Year 2011

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education 2011

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 36 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple- choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

A hyperbola has asymptotes $y = 2x + 5$ and $y = 1 - 2x$. Its domain is R and its range is $(-\infty, -1] \cup [7, \infty)$. Its equation could be

A. $\dfrac{(x+1)^2}{4} - \dfrac{(y-3)^2}{16} = 1$

B. $\dfrac{(y-3)^2}{4} - (x+1)^2 = 1$

C. $\dfrac{(y-3)^2}{16} - \dfrac{(x+1)^2}{4} = 1$

D. $\dfrac{(x-1)^2}{4} - \dfrac{(y+3)^2}{16} = 1$

E. $(x-1)^2 - \dfrac{(y+3)^2}{4} = 1$

Question 2

$z = \text{cis}(\theta)$ is a root of a quadratic polynomial with real coefficients.
The quadratic could be

A. $z^2 + 2z\cos(\theta) + 1 = 0$

B. $z^2 + 2z\cos(\theta) - 1 = 0$

C. $z^2 - 2z\cos(\theta) + 1 = 0$

D. $z^2 - 2z\sin(\theta) + 1 = 0$

E. $z^2 + 2z\sin(\theta) - 1 = 0$

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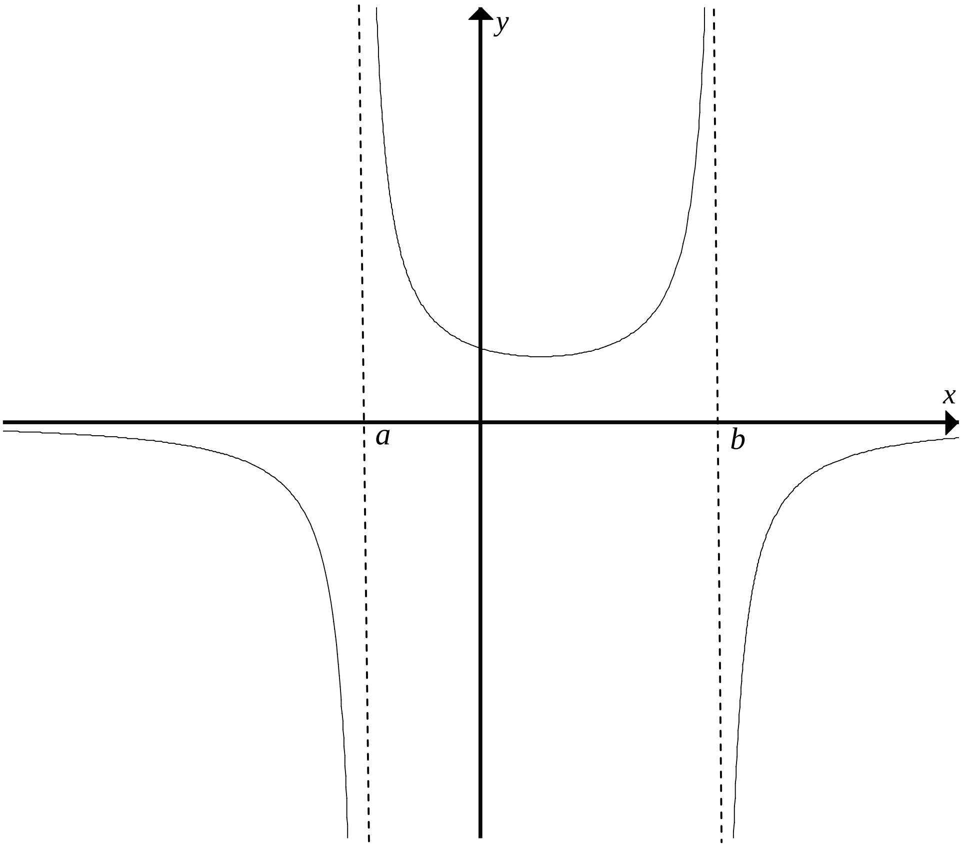
Question 3



The graph shown above, could be that of a function f whose rule is

A. $f(x)=\dfrac{1}{x^2+(a-b)x-ab}$

B. $f(x)=\dfrac{1}{x^2-(a+b)x+ab}$

C. $f(x)=\dfrac{1}{ab-(a-b)x-x^2}$

D. $f(x)=\dfrac{1}{ab-(a+b)x-x^2}$

E. $f(x)=\dfrac{1}{(a+b)x-ab-x^2}$

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Question 4

If $c = a + bi$, which of the following, represents a circle with centre (a,b) and radius r ?

A. $\{z : |z - c| - |\bar{z} - \bar{c}| = r\}$

B. $\{z : |z - c| + |\bar{z} - \bar{c}| = r\}$

C. $\{z : (z - c)(\bar{z} - \bar{c}) = r\}$

D. $\{z : (z - c)(\bar{z} - \bar{c}) = r^2\}$

E. $\{z : (z - \bar{c})(\bar{z} - c) = r^2\}$

Question 5

If $z = a + 1 + ai$ where a is a non-zero real constant, then $\dfrac{2a}{1 - \bar{z}}$ is equal to

A. $\sqrt{2}\,\text{cis}\left(-\dfrac{3\pi}{4}\right)$

B. $\sqrt{2}\,\text{cis}\left(-\dfrac{\pi}{4}\right)$

C. $\sqrt{2}\,\text{cis}\left(\dfrac{\pi}{4}\right)$

D. $\sqrt{2}\,\text{cis}\left(\dfrac{3\pi}{4}\right)$

E. $\dfrac{\sqrt{2}a}{a+1}\text{cis}\left(\dfrac{\pi}{4}\right)$

Question 6

The graph of $y = \dfrac{x^n + a}{x}$ where a and n are real constants, has two turning points, then

A. $n = 2$ and $a > 0$

B. $n = 2$ and $a < 0$

C. $n = 1$ and $a > 0$

D. $n = 3$ and $a < 0$

E. $n = 4$ and $a < 0$

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Question 7

If $u = 4\,\text{cis}(\theta)$, $v = r\,\text{cis}\left(-\frac{2\pi}{3}\right)$ and $uv = 12i$ then

A. $r = 8 \quad \theta = \frac{7\pi}{6}$

B. $r = 8 \quad \theta = \frac{2\pi}{3}$

C. $r = 3 \quad \theta = \frac{2\pi}{3}$

D. $r = 3 \quad \theta = -\frac{5\pi}{6}$

E. $r = -3 \quad \theta = -\frac{\pi}{6}$

Question 8

If $u = \text{cis}(\alpha)$, and $0 < \alpha < \frac{\pi}{2}$, then which of the following is **false?**

A. $\text{Arg}(iu) = \alpha + \frac{\pi}{2}$

B. $\text{Arg}(\overline{u}) = -\alpha$

C. $\text{Arg}(i^2 u) = -\pi + \alpha$

D. $\text{Arg}(u^2) = 2\alpha$

E. $\text{Arg}\left(\frac{1}{u}\right) = \frac{1}{\alpha}$

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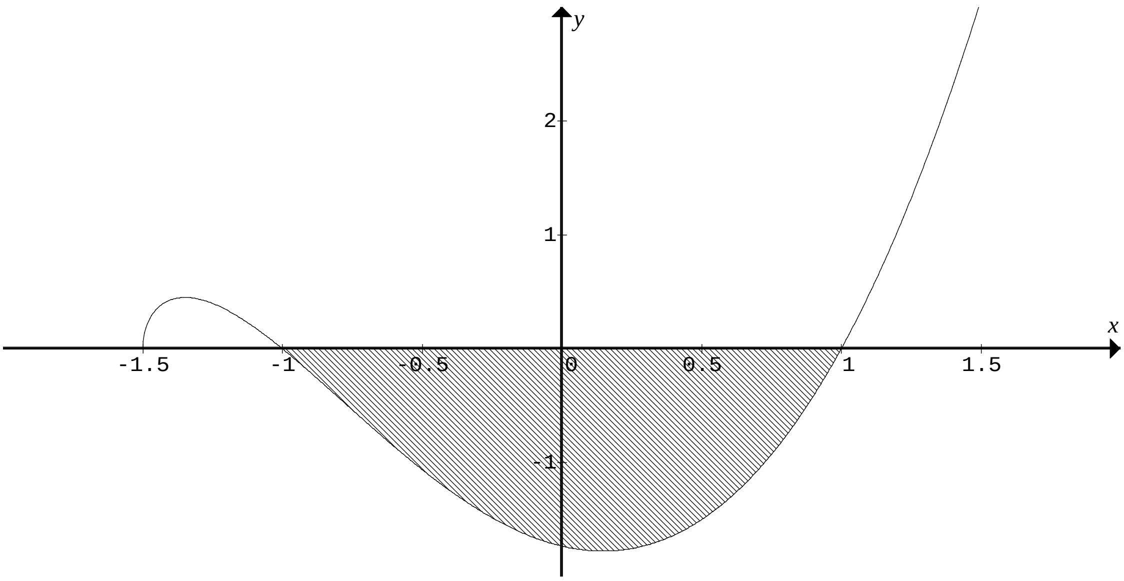
Question 9

Part of the graph with the equation $y=\left(x^2-1\right)\sqrt{2x+3}$ is shown below.



The shaded area that is the area bounded by the curve and the *x*-axis, can be expressed as

A. $\frac{1}{4}\int_0^1 (5-u)(u-1)\sqrt{u}\,du$

B. $\frac{1}{8}\int_1^5 (5-u)(u-1)\sqrt{u}\,du$

C. $\frac{1}{8}\int_1^5 (u-5)(u-1)\sqrt{u}\,du$

D. $\frac{1}{2}\int_1^5 (u-5)(u-1)\sqrt{u}\,du$

E. $\frac{1}{4}\int_0^5 (5-u)(u+1)\sqrt{u}\,du$

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Question 10

A, *B* and *C* are three points in space. To prove that *ABC* is a right-angled isosceles triangle, it is necessary to show that

A. $\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right| = \overrightarrow{AB}.\overrightarrow{AC}$ and $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$

B. $\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right| = \sqrt{2}\,\overrightarrow{AB}.\overrightarrow{AC}$ and $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$

C. $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$ and $\overrightarrow{AB}.\overrightarrow{AC} = 0$

D. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \underset{\sim}{0}$ and $\overrightarrow{BA}.\overrightarrow{BC} = 0$

E. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \underset{\sim}{0}$ and $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$

Question 11

A particle moves so that its position vector at a time *t* is given by
$\underset{\sim}{r}(t) = a\cos(nt)\,\underset{\sim}{i} + b\sin(mt)\,\underset{\sim}{j}$ where *m* and *n* are positive integers, and *a* and *b* are real positive constants. Which of the following is true?

A. If $m = n$ and $a = b$ the path of the particle is an ellipse.

B. If $m \neq n$ and $a = b$ the path of the particle is an ellipse.

C. If $m = n$ and $a \neq b$ the path of the particle is a circle.

D. If $n = 2m$ the path of the particle is a parabola.

E. If $m = 2n$ the path of the particle is a parabola.

Question 12

A unit vector in the opposite direction to $2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ is

A. $-2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$

B. $\frac{1}{2}\left(-2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

C. $\frac{1}{3}\left(-2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

D. $\frac{1}{4}\left(-2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

E. $\frac{1}{5}\left(-2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

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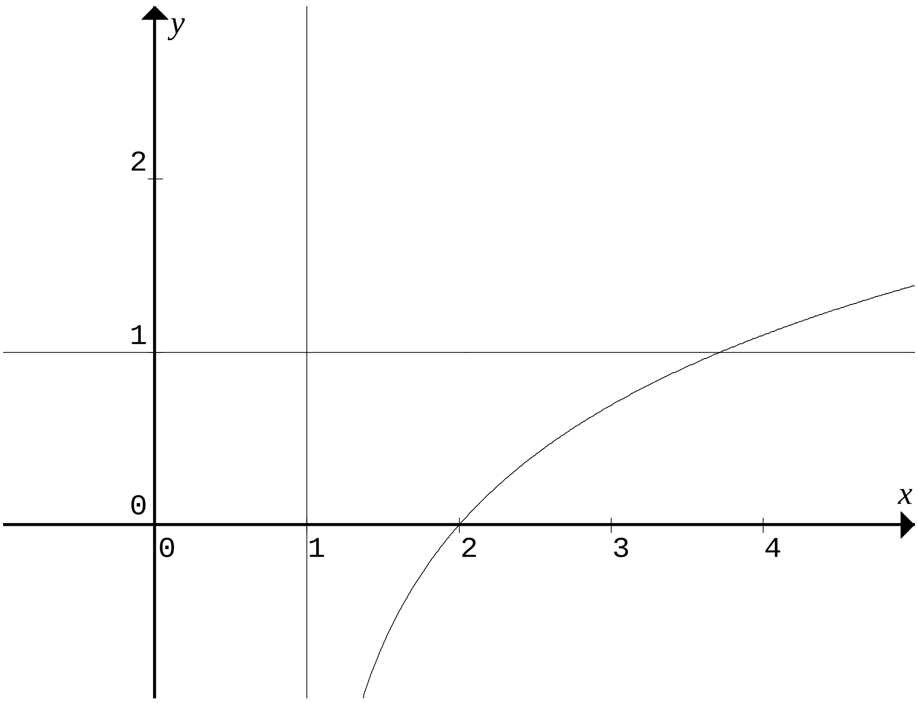
Question 13

The diagram below, show part of the graph of $y = \log_e(x-1)$. The area between the *x*-axis, the curve $y = \log_e(x-1)$, $x = 1$ and $y = 1$ is rotated about the *y*-axis, to form a volume of revolution. The volume is equal to



A. $\pi\int_{1}^{2}\left(\log_e(x-1)^2\right)dx$

B. $\pi\int_{1}^{2+e}\left(\log_e(x-1)^2\right)dx$

C. $\pi\int_{1}^{2+e}\left(\log_e(x-1)^2-1\right)dx$

D. $\pi\int_{0}^{1}e^{2x}dx$

E. $\pi\int_{0}^{1}\left(e^{2x}+2e^{x}\right)dx$

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Question 14

If $\frac{dx}{dt} = \cos\left(\frac{1}{t}\right)$ and $x = 3$ when $t = 1$, then the value of x when $t = 2$ can be found by evaluating

A. $\int_1^2 \cos\left(\frac{1}{u}\right) du$

B. $-\int_1^2 \sin\left(\frac{1}{u}\right) du$

C. $3 - \int_1^2 \sin\left(\frac{1}{u}\right) du$

D. $\int_1^2 \cos\left(\frac{1}{u}\right) du - 3$

E. $\int_1^2 \cos\left(\frac{1}{u}\right) du + 3$

Question 15

A hot air balloon is accelerating upwards with an acceleration of 1 m/s^2. At a particular instant it is 250 metres above ground level and rising upwards with a speed of 3 m/s. A small stone falls from the balloon to the ground. Assuming air resistance is negligible, the time taken, for the stone to hit the ground in seconds, is closest to

A. 7.89

B. 7.54

C. 7.46

D. 7.20

E. 7.14

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Question 16

A tank initially holds 100 litres of a solution, in which 2 kilograms of salt has been dissolved. Another solution containing 10 kilograms of salt per litre is poured into the tank at a rate of 3 litres per minute. The well-stirred mixture leaves the tank at a rate of 2 litres per minute. If Q kilograms is the amount of salt in the tank a time t minutes, then $Q = C(100+t)^n + 10(100+t)$, where C is a constant. The value of n is equal to

A. -3

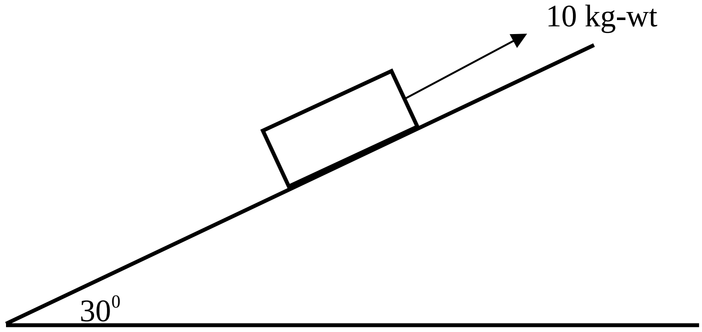
B. -2

C. -1

D. 2

E. 3

Question 17



A box of mass 10 kg is at rest on a plane inclined at angle of 30^0 to the horizontal. A force of magnitude 10 kg-wt acting up and parallel to the plane is applied to the box. For equilibrium to be maintained, the co-efficient of friction between the box and the plane must be

A. at least $\frac{\sqrt{3}}{3}$

B. less than $\frac{\sqrt{3}}{3}$

C. at least $\frac{g-2}{g\sqrt{3}}$

D. less than $\frac{g-2}{g\sqrt{3}}$

E. at least $5g\sqrt{3}$

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Question 18

A body of mass *m* kg moves in a straight line, its velocity is *v* ms^{-1} at a time *t* seconds. The force acting on the body is $f(t)$ newtons. Given that $v = v_1$ when $t = t_1$ and $v = v_2$ when $t = t_2$, it follows that

A. $mv_2 - mv_1 = f(t_2) - f(t_1)$

B. $mv_2 - mv_1 = \int_{t_1}^{t_2} f(t)\,dt$

C. $v_2 - v_1 = m\int_{t_1}^{t_2} f(t)\,dt$

D. $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = f(t_2) - f(t_1)$

E. $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \int_{t_1}^{t_2} f(t)\,dt$

Question 19

A car of mass *m* kg is travelling on a level roadway. The engine exerts a constant propulsive force of *F* newtons and the total resistance to the motion of the car is kv^3 newtons, where *k* is positive constant and *v* is its speed in m/s. The car moves from rest, the distance travelled in metres until it obtains a speed of *V*, is given by

A. $\dfrac{V^2}{2(F - kV^3)}$

B. $\dfrac{mV^2}{2(F - kV^3)}$

C. $\dfrac{m}{2}\displaystyle\int_0^V \frac{v^2}{F - kv^3}\,dv$

D. $\displaystyle\int_0^V \frac{mv}{F - kv^3}\,dv$

E. $\displaystyle\int_0^V \frac{v}{F - kv^3}\,dv$

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Question 20

A particle moves in a straight line. When its displacement from a fixed origin is *x* metres, its acceleration is given by $\dfrac{3bx-2b^2}{x^3}$ m/s^2, where *b* is a non-zero real constant. Given that the particle comes to rest at $x=b$, then it is also at rest at

A. $x=\dfrac{b}{2}$

B. $x=2b$

C. $x=\dfrac{b}{3}$

D. $x=\dfrac{2b}{3}$

E. $x=0$

Question 21

If $f(x)=\cos^{-1}(ax-1)+a\sin^{-1}\left(\sqrt{x}\right)$ and $f'(x)=0$ for $x\in(0,b)$, then

A. $a=1$ and $b=1$

B. $a=1$ and $b=\dfrac{1}{2}$

C. $a=2$ and $b=\infty$

D. $a=2$ and $b=1$

E. $a=2$ and $b=\dfrac{1}{2}$

Question 22

A particle moves, so that at a time *t* seconds, its velocity *v* m/s is given by $v(t)=20\tan^{-1}\left(\dfrac{t}{4}-1\right)$. Over the first 14 seconds, the distance travelled by the particle in metres is closest to

A. 71

B. 124

C. 194

D. 7,088

E. 11,111

END OF SECTION 1

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SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

Let $u = a + bi$ be a complex number where *a* and *b* are positive real numbers.

a. Show that $|u|^2 = u\bar{u}$, where $\bar{u}$ is the conjugate of u.

1 mark

b. The cubic $z^3 + pz^2 + qz + s = 0$ has roots $u, \bar{u}$ and 1.
Express *p*, *q* and *s* in terms of *a* and *b*.

3 marks

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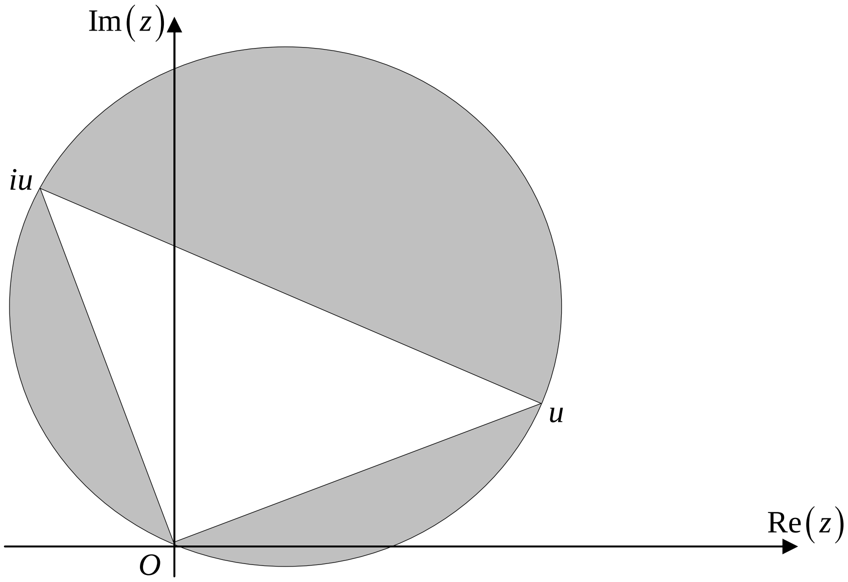
c. Express iu in terms of a and b.

1 mark

The diagram shows a circle, in an argand diagram, which passes through the points, O, u and iu, where O is the origin.



d. Show that $|u|^2 + |iu|^2 = |u - iu|^2$
Explain this result geometrically.

3 marks

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e. The circle can be expressed in the form $\{z : |z - c| = r\}$
Express the complex number c and r in terms of a and b.

2 marks

f. Find in terms of a and b, the exact area of the shaded region.

2 marks
Total 12 marks

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Question 2

a. If $a > b > 0$ and $0 < \theta < 2\pi$, show that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $P\left(a\sec(\theta), b\tan(\theta)\right)$ is given by

$$y = \frac{bx}{a\sin(\theta)} - \frac{b\cos(\theta)}{\sin(\theta)}$$

2 marks

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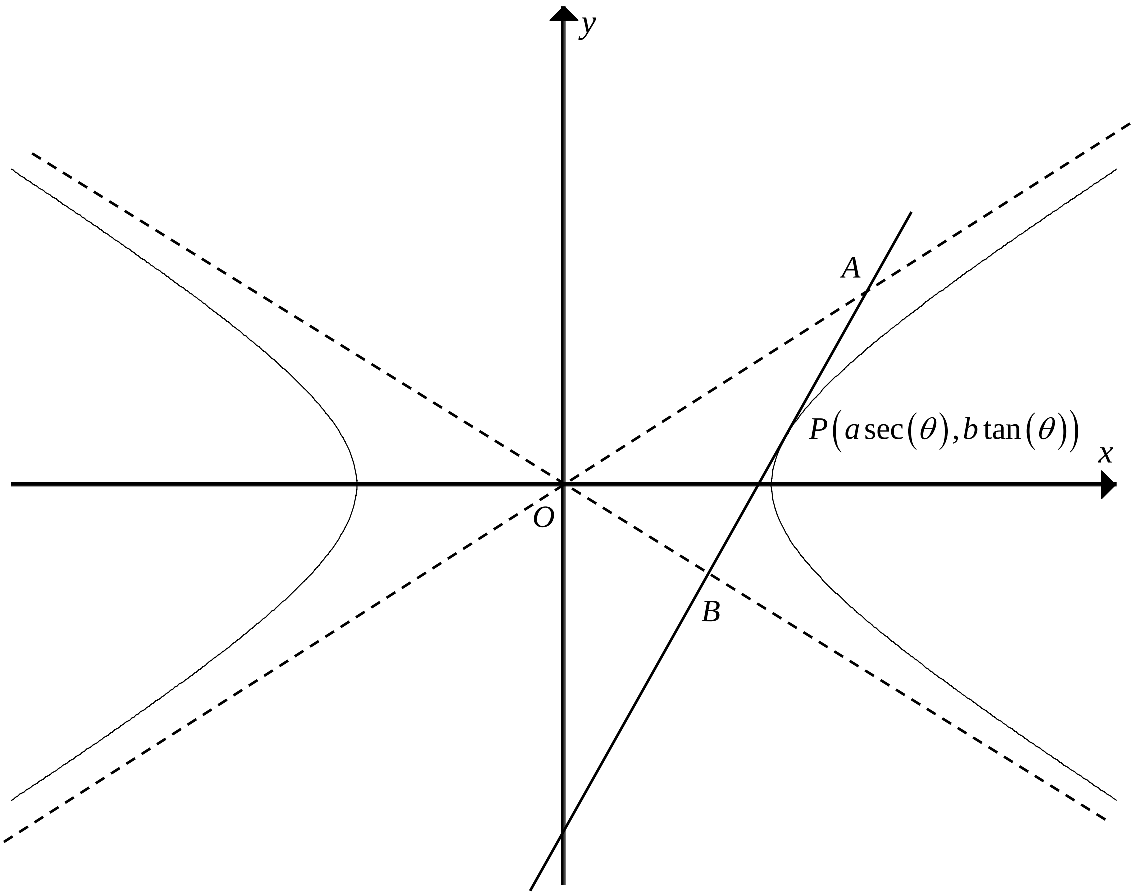
b. The tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at P, crosses the asymptotes to the hyperbola at two points A and B as shown below.

Show that the coordinates are given by

$$A\left(\frac{a\cos(\theta)}{1-\sin(\theta)},\frac{b\cos(\theta)}{1-\sin(\theta)}\right) \text{ and } B\left(\frac{a\cos(\theta)}{1+\sin(\theta)},\frac{-b\cos(\theta)}{1+\sin(\theta)}\right)$$



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4 marks

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c. Use vectors to show that the angle *AOB*, where *O* is the origin is given by $\cos^{-1}\left(\dfrac{a^2-b^2}{a^2+b^2}\right)$

4 marks

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d. Hence find the area of the triangle *AOB*, in terms of a and b only.

2 marks
Total 12 marks

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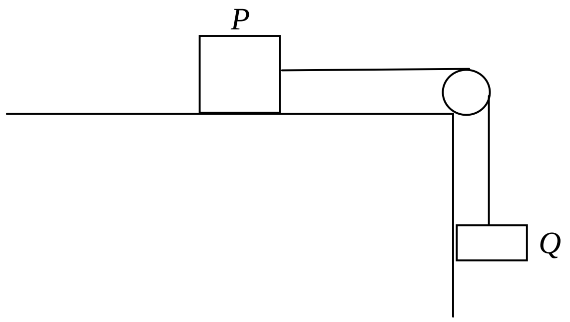
Question 3

Two particles P and Q are connected by a light inextensible string which passes over a smooth fixed pulley, as shown in the diagram below.



The particle P is of mass $2m$ kg and the particle Q has a mass of m kg. The particle P is in contact with a horizontal surface and particle Q hangs vertically. The surface can be modelled as either smooth or rough.

a. If the surface is smooth and the system is released from rest, with the string taut, the particle Q moves vertically downwards. After half a second, the particle Q hits the ground, find its initial height above the ground, giving your answer in metres correct to two decimal places.

3 marks

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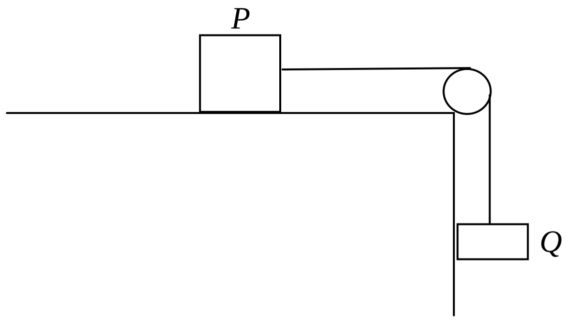
b. If the surface is rough, the coefficient of friction between *P* and the surface is μ. The particle *Q* hangs with the string taut. On the diagram below, mark in and label all the forces acting on the particles.



1 mark

i. Find the tension in the string in newtons, if the particle *Q* is at rest.

1 mark

ii. Find the possible range of values of μ.

4 marks
Total 9 marks

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Question 4

Given the points $A(2,1,4)$, $B(\alpha,-2,-1)$ and $C(-5,8,11)$ relative to an origin O.
Find the value of the scalar α in each of the cases below, if

a. the vectors $\overrightarrow{OA}$ is perpendicular to the vector $\overrightarrow{OB}$.

1 mark

b. the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ form a linearly dependent set of vectors.

3 marks

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c. The vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are equal in length.

2 marks

d. the scalar resolute of $\overrightarrow{OA}$ parallel to $\overrightarrow{OB}$ is equal to $-\frac{10}{3}$

2 marks

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e. The vector $\overrightarrow{OB}$ makes an angle of $\cos^{-1}\left(-\frac{2}{3}\right)$ with the x-axis.

2 marks
Total 10 marks

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Question 5

A golf ball is hit in a vertical plane and has a position vector given by $\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j}$ where $y \geq 0$ for $0 \leq t \leq T$ and t is the time in seconds. $\underset{\sim}{i}$ is a unit vector of one metre horizontally forward in the x direction and $\underset{\sim}{j}$ is a unit vector of one metre vertically upwards, in the y direction, above ground level. Initially the golf ball is hit from ground level, with a speed of V m/s at an angle of α to the horizontal. Its velocity vector at a time t is given by $\dot{\underset{\sim}{r}}(t) = \left(V\cos(\alpha) - k\,x(t)\right)\underset{\sim}{i} + \left(V\sin(\alpha) - gt\right)\underset{\sim}{j}$ where k is a positive constant.

a. Show by integration that $x(t) = \dfrac{V\cos(\alpha)}{k}\left(1 - e^{-kt}\right)$

4 marks

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b. Show that the golf ball hits the ground after a time of $T = \dfrac{2V\sin(\alpha)}{g}$

1 mark

c. Find in terms of V, α and k, the angle at which the golf ball strikes the ground.

3 marks

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d. Given that $\ddot{\underset{\sim}{r}}(t) = -k\,\dot{x}(t)\,\underset{\sim}{i} - g\,\underset{\sim}{j}$ explain the significance of these terms.

1 mark

Suppose now we are given that $V = 49$ $\alpha = 25^0$ and $k = 0.2$

e. Find the range, that is, the horizontal distance that the golf ball travels before hitting the ground. Give your answer correct to two decimal places.

1 mark

f. Find the time when the golf ball reaches its maximum height. Find the maximum height reached and the horizontal distance travelled at this time. Give all answers correct to two decimal places.

3 marks

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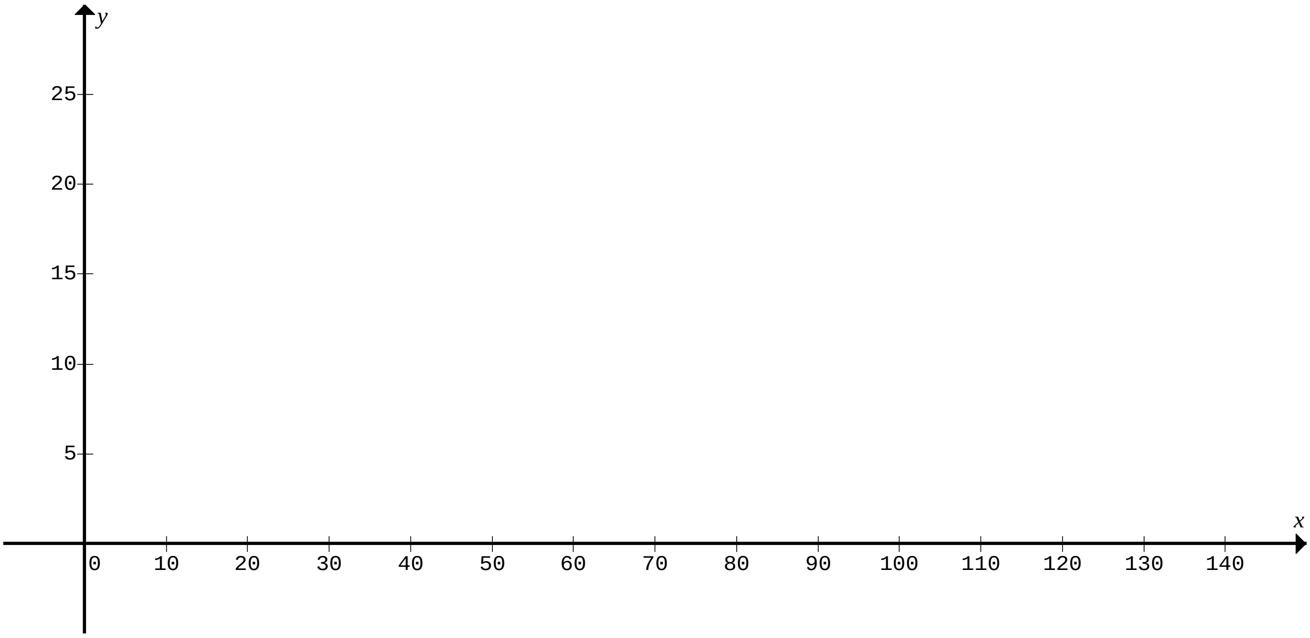
g. Draw the path of the golf ball *y* versus *x*, on the axes below.



2 marks
Total 15 marks

END OF EXAMINATION

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EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

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SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.   This formula sheet is provided for your reference.

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<u>Specialist Mathematics Formulas</u>

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

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function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \,.\, \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u + v)t$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

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Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c,\ \text{ for }\ x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET

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ANSWER SHEET

STUDENT NUMBER

Figures										Letter
Words										

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SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**

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