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Students Name:………………………………………

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2011

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the ten questions are indicated throughout.
Students may not bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 12-14 of this exam.

Question 1

Let $f(z) = z^3 + z^2 - 4z + a,\ z \in C$.

a. If $z+3$ is a factor of $f(z)$, show that $a = 6$.

1 mark

b. Find all the solutions to the equation $f(z) = 0$.

3 marks

Question 2

A family of curves is given by the relation $kx^2 - 2x^2y + y^3 - 6x = 2$ where $k \in R$.

a. Find $\frac{dy}{dx}$ in terms of x, y and k.

2 marks

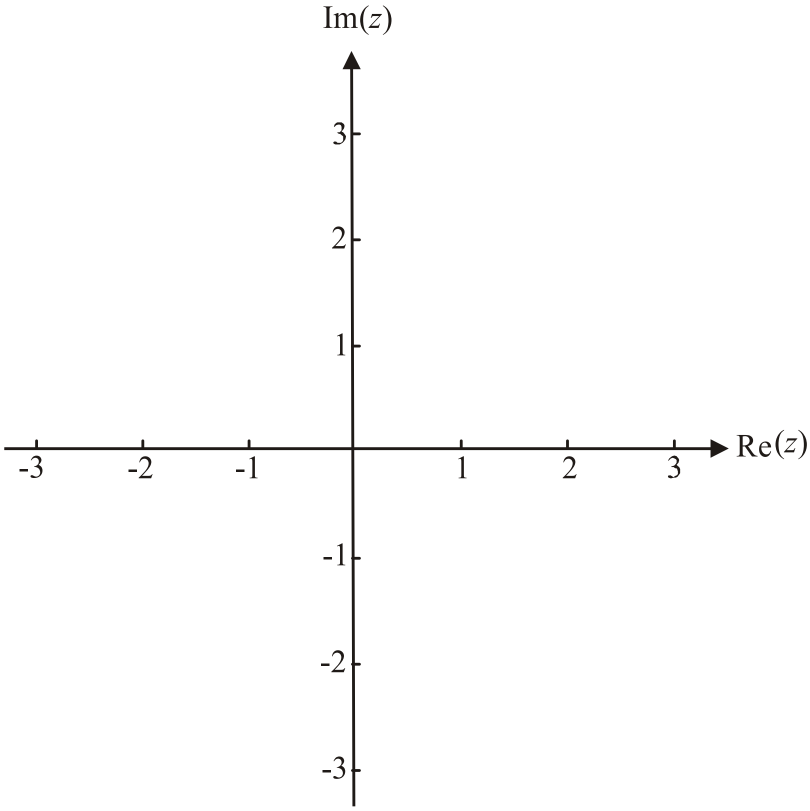
b. Find the gradient of the curve at the point $(1,0)$.

2 marks

Question 3

On the Argand diagram below, shade the region given by

$$\{z : |z-1-i| \le |z|\} \cap \left\{z : 0 \le \text{Arg}(z) \le \frac{\pi}{2}\right\}.$$

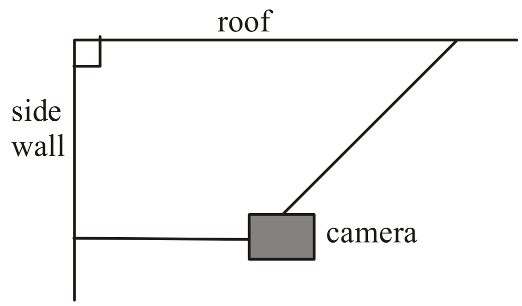


3 marks

Question 4

A camera of mass 6kg is attached to and held in position by two taut wires, one of which is suspended from the roof of a sports stadium. The other wire is attached to the side wall of the stadium and is horizontal.
The tension, in newtons, in the horizontal wire is half the tension in the other wire.

a. Label all the forces acting on the camera on the diagram below.



1 mark

b. Find the tension in each of the wires.

2 marks

c. Find the angle that the wire suspended from the roof makes with the vertical.

1 mark

Question 5

Let $f(x) = 2\arctan\left(\frac{2x+1}{3}\right) - 1$

a. Find the maximal domain and range of *f*.

2 marks

b. Find $f'(x)$.

2 marks

Question 6

The acceleration, in ms^{-2}, of a particle moving in a straight line at time t seconds with velocity $v\text{ms}^{-1}$ is given by $a=\sqrt{9-v^2}$ where $v=\frac{3}{2}$ at time $t=\frac{\pi}{3}$.

Find an expression for v in terms of t.

3 marks

Question 7

a. Find $\int \frac{1}{x^2 - 2x - 3}\,dx$.

2 marks

b. Evaluate $\int_0^1 \frac{x}{2-x}\,dx$.

3 marks

Question 8

Given that $\cot(2\theta)=\frac{\sqrt{5}}{20}$, $0<\theta<\frac{\pi}{4}$, find the exact value of $\cos(\theta)$.

3 marks

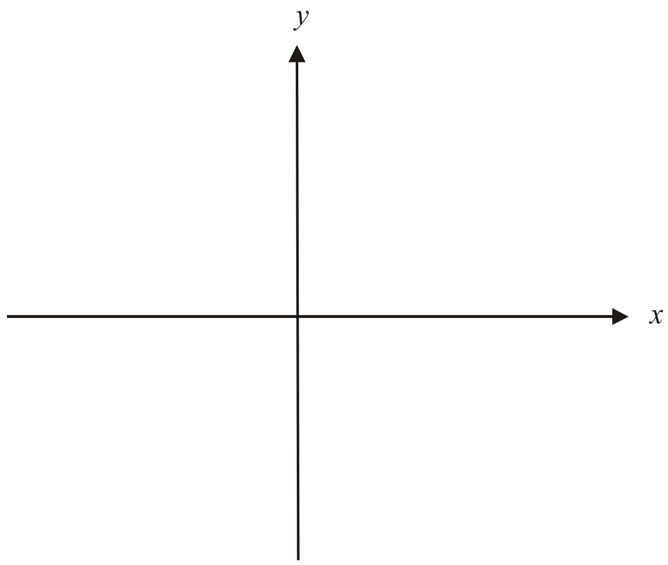
Question 9

The position vector of a particle is given by $\underset{\sim}{r}(t) = \arccos\left(\frac{t}{5}\right)\underset{\sim}{i} + 2t\,\underset{\sim}{j}$ for $t \in [0, 5]$.

a. Find the Cartesian equation of the path followed by this particle.

2 marks

b. On the set of axes below, sketch the path of the particle.



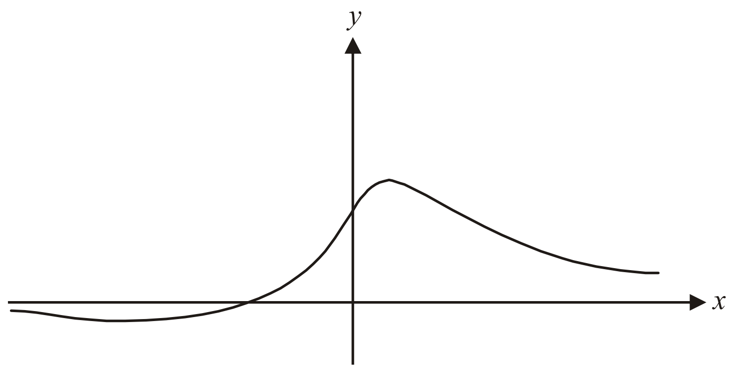
2 marks

c. Find the speed of the particle at $t = 2$.

2 marks

Question 10

The graph of the function $y = \frac{x+1}{x^2+1}$ is shown below.



Find the area enclosed by the graph of the function and the x and y axes. Express your answer in the form $\frac{\pi}{a} - \ln\sqrt{b}$, where a and b are integers.

4 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\qquad$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\qquad$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\qquad$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\qquad$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\qquad$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ $\qquad$ $-\pi<\operatorname{Arg} z\le\pi$

$z_1z_2=r_1r_2\operatorname{cis}(\theta_1+\theta_2)$ $\qquad$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\underset{\sim}{r_1} . \underset{\sim}{r_2} = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$