TSSM
Creating VCE Success

2010 Trial Examination

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STUDENT NUMBER

Figures									Letter:
Words									

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 22 pages.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks are **not** deducted for incorrect answers.
If more than 1 answer is completed for any question, no mark will be given.
Take the **acceleration due to gravity**, to have magnitude g m/s^2, where g = 9.8

Question 1

The equation $x^2 - 4y^2 - 4mx - 8my = 4$ represents

A. an ellipse with centre at the point $(-2m, m)$ and the semi-axes $a = 2m$ and $b = m$
B. a hyperbola with centre at the point $(2m, -m)$ and the semi-axes $a = 2$ and $b = 1$
C. an ellipse with centre at the point $(-2m, m)$ and the semi-axes $a = 1$ and $b = 2$
D. a hyperbola with centre at the point $(2m, -m)$ and the semi-axes $a = 4$ and $b = 1$
E. a hyperbola with centre at the point $(-2m, m)$ and the semi-axes $a = 1$ and $b = 2$

Question 2

If the function $f(x) = Ax^2 - x + B\ln x, x > 0,\ A, B \in R$ has a non-stationary point of inflection for $x = 1$ and is increasing over its implied domain, then

A. $A > \frac{1}{4}, B > \frac{1}{2}$

B. $0 < A < \frac{1}{4}, 0 < B < \frac{1}{2}$

C. $0 < A < \frac{1}{2}, B > \frac{1}{4}$

D. $A > \frac{1}{2}, 0 < B < \frac{1}{4}$

E. $0 < A < \frac{1}{2}, 0 < B < \frac{1}{4}$

SECTION 1 - continued

Question 3

The implied domain of $f(x) = \sqrt{4a^2x^3 - x},\ a \in R^+$, is

A. $\left(-\frac{1}{2a},\ 0\right) \cup \left(\frac{1}{2a}, +\infty\right)$

B. $\left(-\frac{1}{4a^2},\ 0\right) \cup \left(\frac{1}{4a^2}, +\infty\right)$

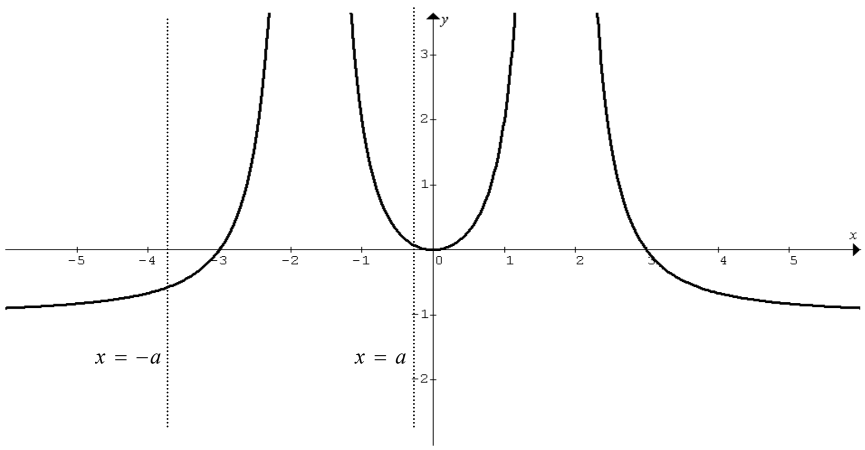
C. $\left[-\frac{1}{2a}, \frac{1}{2\text{a}}\right]$

D. $\left[-\frac{1}{2a},\ 0\right] \cup \left[\frac{1}{2a}, +\infty\right)$

E. The domain depends on the sign of *a.*

Question 4

This graph represents $y = f'(x)$. $f'(x) = 0$ for $x = -3, x = 0,$ and $x = 3$.



Which one of the following statements must be true?

A. $f(x)$ is increasing for $x \in (-\infty, -a) \cup (0, a)$

B. $f(x)$ has a local maximum at $x = -3$.

C. $f''(-4) < 0$

D. $f(x)$ has a stationary point of inflection at $x = 0$

E. $f''(x) > 0$ for $x \in (-a, a)$

SECTION 1 - continued
TURN OVER

Question 5

Let $S = \{z : |z| \le 3\} \cap \{z : |z - 3 - 3i| \le 3\}$. Which one of the following is correct?

A. $Arg(z) \ge \frac{\pi}{2}$

B. $3(\sqrt{2} - 1) \le |z| \le 3$

C. $Arg(z) = \frac{\pi}{4}$

D. $\sqrt{3} - 1 \le |z| \le 3$

E. $\text{Re}(z) - \text{Im}(z) = 3$

Question 6

If $|z| - z = 1 + 2i$, then

A. $\text{Re}(z) + \text{Im}(z) = \frac{1}{2}$

B. $|z| = \sqrt{5}$

C. $Arg(z) = \tan^{-1}(2)$

D. $\text{Re}(z) < \text{Im}(z)$

E. $|z| = \frac{5}{2}$

Question 7

If a polynomial with real coefficients has solutions $z = 1 - ai$ and $z = 2$ over the set of complex numbers, where a is real, then its quadratic factor must be

A. $z^2 + 4$

B. $z^2 - 2aiz + 1$

C. $z^2 - 2z + 1 + a^2$

D. $z^2 - 4$

E. $z^2 + 2z + 1 - a^2$

Question 8

Let $\mathbf{a} = 6\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$. The value of n for which vectors $\mathbf{a} + n\mathbf{b}$ and $\mathbf{c}$ are perpendicular is

A. $-\frac{3}{2}$

B. 4

C. $\frac{3}{2}$

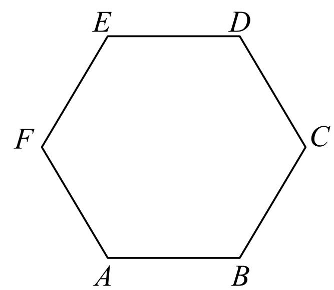
D. -1

E. 1

SECTION 1 - continued

Question 9

The diagram shows a regular hexagon *ABCDEF*. If $\overrightarrow{AB} = \mathbf{u}$ and $\overrightarrow{BC} = \mathbf{v}$, then vector $\overrightarrow{AE}$ can be expressed as



A. $2\mathbf{u} - \mathbf{v}$
B. $2\mathbf{u} + \mathbf{v}$
C. $-\mathbf{u} + 2\mathbf{v}$
D. $\mathbf{u} + 2\mathbf{v}$
E. $2\mathbf{u} - 2\mathbf{v}$

Question 10

$\tan\left(\cos^{-1}\frac{1}{\sqrt{1+a^2}} - \cos^{-1}\frac{a}{\sqrt{1+a^2}}\right),\ a > 0$ is equal to

A. $\frac{a^2-1}{2a}$
B. a
C. $\frac{1-a^2}{2a}$
D. 1
E. undefined

Question 11

The value of p for which the function $y = \ln\frac{1}{1+x}$ satisfies the differential equation

$x\frac{dy}{dx} + p = e^y$ is

A. e
B. $-e$
C. -1
D. 1
E. $\frac{1}{e}$

SECTION 1 – continued
TURN OVER

Question 12

The gradient of the normal to the curve defined parametrically by the equations $x = a\cos t,\ y = b\sin t,\ 0 \le t \le 2\pi$, at the point where $t = \frac{3\pi}{4}$ is

A. $-\frac{a}{b}$

B. $\frac{a}{b}$

C. ab

D. $-\frac{b}{a}$

E. $\frac{b}{a}$

Question 13

Using a suitable substitution, $\int_0^3 \frac{2x-1}{\sqrt{x+2}}\,dx$ is

A. $\frac{1}{2}\int_0^3 u\sqrt{\frac{2}{u+5}}\,du$

B. $\int_2^5 \left(2\sqrt{u} - \frac{5}{\sqrt{u}}\right)du$

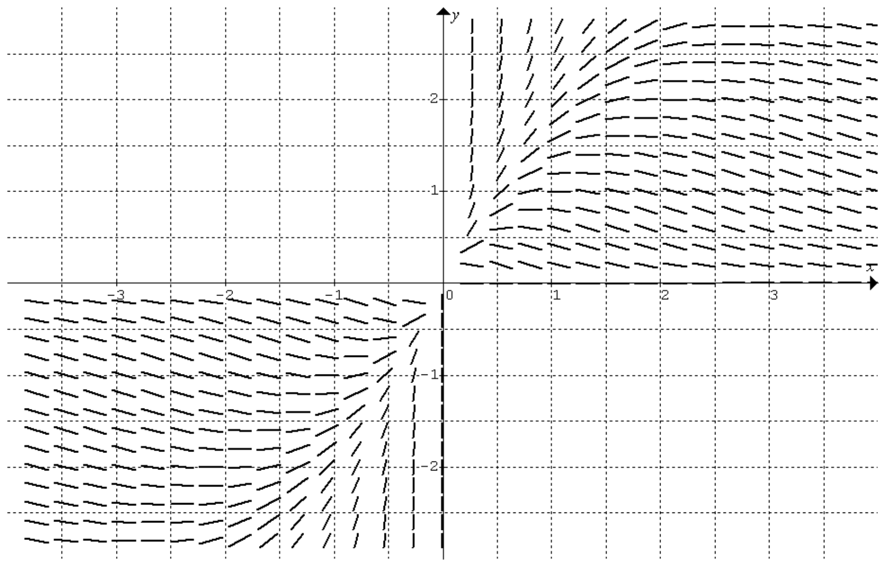
C. $\int_2^5 \left(5\sqrt{u} - \frac{2}{\sqrt{u}}\right)du$

D. $\int_{-1}^5 \left(\frac{u^2-1}{\sqrt{u}}\right)du$

E. $\int_{-1}^5 \left(\frac{u^2}{\sqrt{u-1}}\right)du$

SECTION 1 - continued

Question 14



The differential equation with the slope field shown above could be

A. $\frac{dy}{dx} = x\ln y$

B. $\frac{dy}{dx} = y\ln x$

C. $\frac{dy}{dx} = ye^{x}$

D. $\frac{dy}{dx} = \frac{y}{x}e^{\frac{y}{x}}$

E. $\frac{dy}{dx} = \frac{y}{x}\ln\left(\frac{y}{x}\right)$

Question 15

The area enclosed by the curve $y^2 = -x^4 + 5x^2 - 4$, correct to three decimal places, is

A. 1.162
B. 2.323
C. 0.581
D. 4.646
E. 3.485

SECTION 1 - continued
TURN OVER

Question 16

$\int \frac{\sin^2 x \cos^5 x}{1+\cos 2x}\,dx$, after the substitution $\sin x = u$, is the same as

A. $\frac{1}{2}\int \left(u^4 - u^2\right)du$

B. $-\int (1-u)\,du$

C. $\frac{1}{2}\int \left(u^2 - u^4\right)du$

D. $-\int \frac{1-u^4}{2u^2}\,du$

E. $\int \left(u^2 - 1\right)du$

Question 17

A tank contains 50 litres of sugar mixture at a concentration of 10 grams of sugar per litre. Three litres of sugar mixture at a concentration 8 grams per litre flow into the tank each minute and two litres of the resultant mixture flow out each minute. The amount of sugar in the tank, *x* grams, at any time, *t* minutes, can be found by solving the differential equation

A. $\frac{dx}{dt} = 24 + \frac{2x}{50-t}, t = 0, x = 50$

B. $\frac{dx}{dt} = 24 - \frac{x}{25}, t = 0, x = 50$

C. $\frac{dx}{dt} = 24 - \frac{2x}{50+t}, t = 0, x = 500$

D. $\frac{dx}{dt} = 6 + \frac{8x}{50-t}, t = 0, x = 500$

E. $\frac{dx}{dt} = 6 - \frac{8x}{50-t}, t = 0, x = 500$

Question 18

An approximation to the solution of the differential equation $\frac{dy}{dx} + \log_e(x+y) = x$, given that point $(1,0)$ lies on the curve, is found by using Euler's method with $h = 0.1$. The value of *y* obtained after two applications of Euler's method is

A. $0.21 - 0.1\log_e 1.2$

B. 0.21

C. $1.2 - 0.1\log_e 1.2$

D. 2.1

E. $0.12 - 1.1\log_e 1.2$

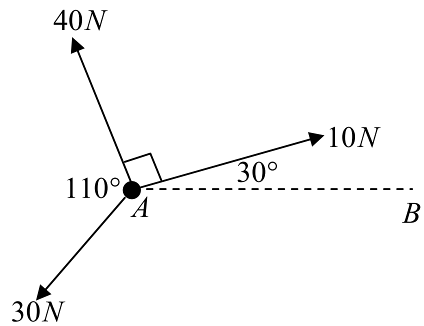
SECTION 1 - continued

Question 19
Two cars leave the same point at the same time. The velocity of the first car is given by $v_1 = 3t^2\ ms^{-1}$, while the velocity of the second car is given by $v_2 = (6t^2 - 10)\ ms^{-1}$. The distance between the cars after 10 seconds is

A. 900m
B. 1000m
C. 100m
D. 1100m
E. 800m

Question 20
The sum of the resolved parts of the forces in the direction perpendicular to *AB*, accurate to the nearest Newton, is

A. 31N
B. 63N
C. 40N
D. 17N
E. 15N



Question 21
A block of mass *m* kg rests on a rough inclined plane. It is kept stationary by a minimum force *F* Newtons acting parallel to the plane. If μ is the coefficient of friction between the plane and the block, α is the angle of inclination, *R* is the normal reaction force and *a* is the acceleration of the block, which one of the following is correct?
A. $F = mg\sin\alpha + \mu R$
B. $F = mg(\sin\alpha - \mu\cos\alpha)$
C. $F = mg(\sin\alpha + \mu\cos\alpha)$
D. $F = m(a + g\sin\alpha - \mu g\cos\alpha)$
E. $R - mg + F + \mu R = ma$

SECTION 1 - continued
TURN OVER

Question 22
The velocity of a particle moving in a horizontal line is given by
$v = \sqrt[3]{2 - x^2}\ ms^{-1}, -\sqrt{2} \le x \le 2.$ If the position of the particle is *x m,* then its acceleration is

A. $a = \dfrac{1}{3v}$

B. $a = -\dfrac{2x}{3v}$

C. $a = \dfrac{v}{x}$

D. $a = -\dfrac{x}{v}$

E. $a = \dfrac{2x}{3v}$

END OF SECTION 1

SECTION 2

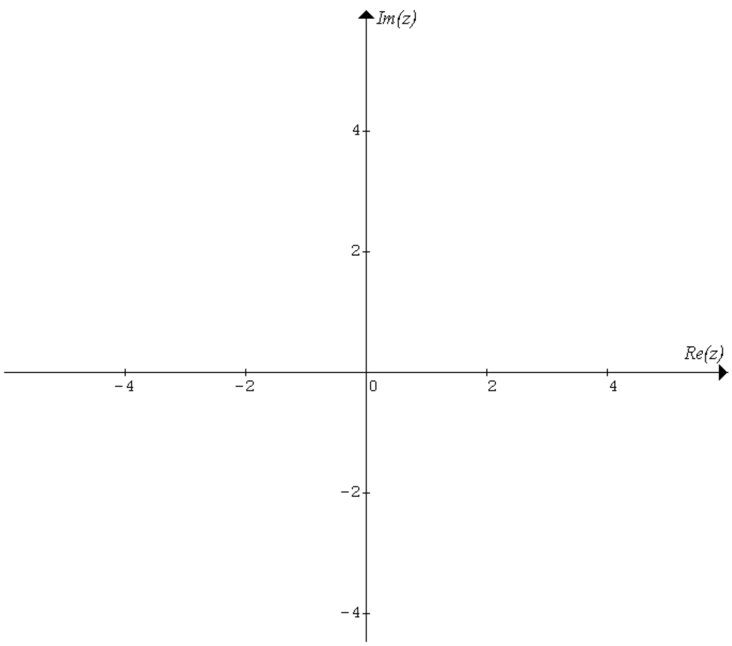
Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.
For questions worth more than one mark, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the **acceleration due to gravity**, to have magnitude g m/s^2 , where $g = 9.8$.

Question 1

Let $A = \left\{ z : Arg(1+z) = \dfrac{\pi}{4} \right\}$ and $B = \{ z : |1+z| = 2 \}$.

a. Sketch the complex regions A and B on the Argand diagram below.



4 marks

SECTION 2 – continued
TURN OVER

b. Write down the Cartesian equations of the regions *A* and *B*.

2 marks

c. Show that $z_0 = \left(\sqrt{2} - 1\right) + i\sqrt{2}$ is the point of intersection of the regions *A* and *B*.

2 marks

d. **i.** Find $\left(1 + z_0\right)^6$ in Cartesian form.

2 marks

SECTION 2 – continued

ii. Find both values of $\sqrt{1+z_0}$ in polar form.

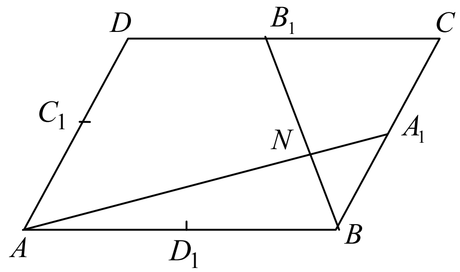
2 marks
Total 12 marks

SECTION 2 – continued
TURN OVER

Question 2

In the diagram below, *ABCD* is a parallelogram. Points A_1, B_1, C_1 and D_1 are the midpoints of *BC, CD, DA* and *AB* respectively. Point *N* is the point of intersection of AA_1 and BB_1.

Let $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.



a. Express $\overrightarrow{AA_1}$ and $\overrightarrow{BB_1}$ in terms of **a** and **b**.

2 marks

b. Use $\overrightarrow{AN} = n\,\overrightarrow{AA_1}$ and $\overrightarrow{BN} = m\,\overrightarrow{BB_1}$ to find the ratio $AN : NA_1$.

4 marks

SECTION 2 – continued

c. The angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ is $60°$ and $|\mathbf{a}| = \frac{2}{3}|\mathbf{b}|$. If $|\mathbf{a}| = k$, find $\overrightarrow{AA_1} \bullet \overrightarrow{BB_1}$ in terms of *k*.

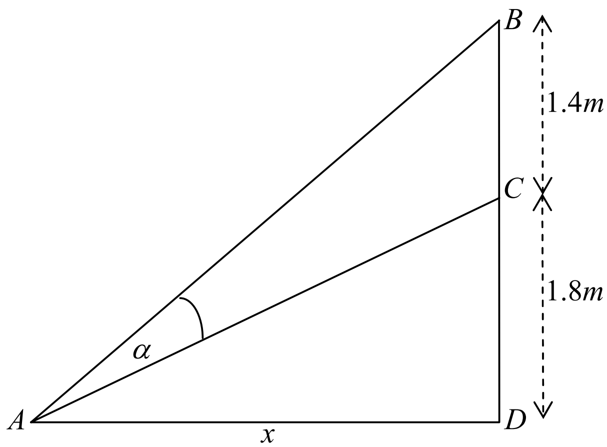
4 marks
Total 10 marks

SECTION 2 – continued
TURN OVER

Question 3

A monitor showing flight information hangs on a vertical wall inside the airport terminal. Its bottom end is 1.8 metres above the eyes of a traveller, which are at point A, as shown in the diagram. The monitor is 1.4 metres high.

Let α be the angle, in degrees, through which the monitor is seen by the traveller and x the distance, in metres, from the traveller to the wall.



a. Show that $\tan\alpha = \dfrac{1.4x}{x^2 + 5.76}$

4 marks

SECTION 2 – continued

b. Let $f(x) = \dfrac{1.4x}{x^2 + 5.76}$.

i. Show that $f'(x) = \dfrac{1.4\left(5.76 - x^2\right)}{(x^2 + 5.76)^2}$.

ii. **Hence,** find the maximum value of angle α, in degrees, correct to two decimal places.

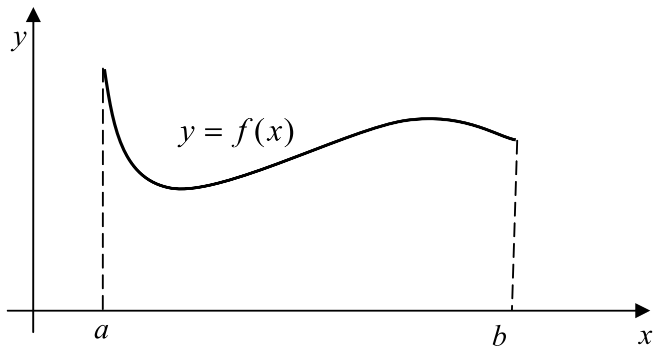
2 + 3 = 5 marks

c. The traveller moves along the line segment *AD,* towards the wall, with a speed of 1.2 ms^{-1}. Find the rate of change of angle α, in degrees per second, when the traveller is 4 metres away from the wall. Give your answer correct to one decimal place.

3 marks
Total 12 marks

SECTION 2 – continued
TURN OVER

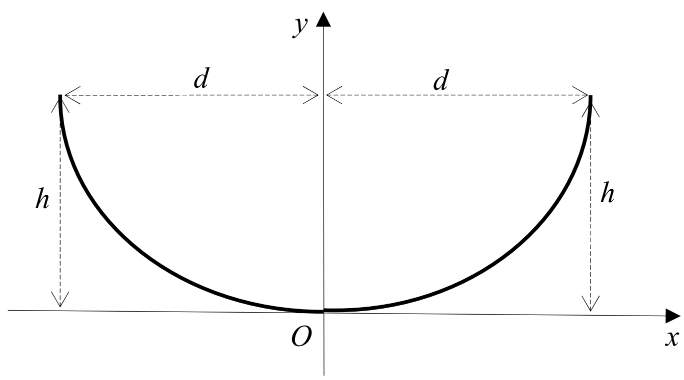
Question 4
In this question, the arc length formula will be used.



$$arc\ \ length = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$$

The length of the curve $y = f(x)$ between $x = a$ and $x = b$ is given by the arc length formula.

A flexible chain of length l hangs loosely between two poles of equal height a distance $2d$ metres apart, so that it sags a distance h metres in the centre.



The curve formed by the chain is called a **catenary**. Using a coordinate system with the lowest point at the origin, the catenary can be described by the equation $y = \dfrac{e^{mx} + e^{-mx} - 2}{2m}$, where m is a constant.

a. Show that $\left(\dfrac{dy}{dx}\right)^2 = \dfrac{e^{2mx} + e^{-2mx} - 2}{4}$.

2 marks

SECTION 2 – continued

b. Use the arc length formula to show that the length of the chain is given by $l = \dfrac{e^{md} - e^{-md}}{m}$.

4 marks

c. In a particular catenary, where $m = \dfrac{1}{32}$, it is required that the height, h, is 20 metres.

Find, correct to two decimal places

i. the distance between the poles.

SECTION 2 – continued
TURN OVER

ii. the length of the cable.

1+1= 2 marks

d. If the length of the cable is 100 metres and the distance between the poles is 80 metres, find the value of m, correct to two decimal places.

2 marks

Total 10 marks

SECTION 2 - continued

Question 5

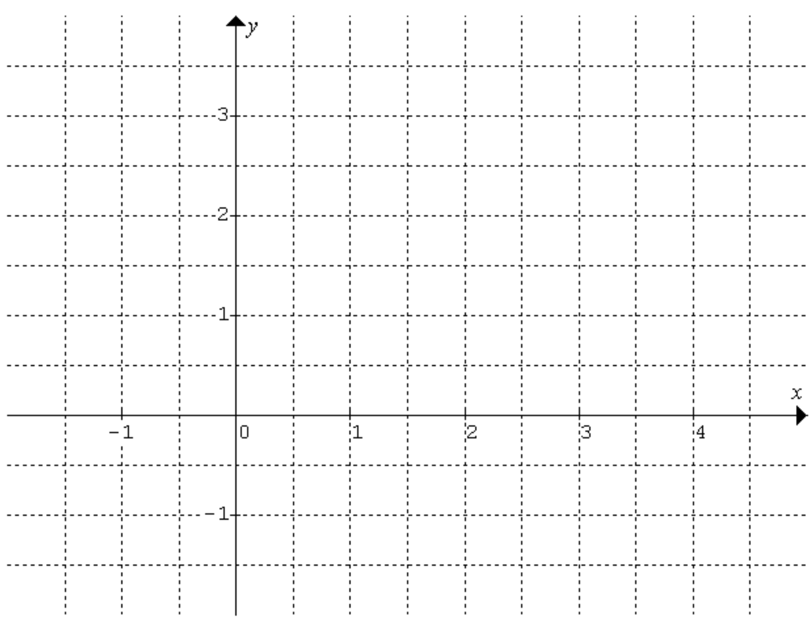
The position vectors of two particles, A and B, with respect to the origin, are given by $\mathbf{r}_A = (1+\cos(kt))\mathbf{i} + (2+2\sin(kt))\mathbf{j}$ and $\mathbf{r}_B = t\,\mathbf{i} + (2t^3 - 3t^2)\mathbf{j}$, where $t \geq 0$ is the time in seconds and k is a positive constant. The distance of the particles from the origin is measured in metres.

a. **i.** Show that the Cartesian equation of the path of particle A is $(x-1)^2 + \dfrac{(y-2)^2}{4} = 1$.

ii. Find the Cartesian equation of the path of particle B.

2+1 = 3 marks

b. On the axes below, sketch the paths of A and B.



2 marks

SECTION 2 – continued

TURN OVER

c. Find the maximum speed of particle A in terms of k.

4 marks

d. Find, correct to three decimal places, the coordinates of the points where the paths of A and B intersect.

2 marks

e. Find the minimum value of k for which the particles will collide, correct to three decimal places.

3 marks
Total 14 marks

END OF QUESTION AND ANSWER BOOK