_1 = 40kg$, placed on a horizontal surface, is acted upon by a driving force F that makes an angle θ with the horizontal. The object is connected to another object of mass $m_2 = 20kg$ by a light inextensible string as in the diagram below.



a. If $\theta = 45^0$ and $F = 300N$, and the surface is frictionless, find the acceleration of the objects, correct to one decimal place.

3 marks

SECTION 2 – Question 5- continued

b. Assume that $\theta = 45^0$ and $F = 300N$ and the coefficient of friction between the surface and the objects is $\mu = \tan\varphi$, where φ is an angle in the first quadrant. The objects are moving with a horizontal acceleration to the right. Show that the acceleration of the objects is

$$a = \frac{F\cos(\theta - \varphi)}{(m_1 + m_2)\cos\varphi} - g\tan\varphi$$

5 marks

SECTION 2 – Question 5- continued
TURN OVER

c. Assume that $F = 300N$ and $\mu = \tan 10^0$. The angle θ can range between 0^0 and 90^0.

i. Find the value of the angle θ for which the acceleration of the objects is maximum.

1 mark

ii. Hence find, correct to one decimal place, the maximum value of the acceleration.

1 mark

iii. Assume that $F = 300N$ and $\mu = \tan 10^0$. Find the value of the angle θ_2 for which the acceleration of the objects is the same as for the angle $\theta_1 = 15^0$.

2 marks

SECTION 2 –Question 5- continued

d. Assume that $\theta = 45^0$ and $\mu = \tan 10^0$. The force F, initially 300 N, starts increasing until the object is on verge of losing contact with the surface. Find, correct to one decimal place, the value of the force F when the object is on verge of losing contact with the surface.

2 marks
Total 14 marks

END OF QUESTION AND ANSWER BOOK