Year 2011

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2011

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 20 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Let *z* be the complex number $z = \sqrt{5} + bi$. If $\text{Arg}(z) = \theta$, $|z| = 3$ and $\text{Im}(z) < 0$, find the value of the real constant *k*, if $\tan(2\theta) = k\sqrt{5}$.

3 marks

Question 2

Given the equation $P(z) = z^3 + \left(3i - 3\sqrt{3}\right) z^2 + 5z - 15\sqrt{3} + 15i = 0$

a. Verify that $P\left(3\sqrt{3} - 3i\right) = 0$

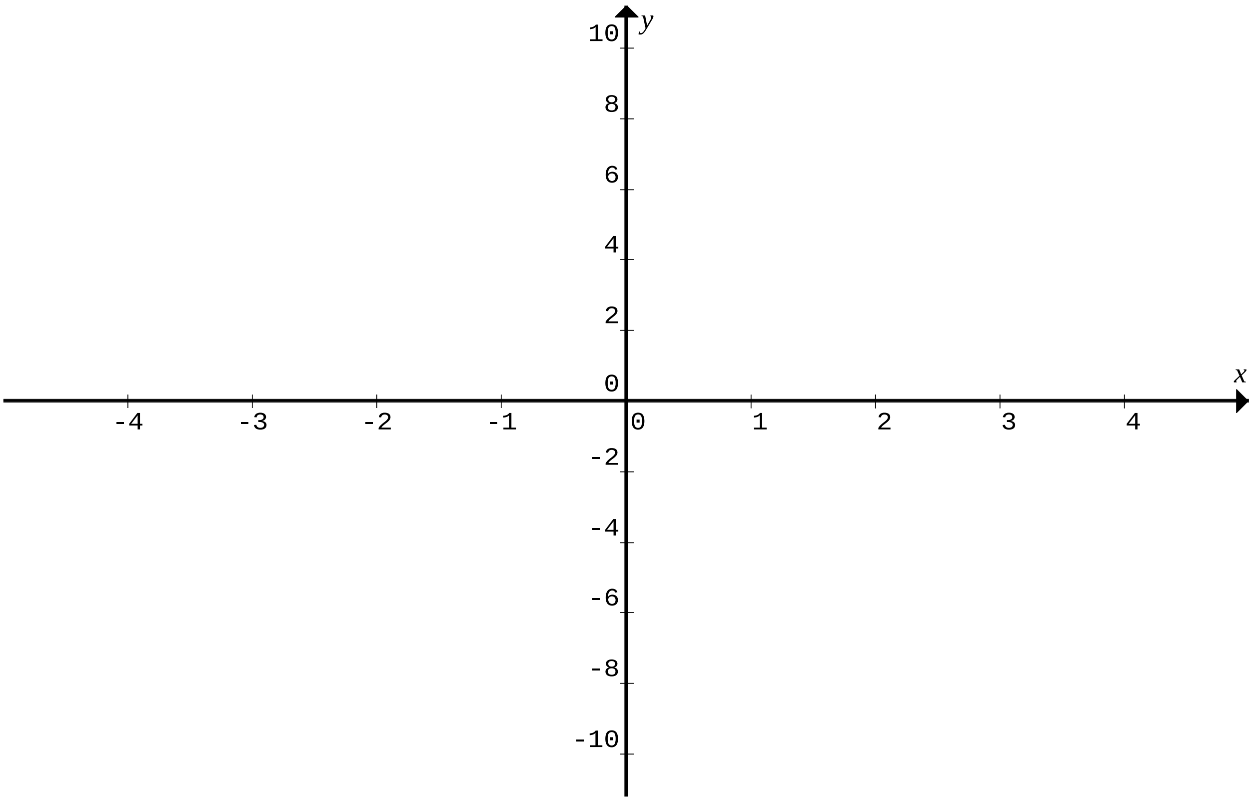
1 mark

b. Hence find all values of z if $z^3 + \left(3i - 3\sqrt{3}\right) z^2 + 5z - 15\sqrt{3} + 15i = 0$

2 marks

Question 3

Sketch the graph of $y = \dfrac{2x^3 - 4}{x}$ on the axes below. Give the exact coordinates of any turning points and intercepts, and state the equations of all asymptotes.



4 marks

Question 4

A particle is moving in a straight line. The mass of the particle is 2 kg and its velocity is v ms^{-1} when its displacement is x metres from the origin. Initially it is one metre to the right of the origin and is moving with an initial velocity of 4 ms^{-1}. The force acting on the particle is given by $4x^3 + 12x$ newtons.

a. Show that $v = x^2 + 3$

2 marks

b. Hence find the time in seconds when the particle is $\sqrt{3}$ metres from the origin.

4 marks

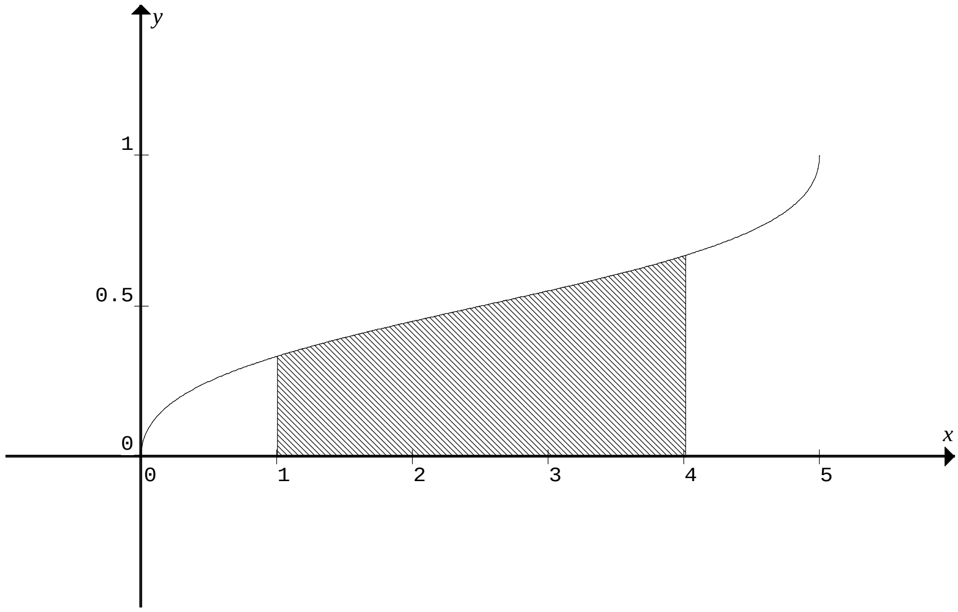
Question 5

a. Let $I=\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx$ use the substitution $u=a+b-x$, to show that

$I=\int_a^b \frac{f(a+b-x)}{f(a+b-x)+f(x)}dx$ and hence show that $I=\frac{1}{2}(b-a)$.

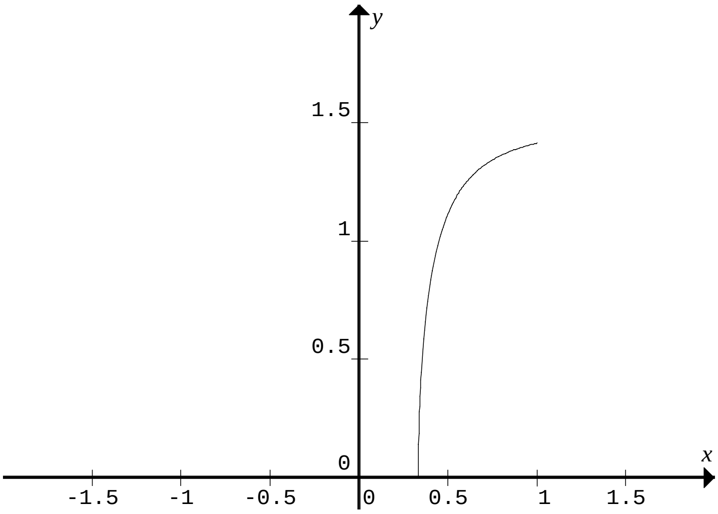
2 marks

b. The shaded area shown below is the area bounded by the graph of $y=\dfrac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}$, the x-axis and $x=1$ and $x=4$. Find the area of the shaded region.



1 mark

Question 6



The graph of $y = \dfrac{\sqrt{9x^2 - 1}}{2x}$ is shown. A vase is formed, when the graph is rotated about the *y*-axis, between the *x*-axis and $y = \sqrt{2}$. Find the volume of the vase.

4 marks

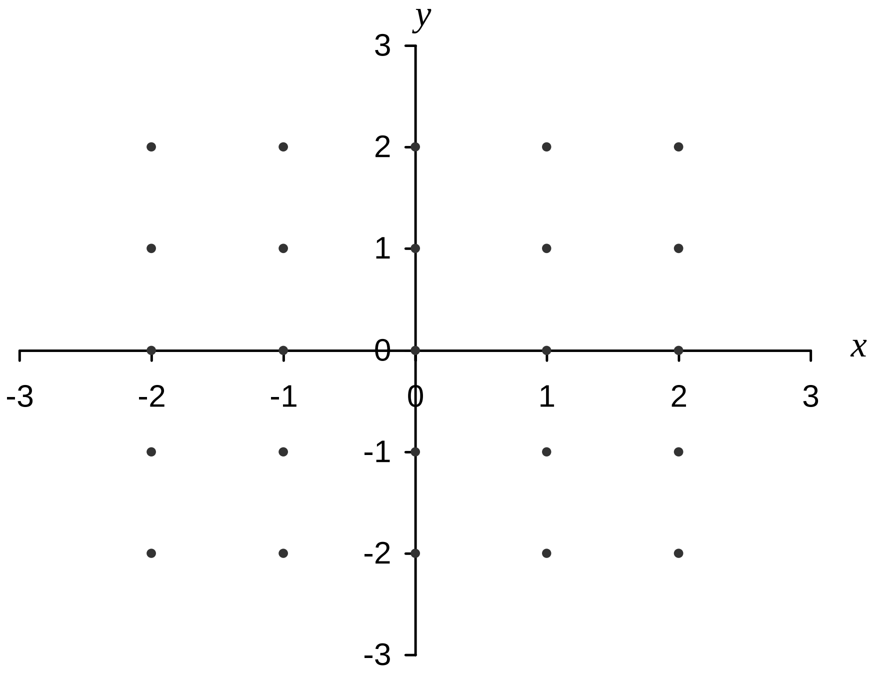
Question 7

a. Use Euler's method to find y_3 if $\dfrac{dy}{dx} = 2x - y$, given that $y_0 = y(0) = 0$ and $h = \dfrac{1}{4}$

Express your answer as a fraction.

2 marks

b. Sketch the slope field of the differential equation $\frac{dy}{dx} = 2x - y$ for $y = -2, -1, 0, 1, 2$ at each of the values $x = -2, -1, 0, 1, 2$ on the axes below.



2 marks

Question 8

a. Differentiate $x\sin^{-1}\left(\frac{2x}{3}\right)$ with respect to *x*.

2 marks

b. Solve the differential equation $\frac{dy}{dx}=\arcsin\left(\frac{2x}{3}\right)$ given that $x=\frac{3}{2}$ when $y=0$.

3 marks

Question 9

A particle is moving along a curve given by

$\underset{\sim}{r}(t) = (t - \sin(t))\underset{\sim}{i} + (1 + \cos(t))\underset{\sim}{j}$ for $0 \le t \le 2\pi$

a. Show that the speed of the particle at a time *t* is given by $2\sin\left(\frac{t}{2}\right)$.

2 marks

b. Find the gradient of curve, when $t = \frac{\pi}{3}$

2 marks

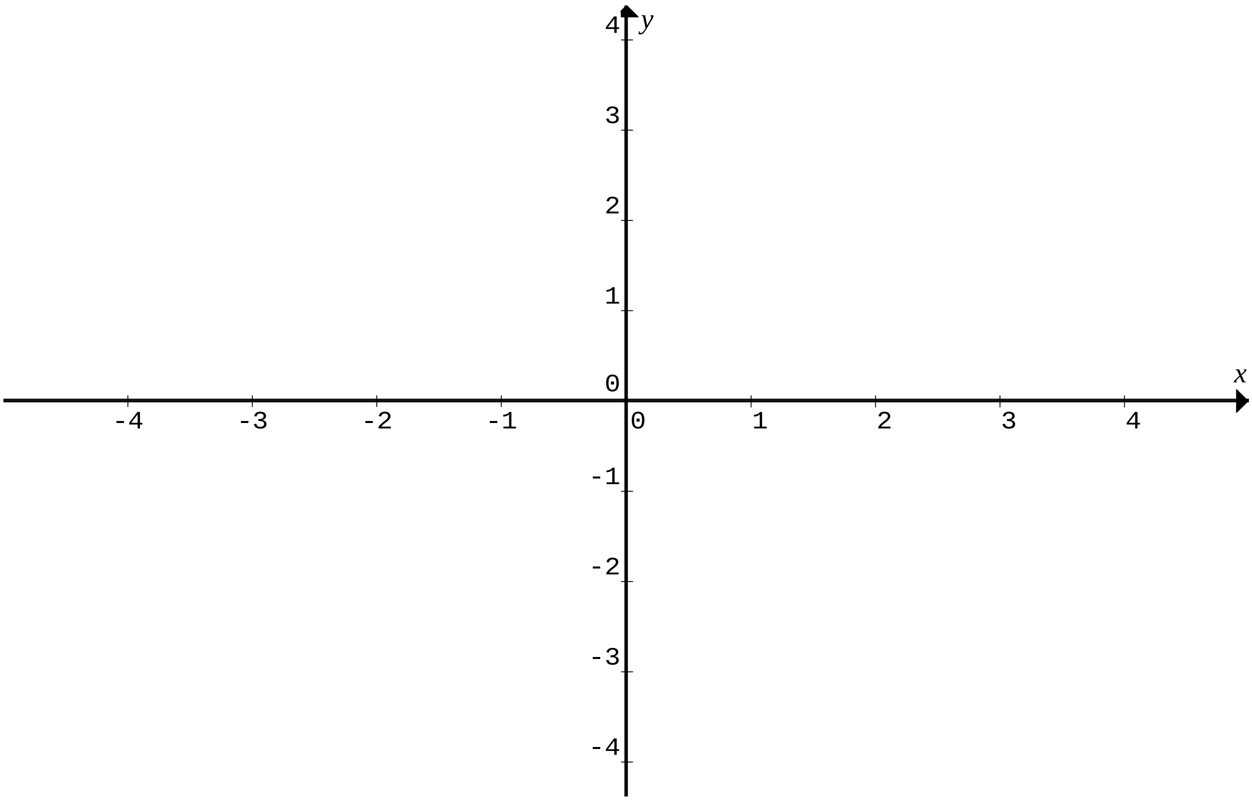
Question 10

a. For the curve $9x^2 + 36x + 4y^2 - 8y + 4 = 0$, find $\frac{dy}{dx}$.

Hence find the coordinates of the points on the curve $9x^2 + 36x + 4y^2 - 8y + 4 = 0$, where the tangent to the curve is parallel to the *x*-axis.

2 marks

b. Sketch the relation $9x^2 + 36x + 4y^2 - 8y + 4 = 0$ on the next page, stating the domain and range.



2 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\text{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r \qquad -\pi < \text{Arg}(z) \le \pi$$

$$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2) \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r \qquad \underset{\sim}{r_1} \cdot \underset{\sim}{r_2} = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e\left(x\right)\right) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e\left(x\right) + c,\ \text{for}\ \ x > 0$

$\frac{d}{dx}\left(\sin\left(ax\right)\right) = a\cos\left(ax\right)$

$\int \sin\left(ax\right) dx = -\frac{1}{a}\cos\left(ax\right) + c$

$\frac{d}{dx}\left(\cos\left(ax\right)\right) = -a\sin\left(ax\right)$

$\int \cos\left(ax\right) dx = \frac{1}{a}\sin\left(ax\right) + c$

$\frac{d}{dx}\left(\tan\left(ax\right)\right) = a\sec^2\left(ax\right)$

$\int \sec^2\left(ax\right) dx = \frac{1}{a}\tan\left(ax\right) + c$

$\frac{d}{dx}\left(\sin^{-1}\left(x\right)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}\left(x\right)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}\left(x\right)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}\left(uv\right) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf\left(x_n\right)$

END OF FORMULA SHEET