

Year 2011

VCE

Specialist Mathematics

Trial Examination 1



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**Victorian Certificate of Education
2011**

STUDENT NUMBER

Figures										Letter
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 20 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

Let z be the complex number $z = \sqrt{5} + bi$. If $\text{Arg}(z) = \theta$, $|z| = 3$ and $\text{Im}(z) < 0$, find the value of the real constant k , if $\tan(2\theta) = k\sqrt{5}$.

3 marks

Question 2

Given the equation $P(z) = z^3 + (3i - 3\sqrt{3})z^2 + 5z - 15\sqrt{3} + 15i = 0$

a. Verify that $P(3\sqrt{3} - 3i) = 0$

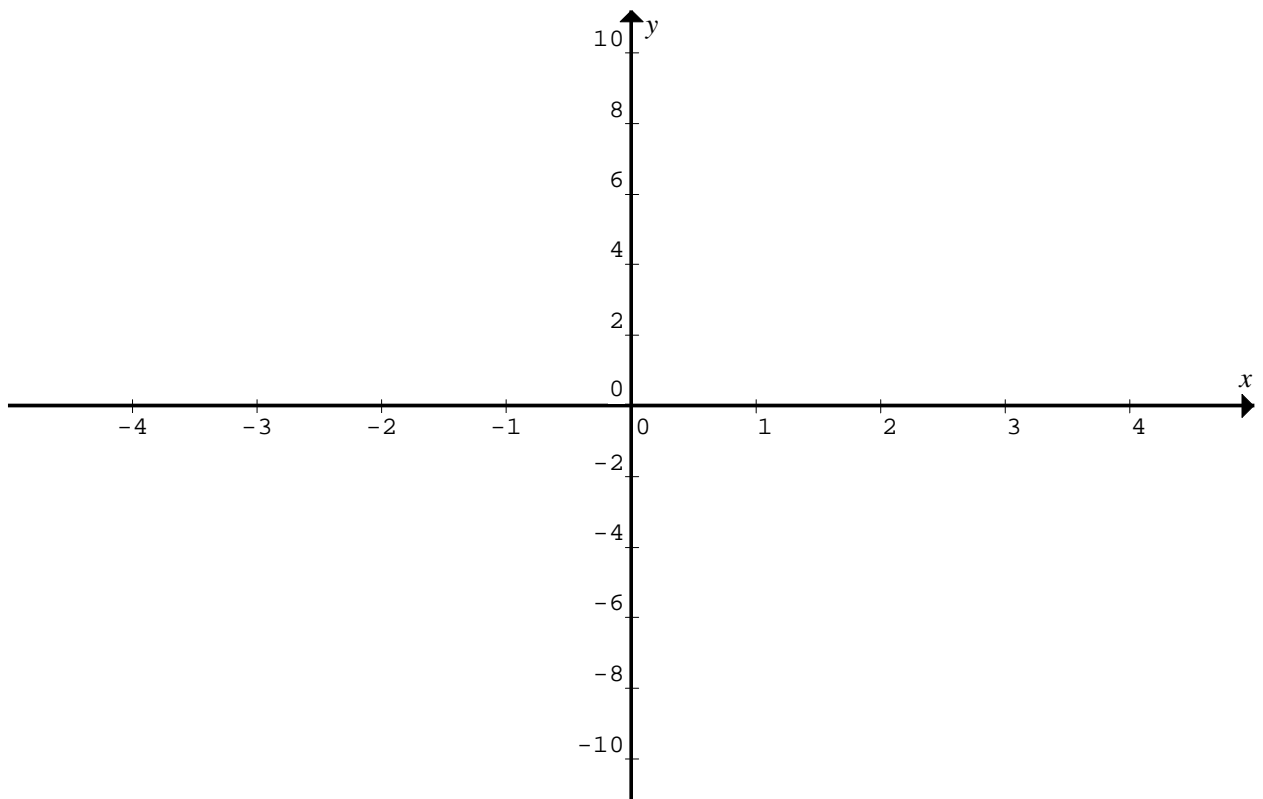
1 mark

b. Hence find all values of z if $z^3 + (3i - 3\sqrt{3})z^2 + 5z - 15\sqrt{3} + 15i = 0$

2 marks

Question 3

Sketch the graph of $y = \frac{2x^3 - 4}{x}$ on the axes below. Give the exact coordinates of any turning points and intercepts, and state the equations of all asymptotes.



4 marks

Question 4

A particle is moving in a straight line. The mass of the particle is 2 kg and its velocity is $v \text{ ms}^{-1}$ when its displacement is x metres from the origin. Initially it is one metre to the right of the origin and is moving with an initial velocity of 4 ms^{-1} . The force acting on the particle is given by $4x^3 + 12x$ newtons.

a. Show that $v = x^2 + 3$

2 marks

b. Hence find the time in seconds when the particle is $\sqrt{3}$ metres from the origin.

4 marks

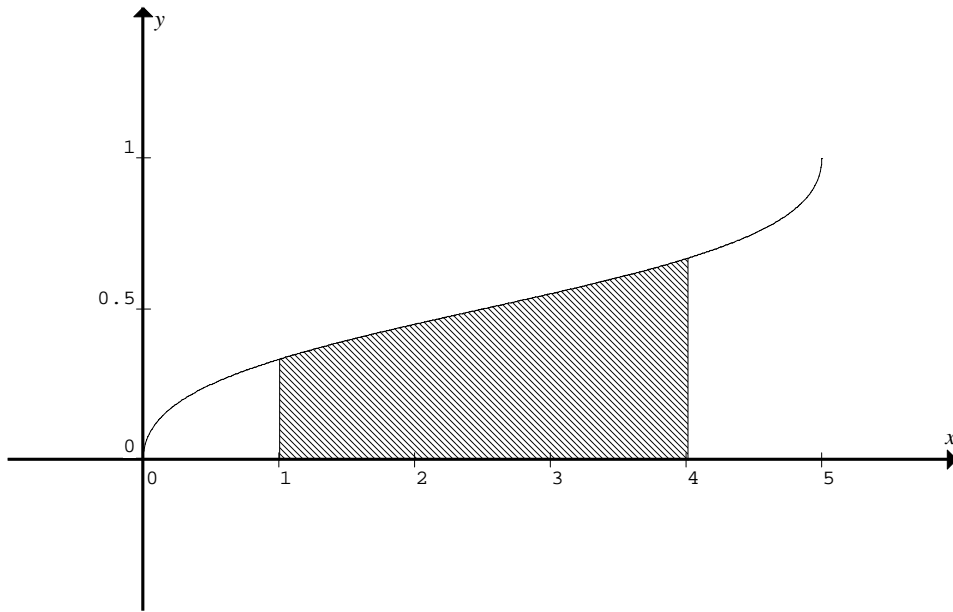
Question 5

a. Let $I = \int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx$ use the substitution $u = a+b-x$, to show that

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x) + f(x)} dx \text{ and hence show that } I = \frac{1}{2}(b-a).$$

2 marks

- b. The shaded area shown below is the area bounded by the graph of $y = \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}}$, the x -axis and $x = 1$ and $x = 4$. Find the area of the shaded region.



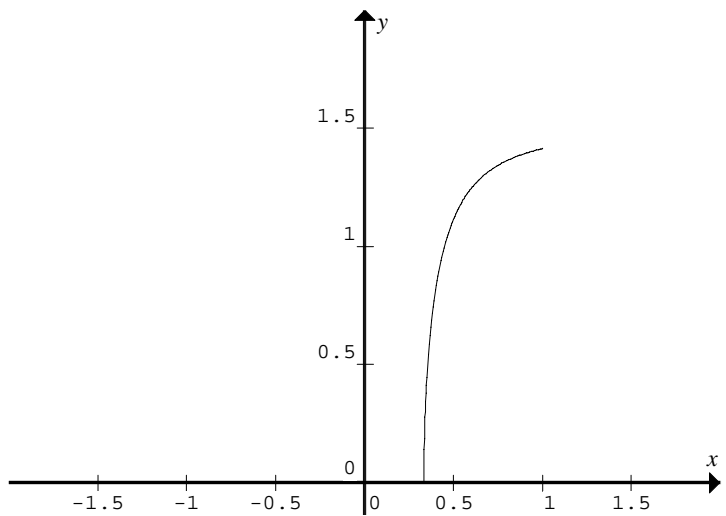
1 mark

Question 6

The graph of $y = \frac{\sqrt{9x^2 - 1}}{2x}$ is

shown. A vase is formed, when the graph is rotated about the y -axis, between the x -axis and $y = \sqrt{2}$.

Find the volume of the vase.



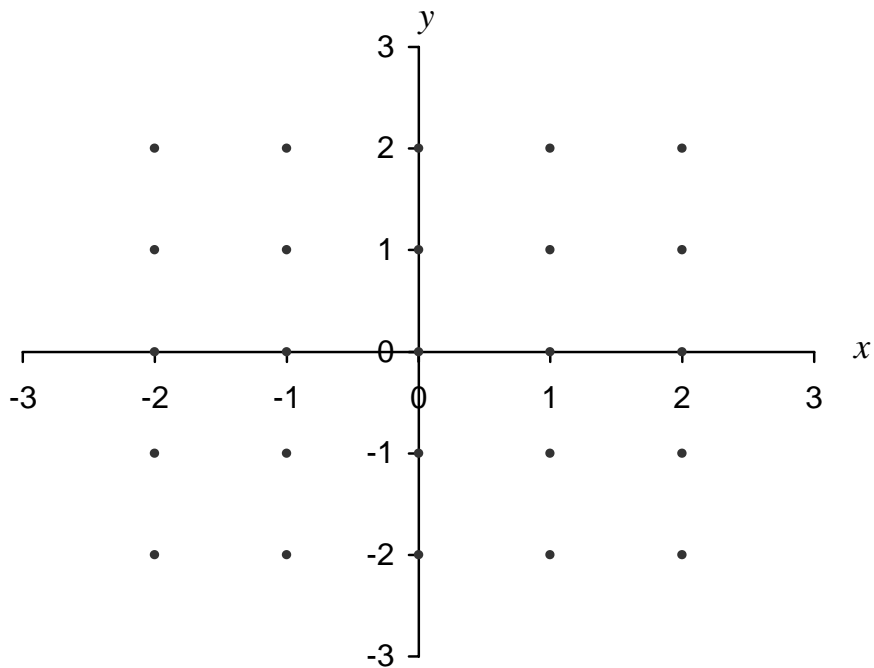
4 marks

Question 7

- a.** Use Euler's method to find y_3 if $\frac{dy}{dx} = 2x - y$, given that $y_0 = y(0) = 0$ and $h = \frac{1}{4}$
Express your answer as a fraction.

2 marks

- b. Sketch the slope field of the differential equation $\frac{dy}{dx} = 2x - y$ for $y = -2, -1, 0, 1, 2$ at each of the values $x = -2, -1, 0, 1, 2$ on the axes below.



2 marks

Question 8

- a. Differentiate $x \sin^{-1}\left(\frac{2x}{3}\right)$ with respect to x .

2 marks

- b. Solve the differential equation $\frac{dy}{dx} = \arcsin\left(\frac{2x}{3}\right)$ given that $x = \frac{3}{2}$ when $y = 0$.

3 marks

Question 9

A particle is moving along a curve given by

$$r(t) = (t - \sin(t))\underline{i} + (1 + \cos(t))\underline{j} \text{ for } 0 \leq t \leq 2\pi$$

- a. Show that the speed of the particle at a time t is given by $2\sin\left(\frac{t}{2}\right)$.

2 marks

- b. Find the gradient of curve, when $t = \frac{\pi}{3}$

2 marks

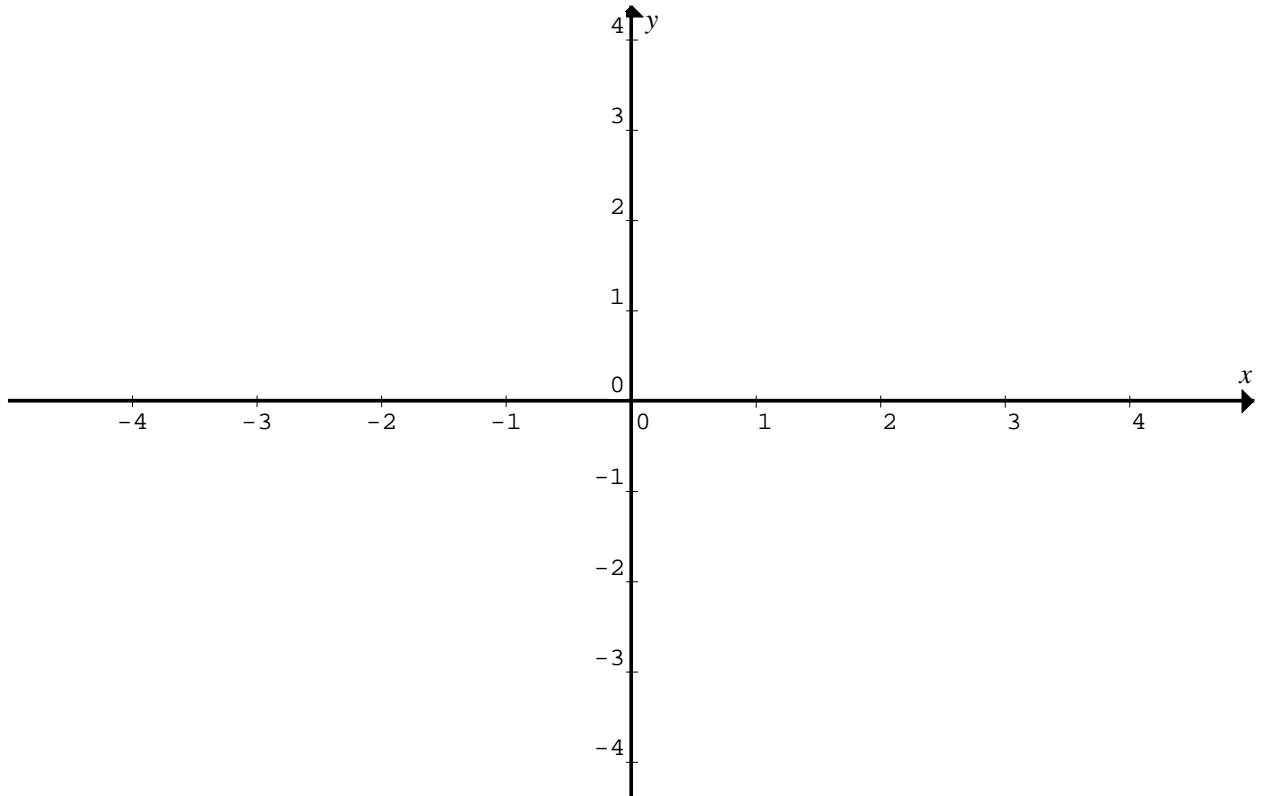
Question 10

- a. For the curve $9x^2 + 36x + 4y^2 - 8y + 4 = 0$, find $\frac{dy}{dx}$.

Hence find the coordinates of the points on the curve $9x^2 + 36x + 4y^2 - 8y + 4 = 0$, where the tangent to the curve is parallel to the x -axis.

2 marks

- b. Sketch the relation $9x^2 + 36x + 4y^2 - 8y + 4 = 0$ on the next page, stating the domain and range.



2 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	\sin^{-1}	\cos^{-1}	\tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2}v^2 \right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET