2011 VCAA Specialist Mathematics Exam 1 Solutions

© Copyright 2011 itute.com   Free download from www.itute.com

Q1 $\int \frac{1+x}{9-x^2}dx = \int\left(\frac{\frac{2}{3}}{3-x} - \frac{\frac{1}{3}}{3+x}\right)dx$ (partial fractions)

$= -\frac{2}{3}\log_e|3-x| - \frac{1}{3}\log_e|3+x| = -\frac{1}{3}\left(2\log_e|3-x| + \log_e|3+x|\right)$

$= -\frac{1}{3}\log_e(3-x)^2|3+x|$

Alternatively, $-\frac{1}{3}\log_e|3-x||9-x^2|$

Q2 $y = kxe^{2x}$, $\frac{dy}{dx} = ke^{2x} + 2kxe^{2x}$, $\frac{d^2y}{dx^2} = 2ke^{2x} + 2\frac{dy}{dx}$,

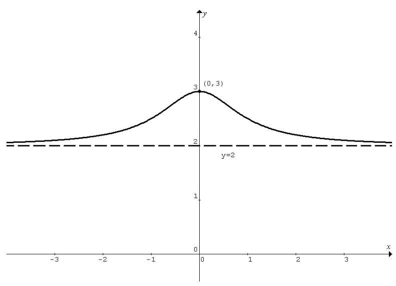
.: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 2ke^{2x} + 5kxe^{2x}$

.: $e^{2x}(15x+6) = 2ke^{2x} + 5kxe^{2x}$

$e^{2x}(15x+6) = e^{2x}(2k+5kx)$, .: $k = 3$

Q3a $f(x) = \frac{2x^2+3}{x^2+1} = \frac{2(x^2+1)+1}{x^2+1} = 2 + \frac{1}{x^2+1}$

Q3b



Q3c Area

$= 2\times\int_0^1\left(2+\frac{1}{1+x^2}\right)dx = 2\left[2x+\tan^{-1}x\right]_0^1 = 2\left(2+\frac{\pi}{4}\right) = 4+\frac{\pi}{2}$

Q4 $z = \frac{1-\sqrt{3}i}{-1+i} = \frac{2cis\left(-\frac{\pi}{3}\right)}{\sqrt{2}cis\left(\frac{3\pi}{4}\right)} = \sqrt{2}cis\left(-\frac{\pi}{3}-\frac{3\pi}{4}\right)$

$= \sqrt{2}cis\left(-\frac{13\pi}{12}\right) = \sqrt{2}cis\left(\frac{11}{12}\pi\right)$, .: $Arg(z) = \frac{11}{12}\pi$

Q5 $x = 4\sin t - 1$, $\frac{dx}{dt} = 4\cos t$, $\sin t = \frac{x+1}{4}$

$y = 2\cos t + 3$, $\frac{dy}{dt} = -2\sin t$, $\cos t = \frac{y-3}{2}$

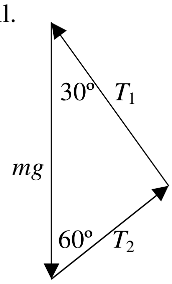
$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sin t}{2\cos t} = \frac{-\frac{x+1}{4}}{y-3}$

At $\left(1, \sqrt{3}+3\right)$, $\frac{dy}{dx} = \frac{-\frac{1}{2}}{\sqrt{3}} = -\frac{1}{2\sqrt{3}} = -\frac{\sqrt{3}}{6}$

Q6 $\int_0^1 e^x\cos\left(e^x\right)dx = \int_0^1\cos(u).\frac{du}{dx}dx = \int_1^e\cos(u)du$

$= [\sin(u)]_1^e = \sin(e) - \sin(1)$

Q7a Add the force vectors head to tail.



$\frac{T_1}{T_2} = \tan 60° = \sqrt{3}$, .: $T_2 = \frac{T_1}{\sqrt{3}}$

Q7b $\cos 60° = \frac{T_2}{mg}$, .: $\frac{1}{2} = \frac{T_2}{9.8m}$

At breaking point, $\frac{1}{2} = \frac{98}{9.8m}$

.: $m = 20$ kg

Q8 Let $\cos ec^2\left(\frac{\pi x}{6}\right) = \frac{4}{3}$, $\sin^2\left(\frac{\pi x}{6}\right) = \frac{3}{4}$, $1-2\sin^2\left(\frac{\pi x}{6}\right) = -\frac{1}{2}$

.: $\cos\left(\frac{\pi x}{3}\right) = -\frac{1}{2}$ where $0 < x < 12$, i.e. $0 < \frac{\pi x}{3} < 4\pi$

.: $\frac{\pi x}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$,

.: $x = 2$, 4, 8, 10 and $y = \frac{4}{3}$

The intersecting points are $\left(2, \frac{4}{3}\right)$, $\left(4, \frac{4}{3}\right)$, $\left(8, \frac{4}{3}\right)$ and $\left(10, \frac{4}{3}\right)$.

Q9a $\tilde{a} = \tilde{i} - \tilde{j} + 2\tilde{k}$, $\tilde{b} = \tilde{i} + 2\tilde{j} + m\tilde{k}$, $\tilde{c} = \tilde{i} + \tilde{j} - \tilde{k}$

$\left|\tilde{b}\right| = 2\sqrt{3}$, $\left|\tilde{b}\right|^2 = 12$, .: $1 + 4 + m^2 = 12$, $m = \pm\sqrt{7}$

Q9b $\tilde{a}.\tilde{b} = 0$, .: $1 - 2 + 2m = 0$, $m = \frac{1}{2}$

Q9ci $3\tilde{c} - \tilde{a} = 3\left(\tilde{i} + \tilde{j} - \tilde{k}\right) - \left(\tilde{i} - \tilde{j} + 2\tilde{k}\right) = 2\tilde{i} + 4\tilde{j} - 5\tilde{k}$

Q9cii $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ are linearly dependent when $3\tilde{c} - \tilde{a} = n\tilde{b}$, where n is a real constant. .: $2\tilde{i} + 4\tilde{j} - 5\tilde{k} = n\left(\tilde{i} + 2\tilde{j} + m\tilde{k}\right)$

.: $n = 2$ and $nm = -5$, .: $m = -\frac{5}{2}$

Q10 $y\log_e x = e^{2y} + 3x - 4$

By implicit differentiation, $\frac{dy}{dx}\log_e x + \frac{y}{x} = 2e^{2y}\frac{dy}{dx} + 3$

$\frac{dy}{dx}\left(\log_e x - 2e^{2y}\right) = 3 - \frac{y}{x}$. At $(1,0)$, $\frac{dy}{dx}(-2) = 3$, .: $\frac{dy}{dx} = -\frac{3}{2}$.

Q11 $V = \int_0^{\frac{\pi}{6}}\pi\sin^2 x dx = \frac{\pi}{2}\int_0^{\frac{\pi}{6}}(1-\cos 2x)dx = \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{6}}$

$\frac{\pi}{2}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \frac{\pi}{24}\left(2\pi - 3\sqrt{3}\right)$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors