THE SCHOOL FOR EXCELLENCE
UNIT 3 & 4 SPECIALIST MATHEMATICS 2010
COMPLIMENTARY WRITTEN EXAMINATION 2 - SOLUTIONS

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MARKING SCHEME (EXTENDED ANSWER QUESTIONS)

- ($A\,4\times\frac{1}{2}\downarrow$) means four answer half-marks rounded **down** to the next integer. Rounding occurs at the end of a part of a question.
- M1 = 1 **M**ethod mark.
- **A**1 = 1 **A**nswer mark (it **must** be this or its equivalent).
- **H**1 = 1 consequential mark (**H**is/**H**er mark…correct answer from incorrect statement or slip).

SECTION 1 – MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11
D	E	B	D	C	E	C	E	D	B	B

12	13	14	15	16	17	18	19	20	21	22
B	C	D	B	C	B	D	A	E	C	B

QUESTION 1 Answer is D

imaginary part $= r\sin(\theta)i = 2\sin\left(\frac{4\pi}{3}\right)i = -2\times\frac{\sqrt{3}}{2}i = -\sqrt{3}\,i$

QUESTION 2 Answer is E

The vector $\overrightarrow{AB} = 4\underset{\sim}{i} + 8\underset{\sim}{j} + 16\underset{\sim}{k} = 2(2\underset{\sim}{i} + 8\underset{\sim}{j} + 16\underset{\sim}{k})$

The vector $\overrightarrow{BC} = (x-4)\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}$

$\because$ The A, B and C are collinear.

$\therefore\ x - 4 = 2$
$x = 6$

QUESTION 3 Answer is B

$\because$ $6i$ and $2i$ are continues lines
$\therefore\ 2 \le \text{Im}(z) \le 6$

$\because$ Region enclosed by angle $\frac{3\pi}{4}$ with a continues line and angle $\frac{\pi}{4}$ with doted line

$\therefore\ \frac{\pi}{4} < \text{Arg}\,z \le \frac{3\pi}{4}$

$\therefore\ \{z : 2 \le \text{Im}(z) \le 6, \frac{\pi}{4} < \text{Arg}\,z \le \frac{3\pi}{4}\}$

QUESTION 4 Answer is D

When $b = 0$.

Ellipse with equation $\frac{x^2}{\left(\frac{36}{a}\right)} + \frac{y^2}{36} = 1$

$\because$ The ellipse only touches the hyperbola with equation $\frac{x^2}{9} - \frac{y^2}{9} = 1$.

$\therefore$ The ellipse and the hyperbola only intersect at the two x-intersects of the hyperbola (3, 0) and (-3, 0)

$\therefore\ \sqrt{\frac{36}{a}} = \sqrt{9}$

$\therefore\ a = 4$

When $b \in [-3,3]$

The ellipse and the hyperbola maximum can have 2 points of intersection.

When $b \in [-6,6]$

The ellipse and the hyperbola can have maximum 3 solutions. Use CAS to solve the equations of the ellipse and the hyperbola when $b = 6$, there are only 3 intersection points shown.

QUESTION 5 Answer is C

$$\left(1+i\sqrt{3}\right)^{\frac{4}{3}} = \left[\left(1+i\sqrt{3}\right)^{4}\right]^{\frac{1}{3}} = \left[2^4 cis\left(\frac{4\pi}{3}\right)\right]^{\frac{1}{3}}$$

Three equivalent representations are taken since we need this expression's cube root.

Therefore:

$$\left(1+i\sqrt{3}\right)^{4} = 16\ \text{cis}\left(\frac{4\pi}{3}\right),\ 16\text{cis}\left(\frac{10\pi}{3}\right),\ 16\text{cis}\left(\frac{16\pi}{3}\right)$$

Therefore:

$$\left(1+i\sqrt{3}\right)^{\frac{4}{3}} = 16^{\frac{1}{3}}\text{cis}\left(\frac{4\pi}{9}\right),\ 16^{\frac{1}{3}}\text{cis}\left(\frac{10\pi}{9}\right),\ 16^{\frac{1}{3}}\text{cis}\left(\frac{16\pi}{9}\right)$$

QUESTION 6 Answer is E

$$\int_{-a}^{a} \frac{1}{4+9x^2}\ dx = 1$$

$$\int_{-a}^{a} \frac{\frac{1}{9}}{\frac{4}{9}+x^2}\ dx = 1$$

$$\frac{1}{6}\int_{-a}^{a} \frac{\frac{2}{3}}{\frac{4}{9}+x^2}\ dx = 1$$

$$\frac{1}{6}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_{-a}^{a} = 1$$

$$\frac{1}{6}\left[\tan^{-1}\left(\frac{3a}{2}\right) - \tan^{-1}\left(\frac{-3a}{2}\right)\right] = 1$$

$$2\tan^{-1}\left(\frac{3a}{2}\right) = 6$$

$$\tan^{-1}\left(\frac{3a}{2}\right) = 3$$

$$\tan(3) = \frac{3a}{2}$$

$$a = \frac{2\tan(3)}{3}$$

QUESTION 7 Answer is C

$$z = cis(\theta)$$

$$z^n - \frac{1}{z^n} = z^n - z^{-n} = cis(n\theta) - cis(-n\theta)$$

$$= 2i\sin(n\theta)$$

QUESTION 8 Answer is E

Use CAS: $\int \frac{3x^2+9}{(2x-1)(x^2+2x+2)}\ dx = \frac{3}{2}\ln(|2x-1|) - 3\tan^{-1}(x+1)$

$\therefore\ a = \frac{3}{2}\quad b = 3\ \ \frac{d}{dx}\left(\tan^{-1}(x+1)\right) = \frac{1}{x^2+2x+2}$

$\therefore g(x) = \frac{1}{x^2+2x+2}$

QUESTION 9 Answer is D

$\sin(t+2t)$ = sin(t)cos(2t)+sin(2t)cos(t)

= sin(t) $\left[1-2\sin^2(t)\right] + 2\sin(t)\cos^2(t)$

$= \sin(t) - 2\sin^3(t) + 2\sin(t)(1-\sin^2(t))$

$= \sin(t) - 2\sin^3(t) + 2\sin(t) - 2\sin^3(t)$

$= -4\sin^3(t) + 3\sin(t)$

QUESTION 10 Answer is B

$$\frac{dt}{d\theta} = \frac{1}{-k(\theta-10)}$$

$$t = \frac{1}{-k}L_n(\theta-10)+c$$

Sub t=0, $\theta = 70$: $c = \frac{L_n(60)}{k}$

$$\therefore t = -\frac{1}{k}L_n(\theta-10) + \frac{L_n(60)}{k} = \frac{1}{k}\left(L_n(60) - L_n(\theta-10)\right)$$

Sub t = 10 $\theta = 40$. Use CAS to find k.

Sub t = 15. Use Solve() in CAS to Solve θ

$\theta = 31.21^0 C$

QUESTION 11 **Answer is B**

When y=2, $x \geq 0$

$$2x^2 = 2$$
$$x = 1$$

$$\therefore \pi\int_0^1 (2-2x^2)^2\,dx$$

$$\pi\int_0^1 \left(4-8x^2+4x^2\right)dx$$

QUESTION 12 **Answer is B**

$$\frac{dt}{dQ} = \frac{1}{4(100-Q)}$$

$$t = -\frac{1}{4}\ln(|100-Q|)+c$$

$$4(c-t) = \ln(|100-Q|)$$

$$e^{4(c-t)} = 100-Q$$

$$Q = -e^{4(c-t)}+100$$

QUESTION 13 **Answer is C**

$$\frac{\underset{\sim}{A}\cdot\underset{\sim}{B}}{|\mathrm{A}|} = \frac{-36+4+6}{\sqrt{76}} = \frac{-26}{\sqrt{76}} \approx -2.97$$

QUESTION 14 **Answer is D**

$$\int_{\frac{\pi}{2}}^{a}\left(\frac{\sin(\theta)}{1-\cos(\theta)}\right)dx = \frac{1}{2}$$

$$\left[L_n(|1-\cos(\theta)|)\right]_{\frac{\pi}{2}}^{a} = \frac{1}{2}$$

$$L_n(|1-\cos(a)|) = \frac{1}{2}$$

$$e^{\frac{1}{2}} = 1-\cos(a)$$

$$\cos(a) = 1-e^{\frac{1}{2}}$$

$$a = \cos^{-1}\left(1-\sqrt{e}\right)$$

QUESTION 15 **Answer is B**

$\because \underset{\sim}{a} \perp$ line $x + y = 1$

$$\frac{1-\sin(\theta)}{2\sin(\theta)} = 1$$

$$1-\sin(\theta) = 2\sin(\theta)$$

$$(1-\sin(\theta))^2 = 4\sin^2(\theta)$$

$$1-2\sin(\theta)+\sin^2(\theta) = 4\sin^2(\theta)$$

$$0 = 3\sin^2(\theta)+2\sin(\theta)-1$$

$\sin(\theta) = \dfrac{1}{3}$ or $\sin(\theta) = -1$

$\because$ Finding the acute angle

$$\therefore \sin(\theta) = \frac{1}{3}$$

$$\therefore \ \theta = \sin^{-1}\left(\frac{1}{3}\right)$$

QUESTION 16 **Answer is C**

$$\frac{dv}{dt} = -\frac{1}{2}v$$

$$\frac{dt}{dv} = \frac{-2}{v}$$

$$t = -2\ln(|v|) + c$$

$$Sub\ t = 0 \quad v = 40$$

$$c = 2\ln(40)$$

$$t = -2\ln(|v|) + 2\ln(40)$$

$$t = 2\ln\left(\frac{40}{v}\right)$$

$$0.5t = \ln\left(\frac{40}{v}\right)$$

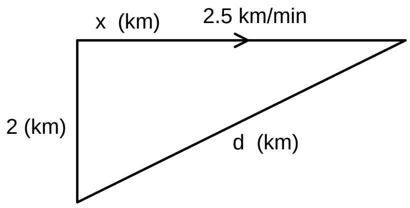
$$e^{0.5t} = \frac{40}{v}$$

$$v = 40e^{-0.5t}$$

QUESTION 17 **Answer is B**

150km/h = 2.5km/min

$$\frac{dd}{dt}=\frac{dx}{dt}\times\frac{dd}{dt}$$
$$=2.5\times\frac{dd}{dt}$$
$$=2.5\times\frac{x}{\sqrt{4+x^2}}$$



$When \quad t=1 \quad x=2.5$

$$\frac{dd}{dt}=\frac{2.5\times 2.5}{\sqrt{10.25}}\Rightarrow 1.95km/\min$$

$$d=\sqrt{2^2+x^2}$$
$$=\sqrt{4+x^2}$$
$$\frac{dd}{dx}=\frac{x}{\sqrt{4+x^2}}$$

QUESTION 18 **Answer is D**

$$r=4e^{0.5t}i-4e^{-0.5t}j+c$$
$$0=4i-4j+c$$
$$c=-4i+4j$$
$$r=\left(4e^{0.5t}-4\right)i-\left(4e^{-0.5t}-4\right)j$$
$$=4\left(e^{0.5t}-1\right)i-4\left(e^{-0.5t}-1\right)i$$

QUESTION 19 **Answer is A**

$$a=\frac{4g-3g}{4+7}\ =\frac{g}{7}$$

QUESTION 20 **Answer is E**

$$a=\frac{F}{m}=\frac{m_2g-\mu N}{m_2-m_1}$$
$$=\frac{m_2g-\mu m_1g}{m_2-m_1}$$
$$=\frac{\left(m_2-\mu m_1\right)g}{m_2-m_1}$$

QUESTION 21 **Answer is C**

Let mass M_1 moving direction as +

Total momentum = $7M_1+(-5M_2)$

$=7M_1-5M_2$

QUESTION 22 **Answer is B**

$$\frac{dx}{dt}=10t-4$$

$$\frac{dy}{dt}=10t$$

$$\frac{dx}{dy}=\frac{dx}{dt}\times\frac{dt}{dy}=\frac{10t-4}{10t}$$

$$Let\ \frac{dx}{dy}=0$$
$$10t-4=0$$
$$t=0.4$$

$Sub\ t=0.4$ into x (t) and y(t)

$x(0.4)=-2.8$
$y(0.4)=5.8$

SECTION 2

QUESTION 1

a. (i) $|1+i| = \sqrt{2} \Rightarrow \left|(1+i)^k\right| = \left(\sqrt{2}\right)^k = \sqrt{2^k}$.

(ii) $x + iy = (1+i)^k$

$\therefore \sqrt{x^2 + y^2} = \sqrt{2^k}$

$\therefore x^2 + y^2 = 2^k$ as required.

b. (i) $1 + i = \sqrt{2}cis\left(\frac{\pi}{4}\right)$

$1 - i = \sqrt{2}cis\left(-\frac{\pi}{4}\right)$

(ii) $(1+i)^{2n} + (1-i)^{2n} = \left(\sqrt{2}cis\left(\frac{\pi}{4}\right)\right)^{2n} + \left(\sqrt{2}cis\left(-\frac{\pi}{4}\right)\right)^{2n}$

$$= \left(\sqrt{2}\right)^{2n} cis\left(\frac{n\pi}{2}\right) + \left(\sqrt{2}\right)^{2n} cis\left(-\frac{n\pi}{2}\right)$$

$\underline{n=2}$: $(1+i)^4 + (1-i)^4 = 4cis(\pi) + 4cis(-\pi) = -8$

$\underline{n=3}$: $(1+i)^6 + (1-i)^6 = 8cis\left(\frac{3\pi}{2}\right) + 8cis\left(-\frac{3\pi}{2}\right) = 0$

$\underline{n=4}$: $(1+i)^8 + (1-i)^8 = 16cis(2\pi) + 16cis(-2\pi) = 32$

(iii) $(1+i)^{2n} + (1-i)^{2n} = \left(\sqrt{2}\right)^{2n} cis\left(\frac{n\pi}{2}\right) + \left(\sqrt{2}\right)^{2n} cis\left(-\frac{n\pi}{2}\right)$

$$= 2^n \cos\left(\frac{n\pi}{2}\right) + i\sin\left(\frac{n\pi}{2}\right) + 2^n \cos\left(-\frac{n\pi}{2}\right) + i\sin\left(-\frac{n\pi}{2}\right)$$

$$= 2^n 2\cos\left(\frac{n\pi}{2}\right) = 2^{n+1}\cos\left(\frac{n\pi}{2}\right)$$

<u>*n* is an odd integer</u>: $\cos\left(\frac{n\pi}{2}\right) = 0$. Therefore $(1+i)^{2n} + (1-i)^{2n} = 0$

<u>*n*/2 is an odd integer</u>: $\cos\left(\frac{n\pi}{2}\right) = -1$. Therefore $(1+i)^{2n} + (1-i)^{2n} = -2^{n+1}$

Note: If *n*/2 is odd then *n* is a multiple of 2 but not a multiple of 4.

<u>*n*/2 is an even integer</u>: $\cos\left(\frac{n\pi}{2}\right) = 1$. Therefore $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1}$

Note: If *n*/2 is even then *n* is a multiple of 4.

c. (i) From De Moivre's Theorem $k = 2$.

(ii) $\dfrac{cis(3\theta)}{cis(2\theta)} = cis(3\theta - 2\theta) = cis(\theta)$.

(iii) $A + iB = cis\left(\dfrac{\pi}{n}\right) + cis\left(\dfrac{2\pi}{n}\right) + cis\left(\dfrac{3\pi}{n}\right) + \cdots + cis\left(\dfrac{2n\pi}{n}\right)$

(iv) $A + iB$ is a geometric series with $a = cis\left(\dfrac{\pi}{n}\right)$, $r = cis\left(\dfrac{\pi}{n}\right)$ and number of terms equal to 2*n*.

From the given formula:

$$A + iB = \frac{cis\left(\frac{\pi}{n}\right)\left(1 - \left[cis\left(\frac{\pi}{n}\right)\right]^{2n}\right)}{1 - cis\left(\frac{\pi}{n}\right)}$$

$$= \frac{cis\left(\frac{\pi}{n}\right)\left(1 - cis(2\pi)\right)}{1 - cis\left(\frac{\pi}{n}\right)}$$

$$= \frac{cis\left(\frac{\pi}{n}\right)(1 - 1)}{1 - cis\left(\frac{\pi}{n}\right)} = 0$$

Therefore $A = 0$ and $B = 0$.

QUESTION 2

a. (i) $\frac{dt}{dv} = \frac{-50}{v(1+v^2)}$

$\therefore t = -\int_{10}^{v} \frac{50}{u(1+u^2)} du + 0$

Substitute $v = 5$: $\therefore t = -\int_{10}^{5} \frac{50}{u(1+u^2)} du = \int_{10}^{5} \frac{50}{v(1+v^2)} dv$

(ii) $t = 0.7318$ seconds.

b. (i) $v\frac{dv}{dx} = \frac{-v(1+v^2)}{50}$

$\frac{dv}{dx} = \frac{-(1+v^2)}{50}$

(ii) $\frac{dx}{dv} = \frac{-50}{1+v^2}$

$x = -\int \frac{50}{1+v^2} dv = -50\tan^{-1}(v) + C$.

Substitute $v = 10$ when $x = 0$: $C = 50\tan^{-1}(10)$.

Therefore $x = 50\tan^{-1}(10) - 50\tan^{-1}(v)$.

(iii) $\frac{x}{50} = \tan^{-1}(10) - \tan^{-1}(v)$

$\therefore \tan\left(\frac{x}{50}\right) = \tan\left(\tan^{-1}(10) - \tan^{-1}(v)\right)$.

Apply the double angle formula $\tan(A-B) = \frac{\tan(A) - \tan(B)}{1+\tan(A)\tan(B)}$:

$\tan\left(\frac{x}{50}\right) = \frac{10-v}{1+10v}$

$\Rightarrow \tan\left(\frac{x}{50}\right) + 10v\tan\left(\frac{x}{50}\right) = 10 - v$

$\Rightarrow 10v\tan\left(\frac{x}{50}\right) + v = 10 - \tan\left(\frac{x}{50}\right)$

$\Rightarrow v = \frac{10 - \tan\left(\frac{x}{50}\right)}{1 + 10\tan\left(\frac{x}{50}\right)}$ as required.

c. (i) $0 = \dfrac{10 - \tan\left(\dfrac{x}{50}\right)}{1 + 10\tan\left(\dfrac{x}{50}\right)} \Rightarrow 10 - \tan\left(\dfrac{x}{50}\right) = 0 \Rightarrow x = 50\tan^{-1}(10) \approx 73.56$.

Therefore the particle is stationary at 73.56 metres to the right of *O*.

(ii) Substitute $v = 0$ into $a = \dfrac{-v(1+v^2)}{50}$: $a = 0$.

Therefore the velocity does not change.
Therefore the velocity remains stationary at 73.56 metres to the right of *O*.

QUESTION 3

a. Require $9 - x^2 \ge 0$ and $-1 \le \dfrac{x}{3} \le 1$. Therefore $-3 \le x \le 3 \Rightarrow D = [-3, 3]$.

b. (i) $V = \pi \int y^2\, dx = \pi \int_0^{2.8} \left(x\sqrt{9 - x^2} + 2\arcsin\left(\dfrac{x}{3}\right)\right)^2 dx$.

(ii) $V \approx 180.5$ cubic units.

c. (i) $f'(x) = \sqrt{9 - x^2} - \dfrac{x^2}{\sqrt{9 - x^2}} + \dfrac{2}{\sqrt{9 - x^2}}$

$= \sqrt{9 - x^2} + \dfrac{2 - x^2}{\sqrt{9 - x^2}}$

$= \dfrac{9 - x^2 + 2 - x^2}{\sqrt{9 - x^2}}$

$= \dfrac{11 - 2x^2}{\sqrt{9 - x^2}}$

(ii) $f''(x) = \dfrac{(-4x)\sqrt{9 - x^2} - \left(\dfrac{-x}{\sqrt{9 - x^2}}\right)(11 - 2x^2)}{9 - x^2}$

$= \dfrac{(-4x)(9 - x^2) + x(11 - 2x^2)}{(9 - x^2)^{3/2}}$

$= \dfrac{2x^3 - 25x}{(9 - x^2)^{3/2}}$

$= \dfrac{x(2x^2 - 25)}{(9 - x^2)^{3/2}}$ as required.

d. $\displaystyle\int_{-p}^{p} \frac{11-2x^2}{\sqrt{9-x^2}}\,dx = \left[x\sqrt{9-x^2} + 2\arcsin\left(\frac{x}{3}\right)\right]_{-p}^{p}$

$$= \left(p\sqrt{9-p^2} + 2\arcsin\left(\frac{p}{3}\right)\right) - \left(-p\sqrt{9-p^2} + 2\arcsin\left(\frac{-p}{3}\right)\right)$$

$$= 2p\sqrt{9-p^2} + 4\arcsin\left(\frac{p}{3}\right) = 2\left(p\sqrt{9-p^2} + 2\arcsin\left(\frac{p}{3}\right)\right).$$

e. The maximum value of $I = p\sqrt{9-p^2} + 2\arcsin\left(\frac{p}{3}\right)$ is required.

From part **(c)**: $\displaystyle\frac{dI}{dp} = \frac{11-2p^2}{\sqrt{9-p^2}}$

$$\frac{dI}{dp} = 0 \Rightarrow \frac{11-2p^2}{\sqrt{9-p^2}} = 0 \Rightarrow p = \pm\sqrt{\frac{11}{2}}\ .$$

From the sign test there is a maximum turning point when $p = \sqrt{\frac{11}{2}}$.

Therefore the maximum value occurs when $p = \sqrt{\frac{11}{2}}$.

f. Inflection points of $f(x)$ occur at values of *x* for which $f'(x)$ has turning points.

Stationary points of $f'(x)$: $f''(x) = \dfrac{x(2x^2-25)}{(9-x^2)^{3/2}} = 0$ when $x = 0$ or $x = \pm\dfrac{5}{\sqrt{2}}$.

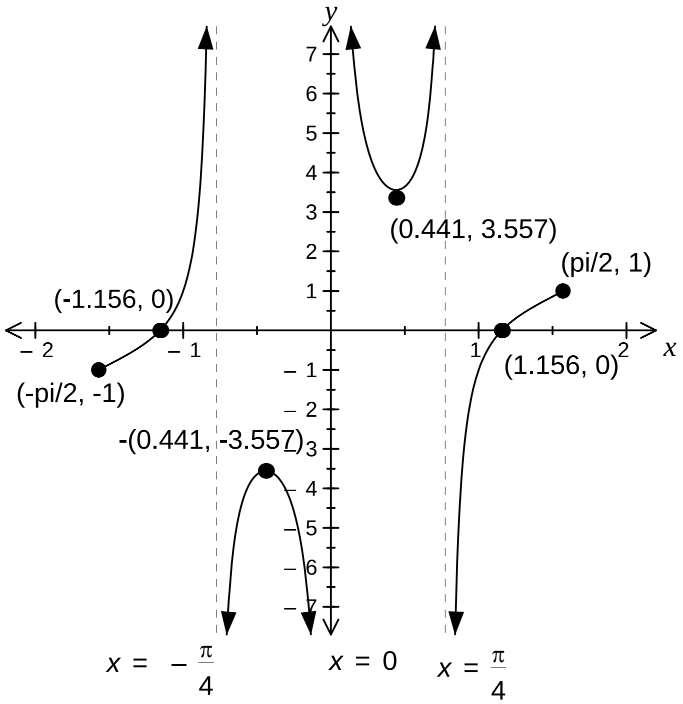
From the sign test $f'(x)$ has a turning point when $x = 0$.

Therefore $f(x)$ has an inflection point at $x = 0$.

$f(x)$ has no inflection point at $x = \pm\dfrac{5}{\sqrt{2}}$ because these values of *x* lie outside the domain *D* (see part (a)).

QUESTION 4

a.



b. $\frac{1}{\sin(x)} + \frac{\sin(2x)}{\cos(2x)} = 0$

$\Rightarrow \frac{1}{\sin(x)} + \frac{2\sin(x)\cos(x)}{2\cos^2(x) - 1} = 0$

$\Rightarrow \frac{2\cos^2(x) - 1 + 2\sin(x)\cos(x)}{2\sin(x)(\cos^2(x) - 1)} = 0$

$\Rightarrow 2\cos^2(x) + 2\sin^2(x)\cos(x) = 0$

$\Rightarrow 2\cos^2(x) + 2(1 - \cos^2(x))\cos(x) = 0$

$\Rightarrow 2\cos^2(x) + 2\cos(x) - 2\cos^3(x) = 0$

$\Rightarrow 2\cos^3(x) - 2\cos^2(x) - 2\cos(x) + 1 = 0$ as required.

c. $\dfrac{dy}{dx}=\dfrac{-\cos(x)}{\sin^2(x)}+\dfrac{2}{\cos^2(2x)}=0$

$\Rightarrow \dfrac{-\cos(x)\cos^2(2x)+2\sin^2(x)}{\sin^2(x)\cos^2(2x)}=0$

$\Rightarrow -\cos(x)\cos^2(2x)+2\sin^2(x)=0$

$\Rightarrow -\cos(x)(2\cos^2(x)-1)^2+2(1-\cos^2(x))=0$

$\Rightarrow -4\cos^5(x)+4\cos^3(x)-\cos(x)+2-2\cos^2(x)=0$

$\Rightarrow 4\cos^5(x)-4\cos^3(x)+2\cos^2(x)+\cos(x)-2=0$ as required.

d. $f(\pi-x)=\dfrac{1}{\sin(\pi-x)}+\dfrac{\sin(\pi-2x)}{\cos(\pi-2x)}=\dfrac{1}{\sin(x)}+\dfrac{\sin(2x)}{-\cos(2x)}=\dfrac{1}{\sin(x)}-\dfrac{\sin(2x)}{\cos(2x)}$.

$f(\pi+x)=\dfrac{1}{\sin(\pi+x)}+\dfrac{\sin(\pi+2x)}{\cos(\pi+2x)}=\dfrac{1}{-\sin(x)}+\dfrac{-\sin(2x)}{-\cos(2x)}=\dfrac{-1}{\sin(x)}+\dfrac{\sin(2x)}{\cos(2x)}$.

Therefore $f(\pi-x)+f(\pi-x)=0$.

QUESTION 5

a. At $t = -1$:

Position of satellite: $\underset{\sim}{r}(-1) = -\underset{\sim}{i} - 10\underset{\sim}{j} - 2\underset{\sim}{k}$.

Position of island: $\underset{\sim}{L} = \underset{\sim}{i} - 12\underset{\sim}{j} - 2\underset{\sim}{k}$

Position of satellite relative to island: $\underset{\sim}{r}(-1) - \underset{\sim}{L} = -2\underset{\sim}{i} + 2\underset{\sim}{j}$.

Distance: $|-2\underset{\sim}{i} + 2\underset{\sim}{j}| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$.

b. Relative position of satellite at time *t*: $\underset{\sim}{r} - \underset{\sim}{L} = (2t)\underset{\sim}{i} + (t^2 + 1)\underset{\sim}{j} + (t^2 - 1)\underset{\sim}{k}$.

Distance: $\left|(2t)\underset{\sim}{i} + (t^2 + 1)\underset{\sim}{j} + (t^2 - 1)\underset{\sim}{k}\right| = \sqrt{(2t)^2 + (t^2 + 1)^2 + (t^2 - 1)^2}$

$$= \sqrt{4t^2 + t^4 + 2t^2 + 1 + t^4 - 2t^2 + 1}$$
$$= \sqrt{2(t^4 + 2t^2 + 1)}$$
$$= \sqrt{2}\sqrt{(t^2 + 1)^2}$$
$$= \sqrt{2}\,|t^2 + 1|$$
$$= \sqrt{2}(t^2 + 1) \quad \text{as required.}$$

c. From part (b) the minimum distance occurs when $t = 0$ and is equal to $\sqrt{2}$.

d. At $t = -1$:

Position of satellite: $\underset{\sim}{r}(-1) = -\underset{\sim}{i} - 10\underset{\sim}{j} - 2\underset{\sim}{k}$.

Position of satellite relative to island: $-2\underset{\sim}{i} + 2\underset{\sim}{j}$.

At $t = -2$:

Position of satellite: $\underset{\sim}{r}(-2) = -4\underset{\sim}{i} + 5\underset{\sim}{j} + 3\underset{\sim}{k}$.

Position of satellite relative to island: $-4\underset{\sim}{i} - 5\underset{\sim}{j} + \underset{\sim}{k}$.

From the dot product:

Angle between vectors $-2\underset{\sim}{i} + 2\underset{\sim}{j}$ and $-4\underset{\sim}{i} - 5\underset{\sim}{j} + \underset{\sim}{k}$ is

$$\cos^{-1}\left(\frac{8 + 10}{\sqrt{8}\sqrt{50}}\right) = \cos^{-1}\left(\frac{18}{20}\right) \approx 25.84^0.$$

Angle to nearest degree is 26^0.

QUESTION 6

a. Net force perpendicular to slope: $0 = N - 1200g\cos(8^0)$ (1)

Net force parallel to slope: $(1200)(0.25) = T - 1200g\sin(8^0) - 0.09N$ (2)

Substitute (1) into (2): $300 = T - 1200g\sin(8^0) - (0.09)(1200g\cos(8^0))$

$\Rightarrow T = 300 + 1200g(\sin(8^0) + 0.09\cos(8^0))$

$\Rightarrow T \approx 2985$ newtons

b. Net force parallel to slope: $0 = T + F - 1200g\sin(8^0)$

$\Rightarrow T = 1200g\sin(8^0) - F$

c. (i) $T = 1200g\sin(8^0) - (0.09)1200g\cos(8^0) \approx 589$ newtons.

(ii) Maximum friction: $\mu N = (0.15)1200g\cos(8^0) \approx 1746.8$ newtons.

Component of weight force of log down the slope: $1200g\sin(8^0) \approx 1636.7$ newtons.

Therefore the log is not about to slide down the slope.

Therefore $T = 0$.