Neap:

Trial Examination 2011

# VCE Specialist Mathematics Units 3 & 4

## Written Examination 2

## Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

**Structure of Booklet**

| Section | Number of questions | Number of questions to be answered | Number of marks |
|---|---|---|---|
| 1 | 22 | 22 | 22 |
| 2 | 5 | 5 | 58 |
| | | | Total 80 |

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

Question and answer booklet of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Answer sheet for multiple-choice questions.

**Instructions**

Detach the formula sheet from the centre of this booklet during reading time.
Write **your name** and your **teacher's name** in the space provided above on this page and in the space provided on the answer sheet for multiple-choice questions.
All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2011 VCE Specialist Mathematics Units 3 & 4 Written Examination 2.

## SECTION 1

**Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude $g$ m/s$^2$, where $g = 9.8$.

### Question 1

Which one of the following statements regarding the graphs of $f(x) = ax^2 + bx + c,\ a \neq 0$ and $g(x) = \dfrac{1}{f(x)}$ is incorrect?

**A.** The graph of $y = f(x)$ and the graph of $y = g(x)$ have the same vertical axis of symmetry.

**B.** The graph of $y = f(x)$ has $y$-intercept $(0, c)$ and the graph of $y = g(x)$ has $y$-intercept $\left(0, \dfrac{1}{c}\right)$.

**C.** The graph of $y = f(x)$ and the graph of $y = g(x)$ always have a turning point at $x = \dfrac{-b}{2a}$.

**D.** The graph of $y = g(x)$ has vertical asymptotes at values of $x$ for which $f(x) = 0$.

**E.** The graph of $y = f(x)$ and the graph of $y = g(x)$ have the same sign.

### Question 2

The set of values of $k$ for which the equation $\dfrac{(4k+1)x^2}{k+1} + \dfrac{(k+3)y^2}{k+1} = 1$ defines an ellipse as

**A.** $k > -\dfrac{1}{4}$

**B.** $k < -3$

**C.** $-3 < k < -\dfrac{1}{4}$

**D.** $k < -3$ or $k > -\dfrac{1}{4}$

**E.** $k > -1$

### Question 3

The graph of the function $g(x) = 4x - \dfrac{4}{x^2}$ has

**A.** two asymptotes and a local maximum at $x = \sqrt[3]{2}$.

**B.** two asymptotes and a local minimum at $x = -\sqrt[3]{2}$.

**C.** two asymptotes and a local maximum at $x = -\sqrt[3]{2}$.

**D.** one asymptote and a local maximum at $x = \sqrt[3]{2}$.

**E.** one asymptote and a local maximum at $x = -\sqrt[3]{2}$.

**Question 4**

The maximal domain of the function $h(x) = 1 - 2\cos^{-1}\left(\frac{x-1}{2}\right)$ is

**A.** $\left[-\frac{1}{2}, \frac{1}{2}\right]$

**B.** $[-1, 3]$

**C.** $[-1, 0]$

**D.** $[-2, 2]$

**E.** $[-1, 1]$

**Question 5**

The trigonometric equation $2\cos(2x) = \sin(x) + 1$ is equivalent to

**A.** $2\sin^2(x) + \sin(x) - 1 = 0$

**B.** $4\sin^2(x) - \sin(x) + 1 = 0$

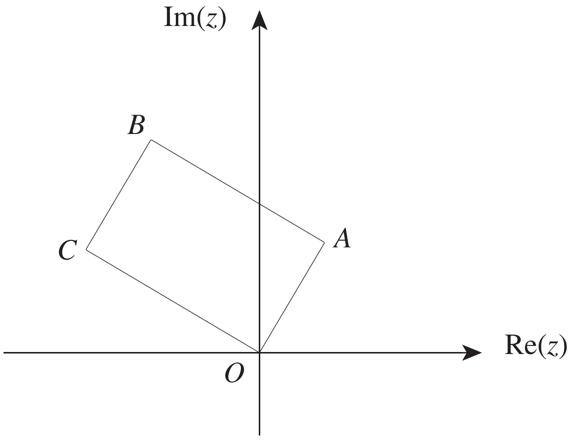
**C.** $4\sin^2(x) - \sin(x) - 1 = 0$

**D.** $4\sin^2(x) + \sin(x) + 1 = 0$

**E.** $4\sin^2(x) + \sin(x) - 1 = 0$

**Question 6**

In the Argand diagram below, $OABC$ is a rectangle where $OC = 2OA$. Vertex $A$ corresponds to the complex number $u$.




The complex number corresponding to vertex $C$ is

**A.** $2iu$

**B.** $2u$

**C.** $-iu$

**D.** $-2u$

**E.** $-2iu$

**Question 7**

In the complex plane, $\text{Re}\left(\frac{z-4}{z}\right) = 0$ where $z = x + yi$ forms a circle with

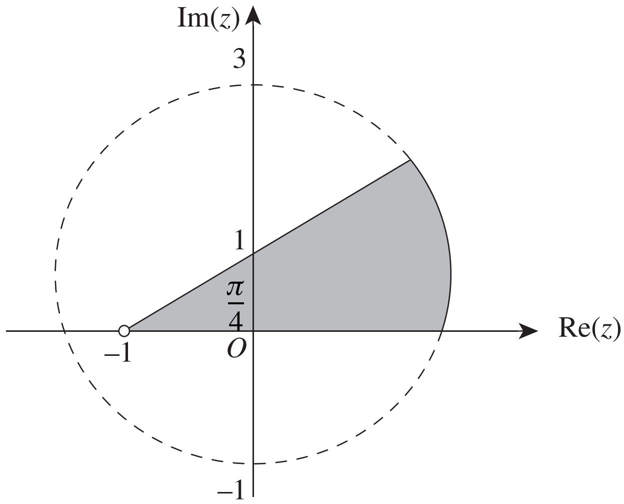
**A.** centre (2, 0) and radius 2.
**B.** centre (0, 2) and radius 2.
**C.** centre (–2, 0) and radius 2, excluding (0, 0).
**D.** centre (0, 2) and radius 2, excluding (0, 0).
**E.** centre (2, 0) and radius 2, excluding (0, 0).

**Question 8**

The number of solutions to the equation $z^{n-1} = i\bar{z}$, where $z \in C$ and $n$ is an integer greater than 2, is

**A.** $2n$
**B.** $n - 2$
**C.** $n - 1$
**D.** $n$
**E.** $n + 1$

**Question 9**




The shaded region of the complex plane is best described by

**A.** $\{z : |z - i| \leq 2\} \cap \left\{0 \leq \text{Arg}(z + 1) \leq \frac{\pi}{4}\right\}$

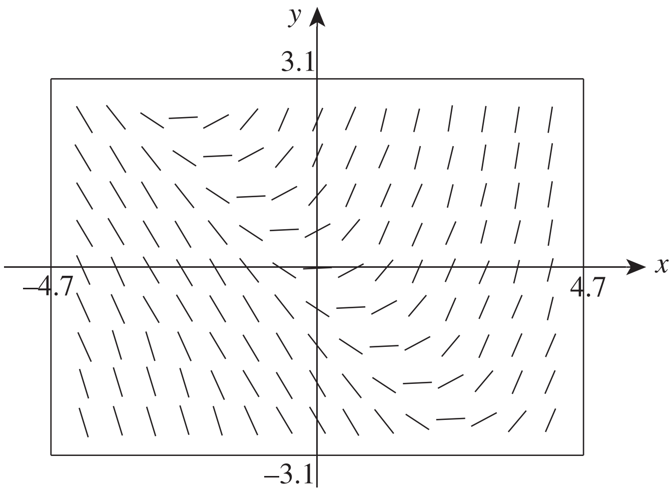
**B.** $\{z : |z + i| \leq 2\} \cap \left\{0 \leq \text{Arg}(z + 1) \leq \frac{\pi}{4}\right\}$

**C.** $\{z : |z - i| \leq 2\} \cap \left\{0 \leq \text{Arg}(z - 1) \leq \frac{\pi}{4}\right\}$

**D.** $\{z : |z + i| \leq 2\} \cap \left\{0 \leq \text{Arg}(z - 1) \leq \frac{\pi}{4}\right\}$

**E.** $\{z : |z - i| \leq 2\} \cap \left\{0 < \text{Arg}(z + 1) < \frac{\pi}{4}\right\}$

**Question 10**




The direction field shown above represents the differential equation

**A.** $\dfrac{dy}{dx} = x + y$

**B.** $\dfrac{dy}{dx} = x + 1$

**C.** $\dfrac{dy}{dx} = \dfrac{x}{y}$

**D.** $\dfrac{dy}{dx} = \log_e(y)$

**E.** $\dfrac{dy}{dx} = x^2$

**Question 11**

Using an appropriate substitution, $\displaystyle\int_2^6 (x + 1)\sqrt{x - 2}\,dx$ can be expressed as

**A.** $\displaystyle\int_0^4 (u + 3)\,du$

**B.** $\displaystyle\int_0^4 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$

**C.** $\displaystyle\int_2^6 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$

**D.** $\displaystyle\int_0^4 \left(u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\right) du$

**E.** $\displaystyle\int_0^6 \left(u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\right) du$

**Question 12**

Given the graph of a function, $y = f(x)$, which one of the following statements regarding the graph of the antiderivative function, $y = F(x)$, is **not** true?

**A.** If the graph of $f$ intersects the $x$-axis from above the $x$-axis to below the $x$-axis for increasing $x$, then the graph of $F$ has a local maximum.

**B.** If the graph of $f$ is above the $x$-axis for $a < x < b$, then the graph of $F$ has a positive gradient for $a < x < b$.

**C.** If the graph of $f$ has a local maximum at $x = a$, then the graph of $F$ has a non-stationary point of inflexion at $x = a$.

**D.** If the graph of $f$ is below the $x$-axis for $a < x < b$, then the graph of $F$ has a negative gradient for $a < x < b$.

**E.** If the graph of $f$ intersects the $x$-axis from below the $x$-axis to above the $x$-axis for increasing $x$, then the graph of $F$ has a local minimum.

**Question 13**

Let $\frac{dy}{dx} = \cos^2(x)$ and $(x_0, y_0) = (0, 1)$.

Using Euler's method, with a step size of 0.25, the value of $y$, correct to two decimal places, when $x = 0.75$ is

**A.** 1.48

**B.** 1.62

**C.** 1.68

**D.** 1.73

**E.** 1.81

**Question 14**

A curve satisfies the differential equation $\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2}$, $x \neq 1$ and $\frac{dy}{dx} = 0$ and $y = 0$ when $x = 0$.

The value of $y$ when $x = 2$ is

**A.** $\log_e(3) - 2$

**B.** $2 - \log_e(3)$

**C.** $3 - \log_e(2)$

**D.** $2 + \log_e(3)$

**E.** $\log_e\left(\frac{2}{3}\right)$

**Question 15**

A unit vector which is perpendicular to $2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ is

**A.** $\frac{1}{\sqrt{2}}(\underset{\sim}{j} - \underset{\sim}{k})$

**B.** $-\frac{1}{\sqrt{2}}(\underset{\sim}{j} + \underset{\sim}{k})$

**C.** $-\frac{1}{\sqrt{2}}(\underset{\sim}{i} + \underset{\sim}{k})$

**D.** $\frac{1}{\sqrt{2}}(\underset{\sim}{i} + \underset{\sim}{k})$

**E.** $\underset{\sim}{j} + \underset{\sim}{k}$

**Question 16**

If $\underset{\sim}{a} = 3\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} - 3\underset{\sim}{k}$, then the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is

**A.** $0$

**B.** $\frac{1}{5}$

**C.** $\frac{21}{\sqrt{43}}$

**D.** $\frac{21}{5}$

**E.** $\frac{3}{5}$

**Question 17**

The velocity of a particle at time $t$ is given by $\dot{\underset{\sim}{r}}(t) = (4 - 2\cos(t))\underset{\sim}{i} - 3\sin(t)\underset{\sim}{j}$. When $t = 0$, $\underset{\sim}{r} = 2\underset{\sim}{j}$.

The position vector, $\underset{\sim}{r}(t)$, at $t = \frac{3\pi}{2}$, is

**A.** $3(2\pi + 1)\underset{\sim}{i} - \underset{\sim}{j}$

**B.** $6\pi\underset{\sim}{i} + \underset{\sim}{j}$

**C.** $2(\pi - 1)\underset{\sim}{i} - \underset{\sim}{j}$

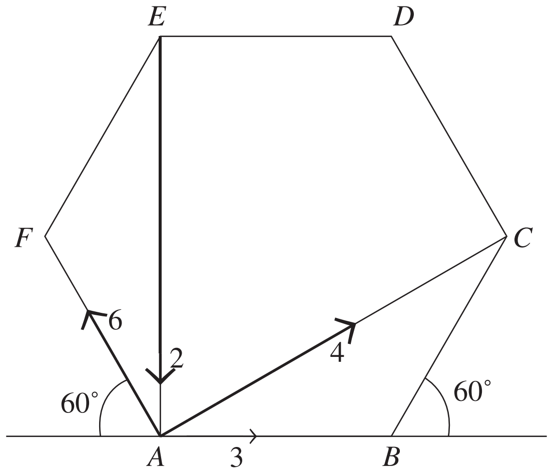
**D.** $3(2\pi - 1)\underset{\sim}{i} - \underset{\sim}{j}$

**E.** $-3\underset{\sim}{i} + 2\underset{\sim}{j}$

**Question 18**

*ABCDEF* is a regular hexagon. Four forces of magnitude 3 N, 4 N, 2 N and 6 N respectively act on a particle at *A* as shown below.




The magnitude of the resultant force acting on the particle is

**A.** $\sqrt{16\cos^2(30°) + 36\sin^2(60°)}$

**B.** $\sqrt{4\cos(30°) + 6\sin(60°)}$

**C.** $4\cos(30°) + 6\sin(60°)$

**D.** $\sqrt{16\cos^2(60°) + 36\sin^2(30°)}$

**E.** $\sqrt{36\sin^2(60°) - 16\cos^2(30°)}$

**Question 19**

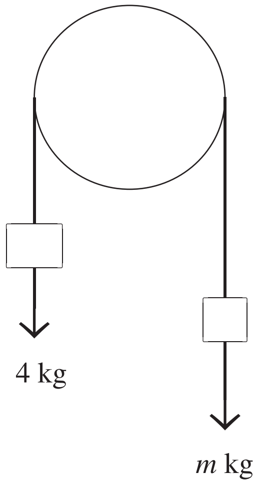
A particle of mass 2 kg moves in a straight line from rest under the action of a resultant force *R* newtons, where $R = 4\pi\sin\left(\frac{\pi t}{3}\right)$ after *t* seconds.

The particle's velocity, in m/s, at $t = 3$, is

**A.** 0
**B.** 6
**C.** 12
**D.** $12\pi$
**E.** 24

**Question 20**

Two particles of mass 4 kg and *m* kg respectively where $m > 4$ are connected by a light inextensible string passing over a smooth fixed pulley. The system is released from rest.




Given that the tension in the string is 5.5*g* newtons, the value of *m* is

**A.** 0.8
**B.** 2
**C.** 2.75
**D.** 4.4
**E.** 8.8

**Question 21**

A particle of mass 10 kg travels in a straight line with constant acceleration. Its initial velocity is 12 m/s and it travels a distance of 80 metres in 5 seconds.

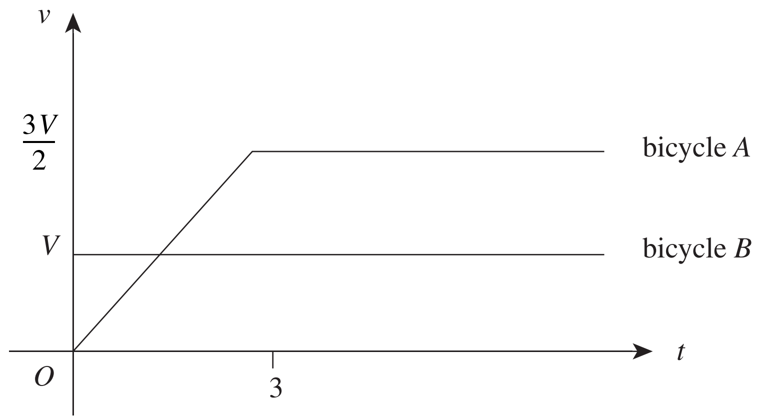
The change in momentum of the particle in kg m/s, in the direction of its motion, is

**A.** 40
**B.** 80
**C.** 120
**D.** 160
**E.** 200

**Question 22**

The velocity-time graph below shows bicycle *A* pursuing bicycle *B*. Both bicycles started at the same position at time $t = 0$ where $t$ is measured in minutes.




Bicycle *A* and bicycle *B* will again share the same position after travelling

**A.** 2 minutes
**B.** 3 minutes
**C.** 4 minutes
**D.** 4.5 minutes
**E.** 5 minutes

## SECTION 2

**Instructions for Section 2**

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ m/s$^2$, where $g = 9.8$.

### Question 1

Two particles, $A$ and $B$, move in the cartesian plane so that at any time $t \geq 0$, they have position vectors:

$$\underset{\sim}{r}_A(t) = 4t\underset{\sim}{i} + 2t\underset{\sim}{j}$$

$$\underset{\sim}{r}_B(t) = (8 - 8\sin(nt))\underset{\sim}{i} + 8\cos(nt)\underset{\sim}{j}, \text{ where } n \text{ is a positive constant}$$

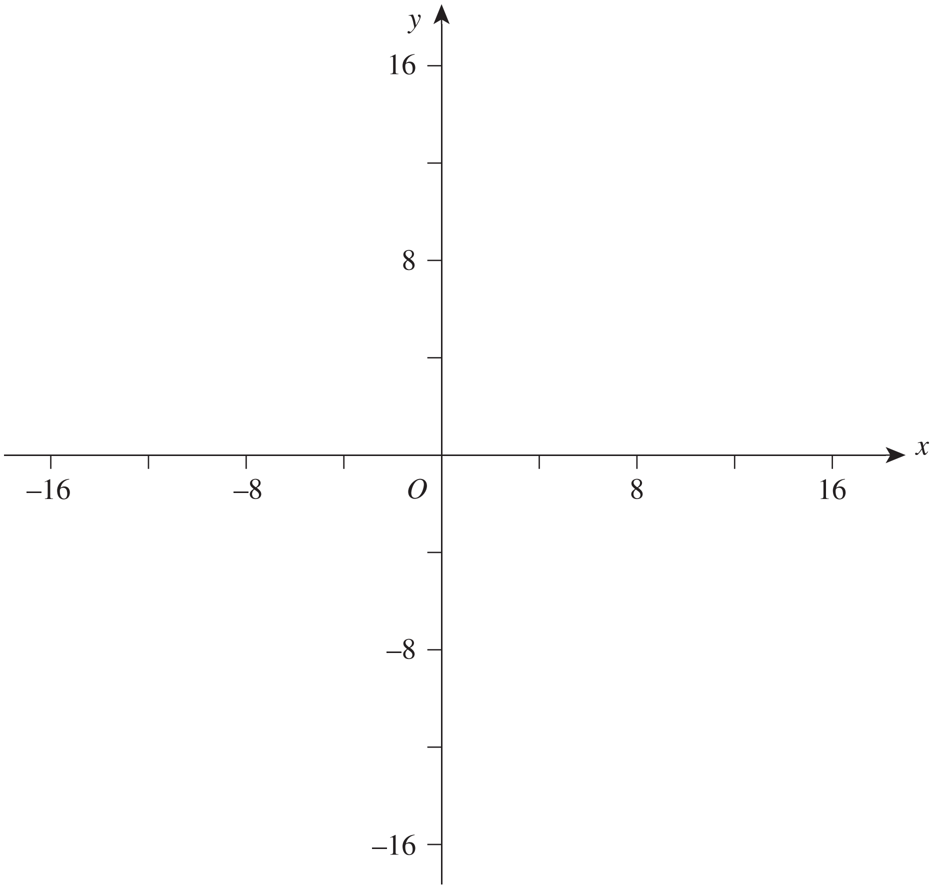
**a.** Show that the cartesian equation of particle $A$ is given by $y = \dfrac{x}{2}$.

1 mark

**b.** Show that the cartesian equation of particle $B$ is given by $(x - 8)^2 + y^2 = 64$.

2 marks

**c.** On the set of axes provided, sketch the path of each particle. Indicate each particle's direction of motion. Justify your answer.




3 marks

**d.** Find the coordinates of the points where the paths of particle *A* and particle *B* meet.

2 marks

**e.** State the value of $t$ for which a collision will occur and hence, find the smallest value of $n$, correct to two decimal places, for which particle $A$ and particle $B$ will collide.

4 marks

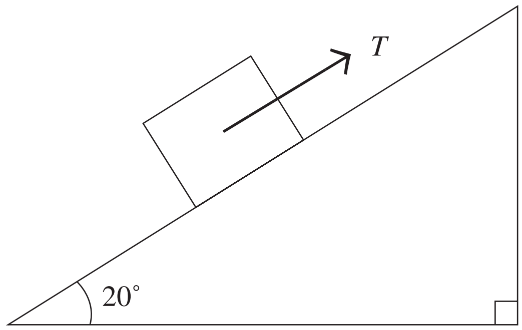
For particle $B$, the velocity vector is $\underset{\sim}{v}_B(t)$ and the acceleration vector is $\underset{\sim}{a}_B(t)$.

**f.** Show that $\underset{\sim}{a}_B(t)$ is always perpendicular to $\underset{\sim}{v}_B(t)$.

3 marks
Total 15 marks

**Question 2**

A block of mass 8 kg sits on a rough plane which is inclined at 20° to the horizontal. The coefficient of friction is 0.3. A rope is attached to the block and is parallel to the plane. The tension in the rope is of magnitude $T$ newtons.




The block is initially held in equilibrium by the rope.

**a.** Show that the normal reaction force between the block and the plane is 73.7 newtons, correct to one decimal place.

2 marks

**b.** Find, correct to one decimal place, the magnitude of the maximum value of the frictional force, $F$, which can act on the block.

1 mark

**c.** Find, correct to one decimal place, the least possible tension in the rope that prevents the block from moving down the plane.

3 marks

The rope is released and the block slides down the plane.

**d.** Find, correct to one decimal place, the acceleration of the block.

3 marks
Total 9 marks

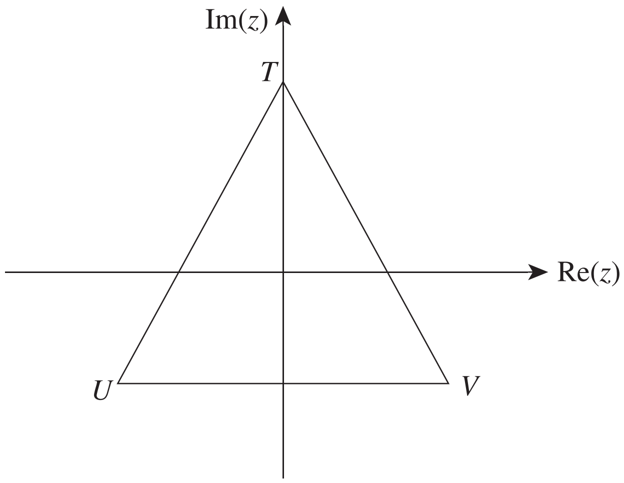
**Question 3**

Consider two points in the complex plane represented by $z$ and $z\text{cis}(\theta)$ where $\theta$ is positive.

**a.** Describe the geometrical relationship between $z$ and $z\text{cis}(\theta)$.

1 mark

In the complex plane, the roots of the equation $z^3 = -8i$ where $z \in C$ form the vertices of triangle *TUV*.




Let the roots of the equation be *t*, *u* and *v*, where vertex *T* corresponds to *t*, vertex *U* corresponds to *u* and vertex *V* corresponds to *v*.

**b.** Express *t*, *u* and *v* in cartesian form.

2 marks

**c.** Show that *TUV* is an equilateral triangle.

1 mark

The relation $S = \{z : |z| = k\}$ describes a circle that passes through *T*, *U* and *V*.

**d.** State the value of *k*.

1 mark

**e.** Verify that $\bar{v} \in S$.

2 marks

The relation *S* can also be described by $\{z : (z - a)(\bar{z} - a) = b\}$, where *a* and *b* are integers.

**f.** Find the values of *a* and *b*.

3 marks
Total 10 marks

**Question 4**

Consider the curve with equation $y = \frac{x^2 - 16}{4}, 4 \le x \le 8$.

A bowl is constructed by rotating this curve through 360° about the $y$-axis. All length measurements are in centimetres.

**a.** Show that the height of the bowl is 12 cm.

1 mark

The bowl is filled with hot water to a depth of $h$ cm, where $0 \le h \le 12$.

**b.** Show that the volume, $V$ cm$^3$, of hot water in the bowl is given by $V = 4\pi\left(\frac{h^2}{2} + 4h\right)$.

2 marks

It is known that $120\pi$ cm$^3$ of water is required for the bowl to be exactly one-quarter full.

**c.** Find the exact depth, $h$ cm, of water in the bowl.

2 marks

Water is poured into the bowl at a rate of $50 \text{ cm}^3$ per second.

**d.** Find the exact rate at which the depth of water rises when the bowl is one-quarter full.

3 marks

Newton's Law of Cooling states that the rate of cooling of a body is proportional to the excess of its temperature, $T$ °C, above that of its surroundings.

The temperature of the water poured into the bowl was 70°C. The temperature of the surroundings, $T_S$ °C, is 20°C.

After 10 minutes, the water temperature was 55°C.

**e.** Find, correct to one decimal place, the water temperature after a further 15 minutes.

5 marks
Total 13 marks

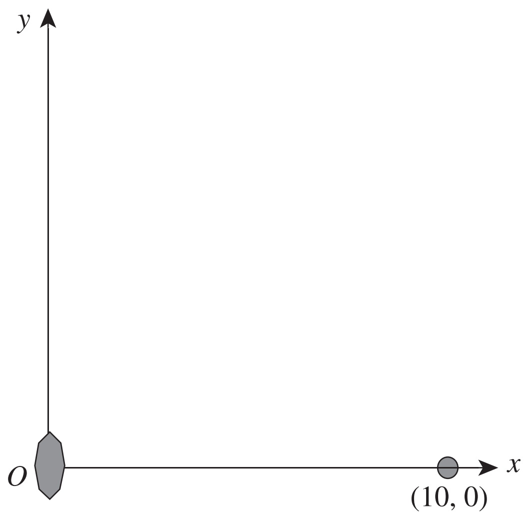
  

**Question 5**

A speedboat and a water skier are positioned alongside a river bank. The skier is connected to the boat by a tightly stretched rope of fixed length 10 metres.

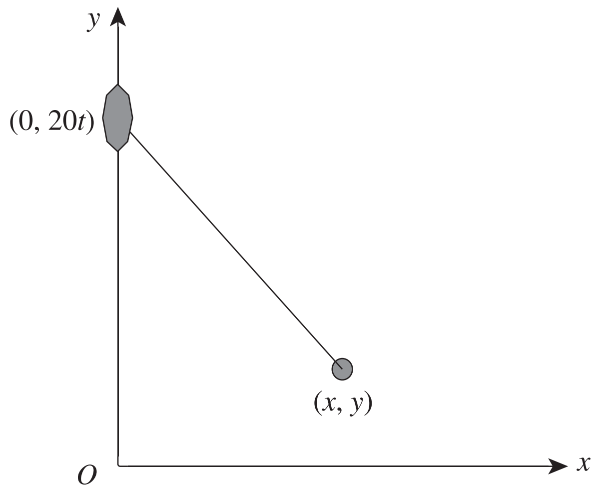
In the diagrams below, the horizontal axis represents the river bank and the vertical axis represents the direction perpendicular to the river bank.

At $t = 0$, the boat is at $O$ and the skier is at $(10, 0)$.




The boat moves at a constant velocity 20 m/s in a direction perpendicular to the bank.

At time $t$ seconds, the boat is at $(0, 20t)$ and the skier is at $(x, y)$.




During the skier's motion, the rope maintains a constant length and is always tangent to the skier's path.

**a.** Show that $\frac{dy}{dx} = \frac{y - 20t}{x}$.

1 mark

**b.** Hence show that $\frac{dy}{dx} = \frac{-\sqrt{100 - x^2}}{x}$.

2 marks

**c.** Given that $u^2 = 100 - x^2$ and $2u\,du = -2x\,dx$, show that $-\int \frac{\sqrt{100 - x^2}}{x} dx = \int \frac{u^2}{100 - u^2} du$.

3 marks

**d.** Express $\int \frac{u^2}{100 - u^2} du$ in the form $\int \left( m + \frac{n}{10 - u} + \frac{n}{10 + u} \right) du$, where $m$ and $n$ are integers.

3 marks

The path of the water-skier is $y(x)$.

**e.** Find $y(x)$ and express it in the form $y(x) = f(x) + n\log_e\left( \frac{g(x)}{h(x)} \right)$, where $n$ is the integer found in **part d.**

2 marks
Total 11 marks

**END OF QUESTION AND ANSWER BOOKLET**