



THE SCHOOL FOR EXCELLENCE (TSFX)

UNIT 4 SPECIALISTS MATHEMATICS 2010

WRITTEN EXAMINATION 1

Reading Time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers.

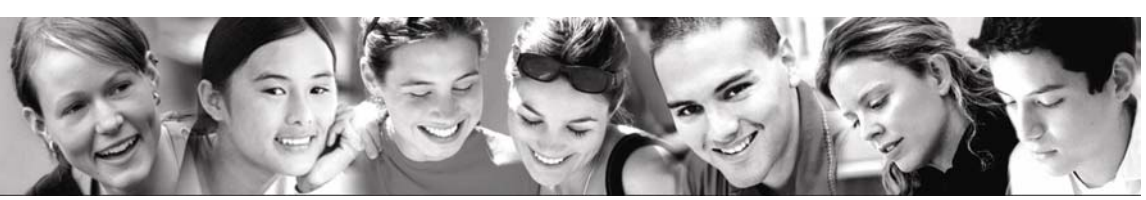
Students are **NOT** permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

All written responses must be in English.

COMPLIMENTS OF THE SCHOOL FOR EXCELLENCE

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PRINTING SPECIFICATIONS

Please ensure that the paper size on your printer is selected as **A4** and that you select "**None**" under "Page Scaling".

Instructions

- Answer **all** questions in the spaces provided.
- In questions where more than 1 mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the acceleration due to gravity to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

QUESTION 1

- (a) Show that by a suitable substitution $\int_0^{\ln 3} \frac{e^x}{e^{2x} + 9} dx$ can be written in the form $\int_1^3 \frac{du}{u^2 + 9}$.

- (b) Hence show that the exact value of $\int_0^{\ln 3} \frac{e^x}{e^{2x} + 9} dx$ is $\frac{\pi - 4 \tan^{-1}\left(\frac{1}{3}\right)}{12}$.

2 + 2 = 4 marks

QUESTION 2

P, Q and R are three distinct points in space which have position vectors \vec{OP} , \vec{OQ} and \vec{OR} respectively.

(a) Write down expressions for \vec{QR} and \vec{RP} in terms of \vec{OP} , \vec{OQ} , \vec{OR} .

(b) If \vec{OP} is perpendicular to \vec{QR} and \vec{OQ} is perpendicular to \vec{RP} , show that \vec{OR} is perpendicular to \vec{QP} .

1 + 3 = 4 marks

QUESTION 3

Consider the relation $e^{xy} - y^2 \log_e x = e$.

- (a) If the point $(1, a)$ satisfies the relation, find a .

- (b) Find $\frac{dy}{dx}$ when $x = 1$.

1 + 3 = 4 marks

QUESTION 4

The acceleration $a \text{ ms}^{-2}$ of an object x metres from a point O is given by $a = \frac{2x}{x^2 + 1}$.

- (a) Find the velocity $v \text{ ms}^{-1}$ of the object in terms of x , given $v = 2$ when $x = 1$.

- (b) If the velocity of the object when $x = 5$ is given by $v = \sqrt{\log_e(a) + b}$, find the exact values of a and b .

3 + 1 = 4 marks

QUESTION 5

Let $z = \frac{-1+i\sqrt{3}}{4}$ and $w = \frac{\sqrt{2}+i\sqrt{2}}{4}$.

(a) Write z and w in the form $r(\cos\theta + i\sin\theta)$ where $0 \leq \theta \leq \pi$.

(b) Show that $zw = \frac{1}{4} \left(\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right) \right)$.

(c) (i) Evaluate zw in the form $a + ib$.

(ii) Hence find the exact values of $\cos\left(\frac{11\pi}{12}\right)$ and $\sin\left(\frac{11\pi}{12}\right)$.

2 + 1 + 2 = 5 marks

QUESTION 6

Car A is travelling on a straight east-west road in a westerly direction at 60 kmh^{-1} . Car B is travelling on a straight north-south road in a northerly direction at 70 kmh^{-1} . The roads intersect at the point O. When car A is $x \text{ km}$ east of O, Car B is $y \text{ km}$ south of O, the distance between the cars is $z \text{ km}$.

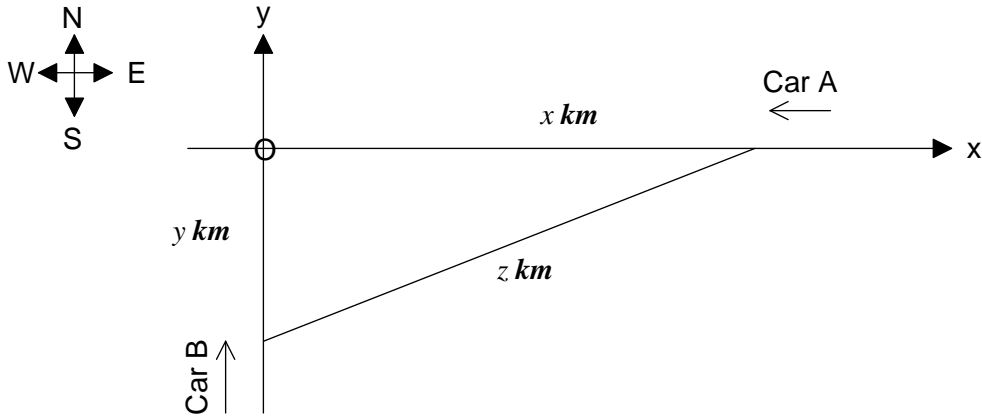


Diagram not to scale.

- (a) Write down the values of $\frac{dx}{dt}$ and $\frac{dy}{dt}$, where t is the time in hours.

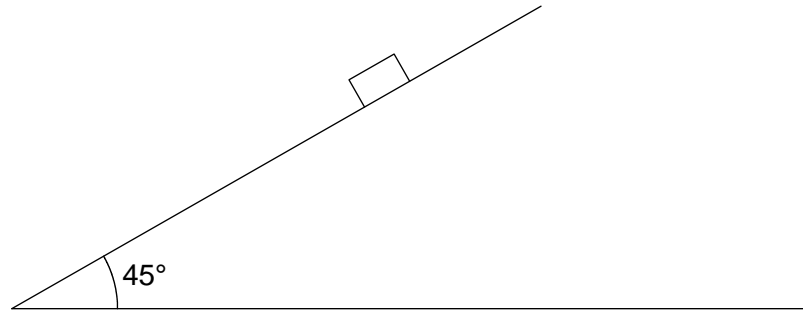
- (b) Differentiate $z^2 = x^2 + y^2$ implicitly with respect to t and hence find the rate of change of z when Car A is 0.8 km east of O and car B is 0.6 km south of O.

1 + 3 = 4 marks

QUESTION 7

An object of mass 2kg resting on a rough surface is found to be on the point of slipping down when the surface is inclined 45° to the horizontal.

- (a) Draw a force diagram labelling the weight force W Newtons, the normal reaction R Newtons and friction μR Newtons.



- (b) Find the value of μ .

- (c) The surface is now horizontal and a force of 9.8 Newtons **north** acts on the object.

Calculate:

- (i) the magnitude and direction of the friction force acting on the object.

- (ii) the resultant acceleration of the object.

- (d) If the surface remains horizontal and the northerly force is increased to 29.4 Newtons, how far will the object travel before its speed becomes 7ms^{-1} ?

1 + 1 + 2 + 1 = 5 marks

QUESTION 8

The position of an object is given by $\vec{r} = x\vec{i} + y\vec{j}$, where \vec{i} is a horizontal unit vector and \vec{j} is a vertical unit vector. If the path of the object is $y = 3x^2 - x^3$ and the horizontal component of the object's velocity is $1/3 \text{ ms}^{-1}$ for all values of $t \geq 0$, find the acceleration at the point where the velocity is horizontal.

3 marks

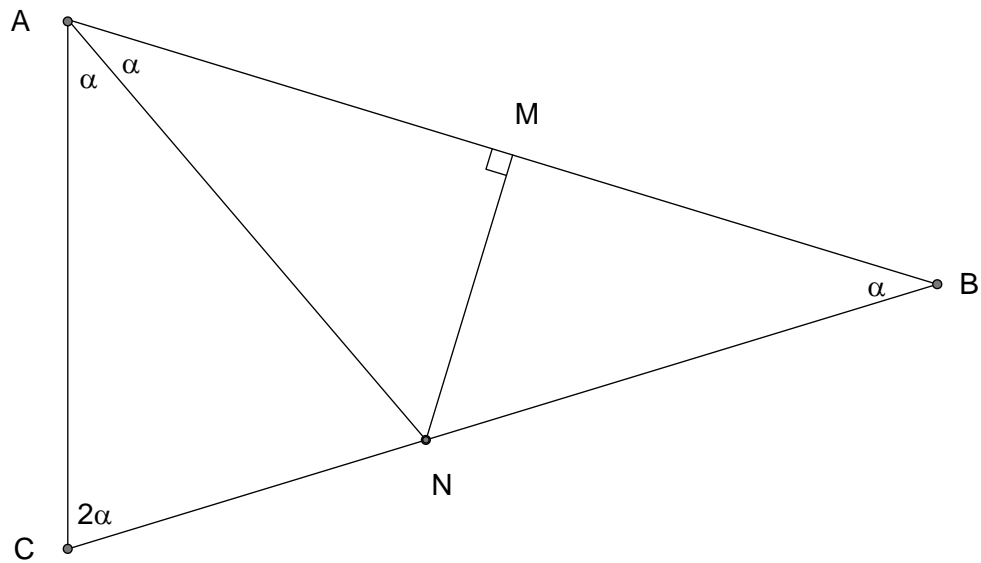
QUESTION 9

The tangent to the curve $y = f(x)$ at the point $A(x, y)$ meets the x -axis at the point $B(x - \frac{1}{2}, 0)$. The curve meets the y -axis at the point $C(0, \frac{1}{e})$.

Find the equation of the curve.

3 marks

QUESTION 10



In the diagram above, $\angle CAN = \angle NAM = \angle MBN = \alpha$, $\angle ACN = 2\alpha$ and MN is perpendicular to AB . The length of AB is 2 units and the length of AC is x units.

- (a) Show that $\alpha = 36^\circ$.

- (b) Explain why $\triangle ABC$ and $\triangle ANC$ are both isosceles triangles.

- (c) Find the value of x and hence calculate the exact value of $\cos\left(\frac{\pi}{5}\right)$.

1 + 1 + 2 = 4 marks

END OF QUESTION AND ANSWER BOOK