VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2010

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter:
Words									

SPECIALIST MATHEMATICS

Written examination 2

Monday 1 November 2010

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

An ellipse has a horizontal semi-axis length of 1 and a vertical semi-axis length of 2. The centre of the ellipse is at the point with coordinates (3, –5).

A possible equation for the ellipse is

A. $(x + 3)^2 + 4(y - 5)^2 = 4$

B. $(x - 3)^2 + 4(y + 5)^2 = 4$

C. $4(x + 3)^2 + (y - 5)^2 = 4$

D. $4(x - 3)^2 + (y + 5)^2 = 4$

E. $4(x - 3)^2 + (y + 5)^2 = 1$

Question 2

Each of the following equations represents a hyperbola.

Which hyperbola **does not** have perpendicular asymptotes?

A. $(x - 1)^2 - (y + 2)^2 = 1$

B. $x^2 - 2x - y^2 + 4y = 4$

C. $(x - 1)^2 - (y + 2)^2 = 9$

D. $(y - 1)^2 - (x + 2)^2 = 1$

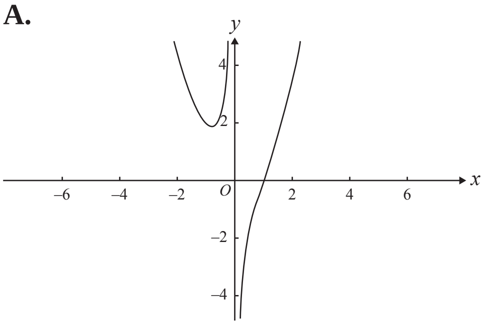
E. $2x^2 - 4x - y^2 - 4y = 4$

Question 3

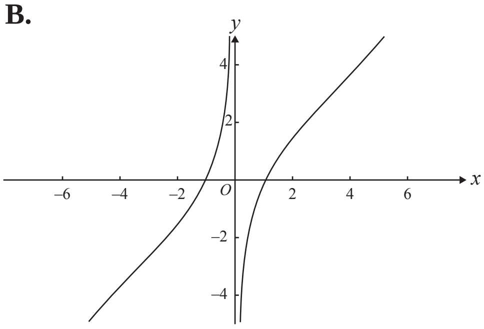
Let $f(x) = \dfrac{x^k + a}{x}$, where k and a are real constants.

If k is an odd integer which is greater than 1 and $a < 0$, a possible graph of f could be

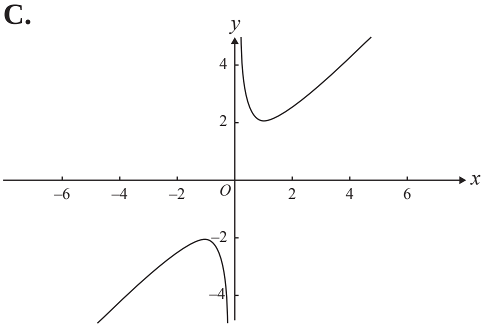
A.



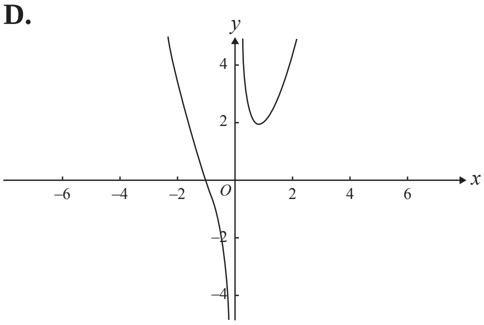
B.



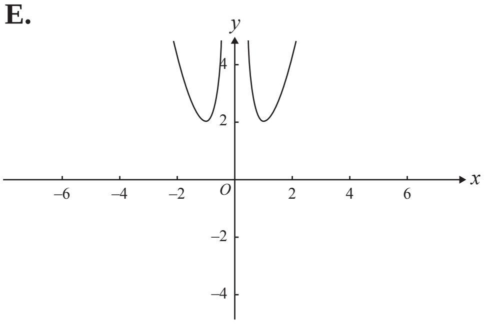
C.



D.



E.



Question 4

The position vector of a particle at time $t \geq 0$ is given by $\underset{\sim}{r} = \sin(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$.

The path of the particle has cartesian equation

A. $y = 2x^2 - 1$

B. $y = 1 - 2x^2$

C. $y = \sqrt{1 - x^2}$

D. $y = \sqrt{x^2 - 1}$

E. $y = 2x\sqrt{1 - x^2}$

Question 5

For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the graphs of the two curves given by $y = 2\sec^2(x)$ and $y = 5|\tan(x)|$ intersect

A. only at the one point $(\arctan(2), 10)$

B. only at the two points $(\pm\arctan(2), 10)$

C. only at the one point $\left(\arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$

D. only at the two points $\left(\pm\arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$

E. at the two points $\left(\pm\arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$, as well as at the two points $(\pm\arctan(2), 10)$

Question 6

Let $z = \text{cis}\left(\frac{5\pi}{6}\right)$.

The imaginary part of $z - i$ is

A. $-\frac{i}{2}$

B. $-\frac{1}{2}$

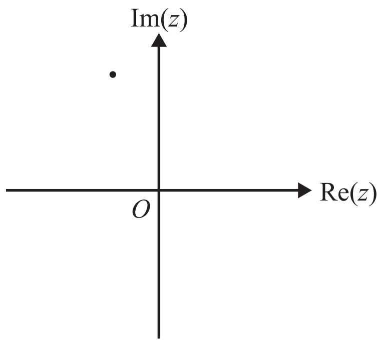
C. $-\frac{\sqrt{3}}{2}$

D. $-\frac{3}{2}$

E. $-\frac{3i}{2}$

Question 7

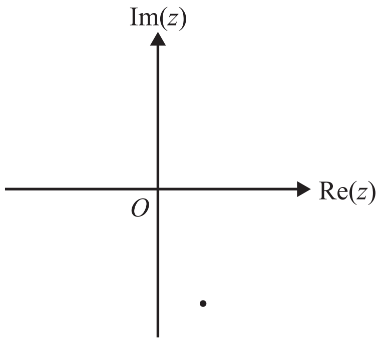
A particular complex number z is represented by the point on the following argand diagram.



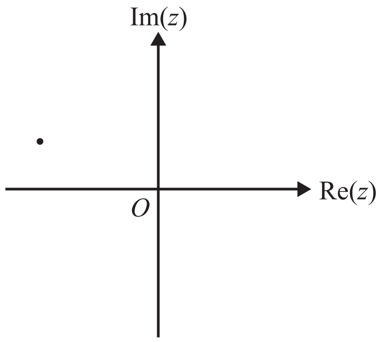
All axes below have the **same scale** as those in the diagram above.

The complex number $i\bar{z}$ is best represented by

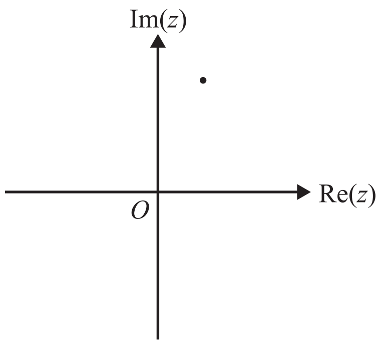
A.



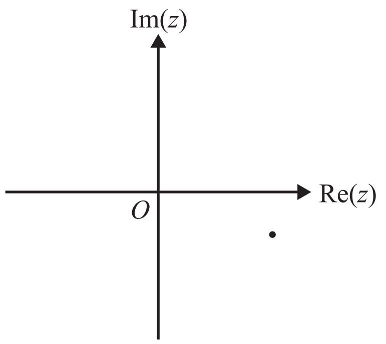
B.



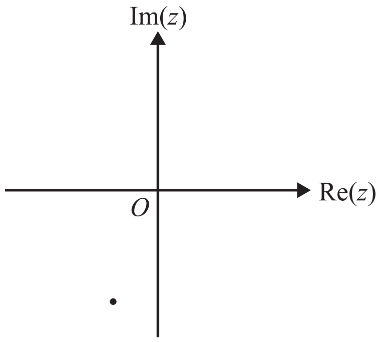
C.



D.



E.



Question 8

The polynomial equation $P(z) = 0$ has one complex coefficient. Three of the roots of this equation are $z = 3 + i$, $z = 2 - i$ and $z = 0$.

The **minimum degree** of $P(z)$ is

A. 1

B. 2

C. 3

D. 4

E. 5

Question 9

Given that $z = 4\text{cis}\left(\frac{2\pi}{3}\right)$, it follows that $\text{Arg}\left(z^5\right)$ is

A. $\frac{10\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{7\pi}{3}$

D. $-\frac{\pi}{3}$

E. $-\frac{2\pi}{3}$

Question 10

On an argand diagram, a set of points which lies on a circle of radius 2 centred at the origin is

A. $\{z \in C : z\bar{z} = 2\}$

B. $\{z \in C : z^2 = 4\}$

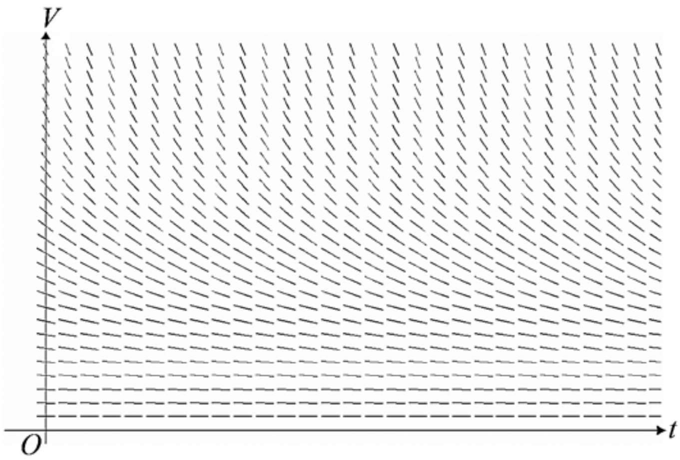
C. $\{z \in C : \text{Re}\left(z^2\right) + \text{Im}\left(z^2\right) = 4\}$

D. $\{z \in C : \left(z + \bar{z}\right)^2 - \left(z - \bar{z}\right)^2 = 16\}$

E. $\{z \in C : \left(\text{Re}(z)\right)^2 + \left(\text{Im}(z)\right)^2 = 16\}$

Question 11

A direction field for the volume of water, V megalitres, in a reservoir t years after 2010 is shown below.



According to this model, for $k > 0$, $\frac{dV}{dt}$ is equal to

A. $-kt^2$

B. $\frac{k}{V}$

C. $-kV^2$

D. kV^2

E. $-\frac{k}{V}$

Question 12

Let $\frac{dy}{dx} = \frac{x+2}{x^2+2x+1}$ and $(x_0, y_0) = (0, 2)$.

Using Euler's method, with a step size of 0.1, the value of y_1 correct to two decimal places is

A. 0.17

B. 0.20

C. 1.70

D. 2.17

E. 2.20

Question 13

The amount of a drug, x mg, remaining in a patient's bloodstream t hours after taking the drug is given by the differential equation

$$\frac{dx}{dt} = -0.15x.$$

The number of hours needed for the amount x to halve is

A. $2\log_e\left(\frac{20}{3}\right)$

B. $\frac{20}{3}\log_e(2)$

C. $2\log_e(15)$

D. $15\log_e\left(\frac{3}{2}\right)$

E. $\frac{3}{2}\log_e(200)$

Question 14

Use a suitable substitution to show that the definite integral $\int_0^2 \frac{x}{\sqrt{x^2-1}}\,dx$ can be simplified to

A. $\frac{1}{2}\int_{-1}^{3} u^{-\frac{1}{2}}\,du$

B. $2\int_{-1}^{3} u^{-\frac{1}{2}}\,du$

C. $\frac{1}{2}\int_{0}^{2} u^{-\frac{1}{2}}\,du$

D. $2\int_{0}^{2} u^{-\frac{1}{2}}\,du$

E. $\int_{0}^{2} u^{-\frac{1}{2}}\,du$

Question 15

The scalar resolute of $\underset{\sim}{a} = 3\underset{\sim}{i} - \underset{\sim}{k}$ in the direction of $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$ is

A. $\frac{8}{\sqrt{10}}$

B. $\frac{8}{9}\left(2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}\right)$

C. 8

D. $\frac{4}{5}\left(3\underset{\sim}{i} - \underset{\sim}{k}\right)$

E. $\frac{8}{3}$

Question 16

The square of the magnitude of the vector $\underset{\sim}{d} = 5\underset{\sim}{i} - \underset{\sim}{j} + \sqrt{10}\,\underset{\sim}{k}$ is

A. 6

B. 34

C. 36

D. 51.3

E. $\sqrt{34}$

Question 17

The angle between the vectors $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j}$ is exactly

A. $\frac{\pi}{6}$

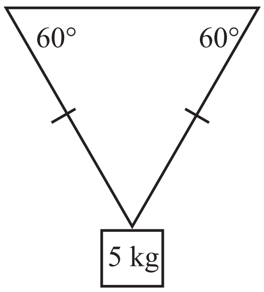
B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

E. π

Question 18



A 5 kg mass is suspended from a horizontal ceiling by two strings of equal length. Each string makes an angle of 60° to the ceiling, as shown in the above diagram. Correct to one decimal place, the tension in each string is

A. 24.5 newtons

B. 28.3 newtons

C. 34.6 newtons

D. 49.0 newtons

E. 84.9 newtons

Question 19

An object is moving in a northerly direction with a constant acceleration of 2 ms^{-2}. When the object is 100 m due north of its starting point, its velocity is 30 ms^{-1} in the northerly direction.

The exact initial velocity of the object could have been

A. $10\sqrt{5}$ ms^{-1}

B. $5\sqrt{10}$ ms^{-1}

C. $10\sqrt{7}$ ms^{-1}

D. $-10\sqrt{7}$ ms^{-1}

E. $7\sqrt{10}$ ms^{-1}

Question 20

The acceleration, a ms^{-2}, of a particle moving in a straight line is given by $a = \frac{v}{\log_e(v)}$, where v is the velocity of the particle in ms^{-1} at time t s. The initial velocity of the particle was 5 ms^{-1}.

The velocity of the particle, in terms of t, is given by

A. $v = e^{2t}$

B. $v = e^{2t} + 4$

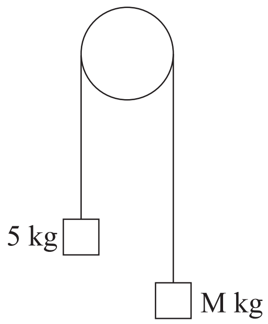
C. $v = e^{\sqrt{2t} + \log_e(5)}$

D. $v = e^{\sqrt{2t + (\log_e 5)^2}}$

E. $v = e^{-\sqrt{2t + (\log_e 5)^2}}$

Question 21

A light inextensible string passes over a smooth, light pulley. A mass of 5 kg is attached to one end of the string and a mass of M kg is attached to the other end, as shown below.



The M kg mass accelerates downwards at $\frac{7}{5}$ ms^{-2}.

The value of M is

A. $5\frac{2}{7}$

B. $6\frac{2}{5}$

C. $6\frac{2}{3}$

D. $8\frac{2}{5}$

E. $9\frac{4}{5}$

Question 22

A particle of mass m moves in a straight line under the action of a resultant force F where $F = F(x)$. Given that the velocity v is v_0 where the position x is x_0, and that v is v_1 where x is x_1, it follows that

A. $v_1 = \sqrt{\frac{2}{m}} \int_{x_0}^{x_1} \sqrt{F(x)}\, dx + v_0$

B. $v_1 = \sqrt{2} \sqrt{\int_{x_0}^{x_1} F(x)\, dx + v_0^2}$

C. $v_1 = \sqrt{2} \int_{\sqrt{x_0}}^{\sqrt{x_1}} F(x)\, dx + v_0$

D. $v_1 = \sqrt{\frac{2}{m} \int_{x_0}^{x_1} F(x)\, dx + v_0^2}$

E. $v_1 = \sqrt{\frac{2}{m} \int_{x_0}^{x_1} \left(F(x) + v_0^2\right) dx}$

END OF SECTION 1
TURN OVER

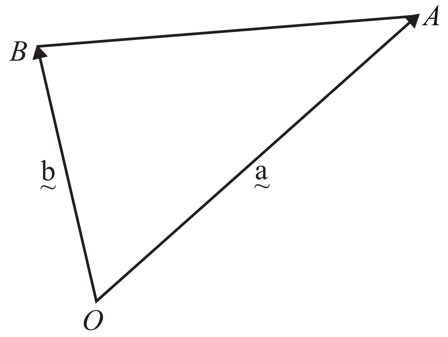
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The diagram below shows a triangle with vertices O, A and B. Let O be the origin, with vectors $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.



a. Find the following vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

i. $\overrightarrow{MA}$, where M is the midpoint of the line segment OA

ii. $\overrightarrow{BA}$

iii. $\overrightarrow{AQ}$, where Q is the midpoint of the line segment AB.

1 + 1 + 1 = 3 marks

b. Let N be the midpoint of the line segment OB. Use a vector method to prove that the quadrilateral $MNQA$ is a parallelogram.

3 marks

Now consider the **particular** triangle OAB with $\overrightarrow{OA} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \sqrt{3}\underset{\sim}{k}$ and $\overrightarrow{OB} = \alpha\underset{\sim}{i}$ where α, which is greater than zero, is chosen so that triangle OAB is isosceles, with $|\overrightarrow{OB}| = |\overrightarrow{OA}|$.

c. Show that $\alpha = 4$.

1 mark

d. **i.** Find $\overrightarrow{OQ}$, where Q is the midpoint of the line segment AB.

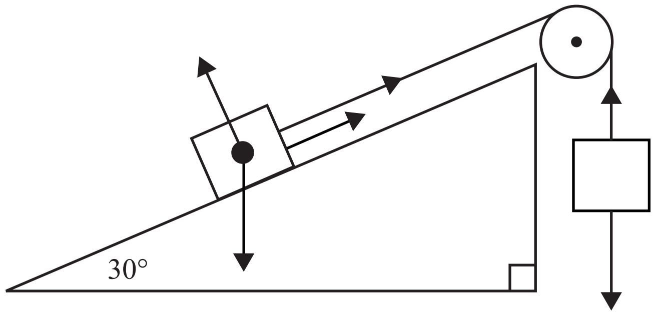
ii. Use a vector method to show that $\overrightarrow{OQ}$ is perpendicular to $\overrightarrow{AB}$.

1 + 3 = 4 marks

Total 11 marks

Question 2

A block of m kg sits on a rough plane which is inclined at 30° to the horizontal. The block is connected to a mass of 10 kg by a light inextensible string which passes over a frictionless light pulley.



a. If the block is on the point of moving **down the plane**, clearly label the forces which are shown in the diagram.

2 marks

b. Write down, **but do not attempt to solve**, equations involving the forces acting on the block, and the forces acting on the 10 kg mass, for the situation in **part a.**

3 marks

c. Given that the coefficient of friction is $\frac{1}{4}$, show that the mass of the block in kg is given by $m = \frac{80}{4 - \sqrt{3}}$.

2 marks

In order to pull the block up the plane, the mass hanging from the string is replaced by a 20 kg mass, and a lubricant is spread on the inclined plane to decrease the coefficient of friction.

d. If the block is now on the point of moving **up the plane**, show that the new value of the coefficient of friction, correct to three decimal places, is $\mu_1 = 0.077$.

2 marks

Before the block actually starts to move up the lubricated plane where $\mu_1 = 0.077$, the string breaks and the block begins to slide down the plane.

e. Find the velocity of the block three seconds after the string breaks. Give your answer in ms^{-1} correct to one decimal place.

3 marks

Total 12 marks

Question 3

The population of a town is initially 20 000 people. This population would increase at a rate of one per cent per year, except that there is a steady flow of people arriving at and leaving from the town. The population P after t years may be modelled by the differential equation

$$\frac{dP}{dt} = \frac{P}{100} - k \text{ with the initial condition, } P = 20\,000 \text{ when } t = 0,$$

where k is the number of people leaving per year minus the number of people arriving per year.

a. Verify by substitution that for $k = 800$,

$$P = 20\,000(4 - 3e^{0.01t})$$

satisfies both the differential equation and the initial condition.

3 marks

b. For $k = 800$, find the time taken for the population to decrease to zero. Give your answer correct to the nearest whole year.

2 marks

The differential equation which models the population growth can be expressed as

$$\frac{dt}{dP} = \frac{100}{P - 100k} \text{ with } P = 20\,000 \text{ when } t = 0.$$

c. Show by integration that for $k < 0$, the solution of this differential equation is

$$P = (20\,000 - 100k)e^{0.01t} + 100k.$$

2 marks

It can be shown that the solution given in **part c.** is valid for all real values of k.

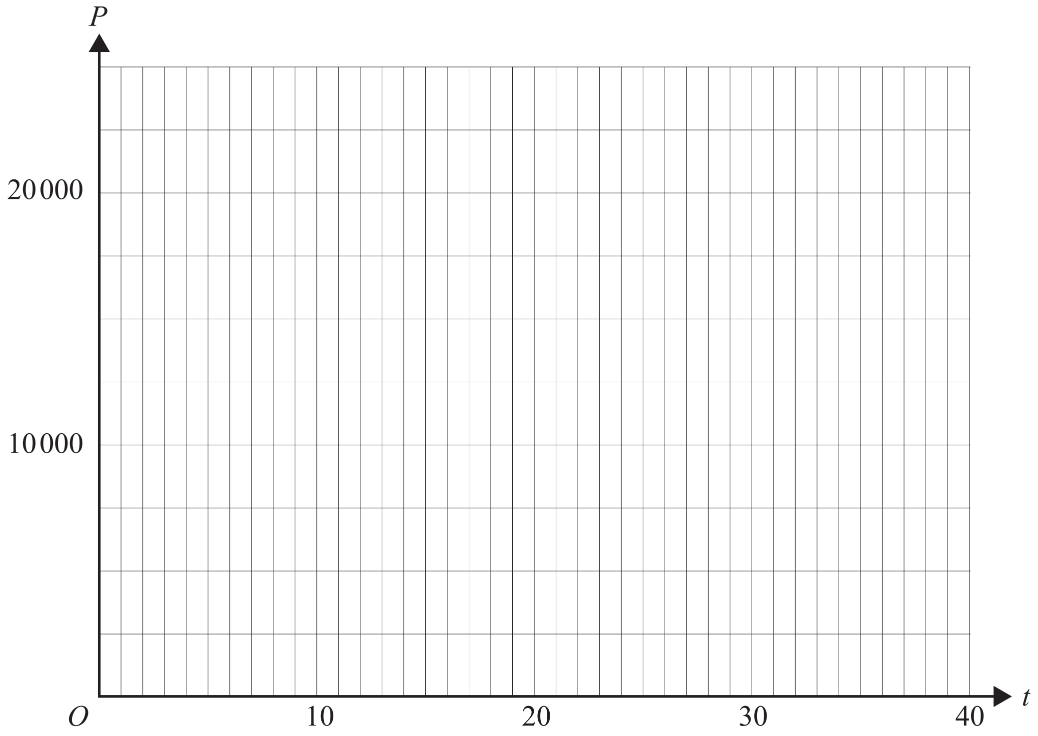
d. For each of the values

i. $k = 800$

ii. $k = 200$

iii. $k = 100$

sketch the graph of P versus t on the set of axes below while the population exists, for $0 \le t \le 40$.



3 marks

e. **i.** Find the value of k if the population has increased to 22 550 after twelve years.

ii. Use the definition of k to interpret your answer to **part i.** in the context of the population model.

1 + 1 = 2 marks

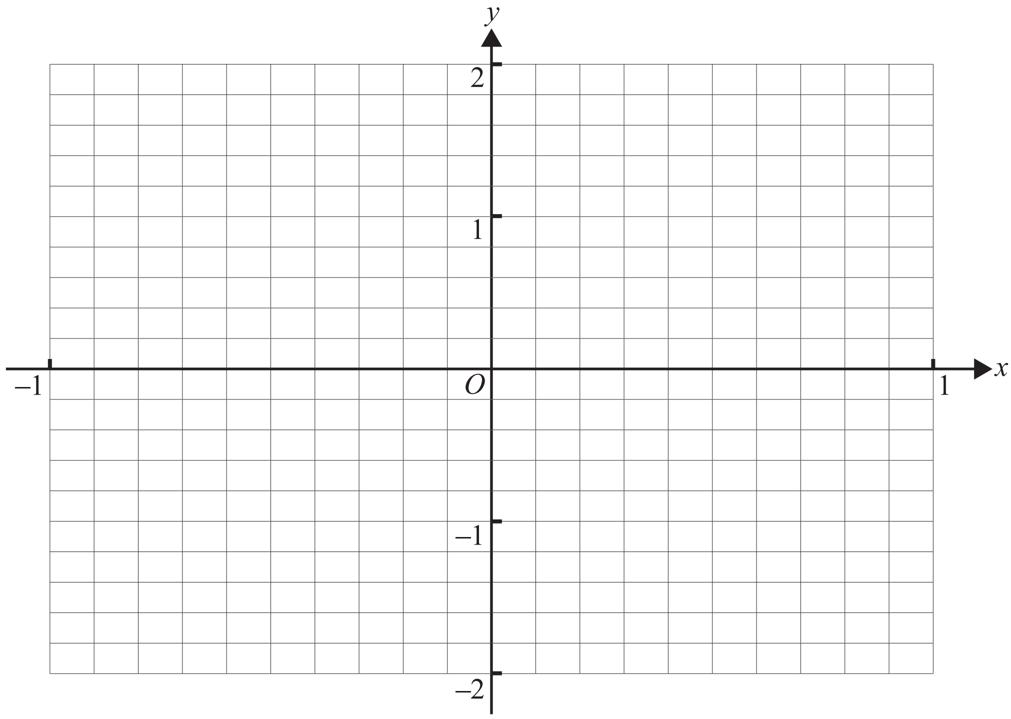
Total 12 marks

Working space

Question 4

Consider the function f with rule $f(x) = \sin^{-1}(2x^2 - 1)$.

a. Sketch the graph of the relation $y = f(x)$ on the axes below. Label the endpoints with their **exact coordinates,** and label the x and y intercepts with their exact values.



3 marks

b. **i.** Write down a **definite integral** in terms of y, which when evaluated will give the volume of the solid of revolution formed by rotating the graph drawn above about the y-axis.

ii. Find the exact value of the definite integral in **part i.**

2 + 1 = 3 marks

c. Use calculus to show that

$$f'(x) = \frac{2}{\sqrt{1-x^2}}, \text{ for } x \in (0, a) \text{ and find the value of } a.$$

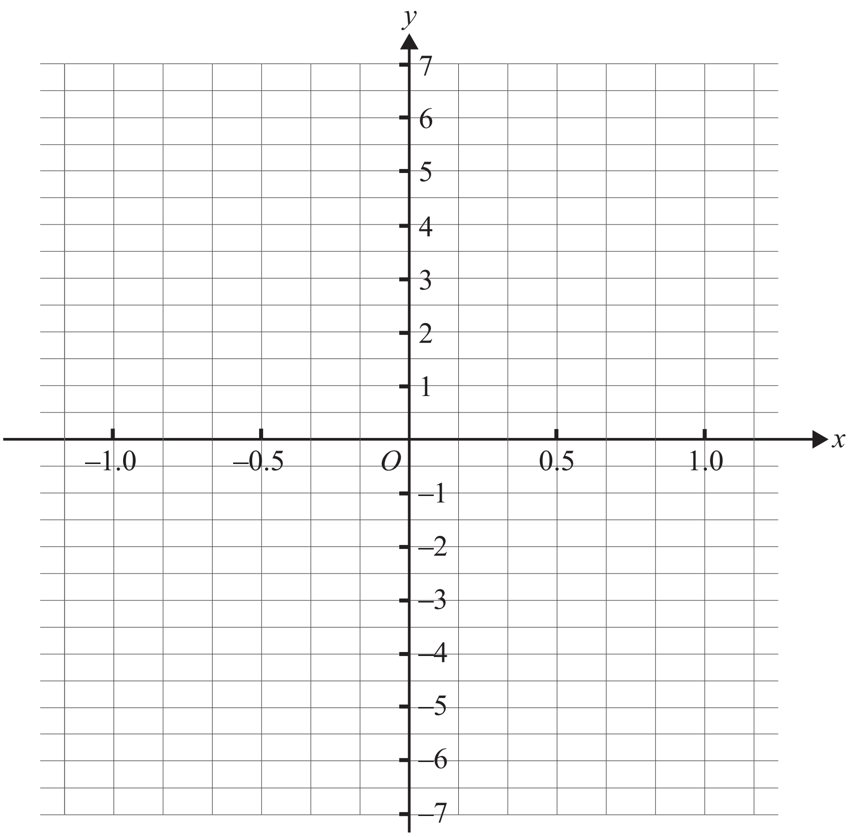
3 marks

d. Complete the following to specify $f'(x)$ as a hybrid function over the maximal domain of f'.

$$f'(x) = \begin{cases} \_\_\_\_\_\_\_\_\_\_ & \text{for } x \in \_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_ & \text{for } x \in \_\_\_\_\_ \end{cases}$$

2 marks

e. Sketch the graph of the hybrid function f' on the set of axes below, showing any asymptotes.



2 marks

Total 13 marks

Question 5

Let $u = 6 - 2i$ and $w = 1 + 3i$ where $u, w \in C$.

a. Given that $z_1 = \dfrac{(u+w)\overline{u}}{iw}$, show that $|z_1| = 10\sqrt{2}$.

1 mark

b. The complex number z_1 can be expressed in polar form as

$$z_1 = 200^{\frac{1}{2}} \operatorname{cis}\left(-\frac{3\pi}{4}\right).$$

Find all distinct complex numbers z such that $z^3 = z_1$.

Give your answers in the form $a^{\frac{1}{n}} \operatorname{cis}\left(\dfrac{b\pi}{c}\right)$, where a, b, c and n are integers.

3 marks

Let the argument of u be given by Arg $(u) = -\alpha$. (**You are not required to find $\boldsymbol{\alpha}$.**)

c. By expressing iw in polar form in terms of α, show that

$$\frac{\overline{u}}{iw} = 2\operatorname{cis}(2\alpha - \pi).$$

3 marks

d. Use the relation given in **part a.** to find Arg $(u + w)$ in terms of α.

3 marks

Total 10 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r \operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET