

Trial Examination 2010

## VCE Specialist Mathematics Units 3 & 4

Written Examination 1

### Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

#### Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

Question and answer booklet of 10 pages. The question and answer booklet has a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

#### Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Write **your name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

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**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$ .

**Question 1**

Find all solutions to the equation  $z^3 - \sqrt{2}z^2 + z - \sqrt{2} = 0$ ,  $z \in \mathbb{C}$ .

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2 marks

**Question 2**

A particle moves so that its position vector at time  $t$  is given by

$$\mathbf{r}(t) = -3 \sin\left(\frac{t}{2}\right)\mathbf{i} + \left(4 \cos\left(\frac{t}{2}\right) - 1\right)\mathbf{j}, \quad 0 \leq t \leq \pi.$$

- a. Show that the cartesian equation of the particle's path is given by  $\frac{x^2}{9} + \frac{(y+1)^2}{16} = 1$ .

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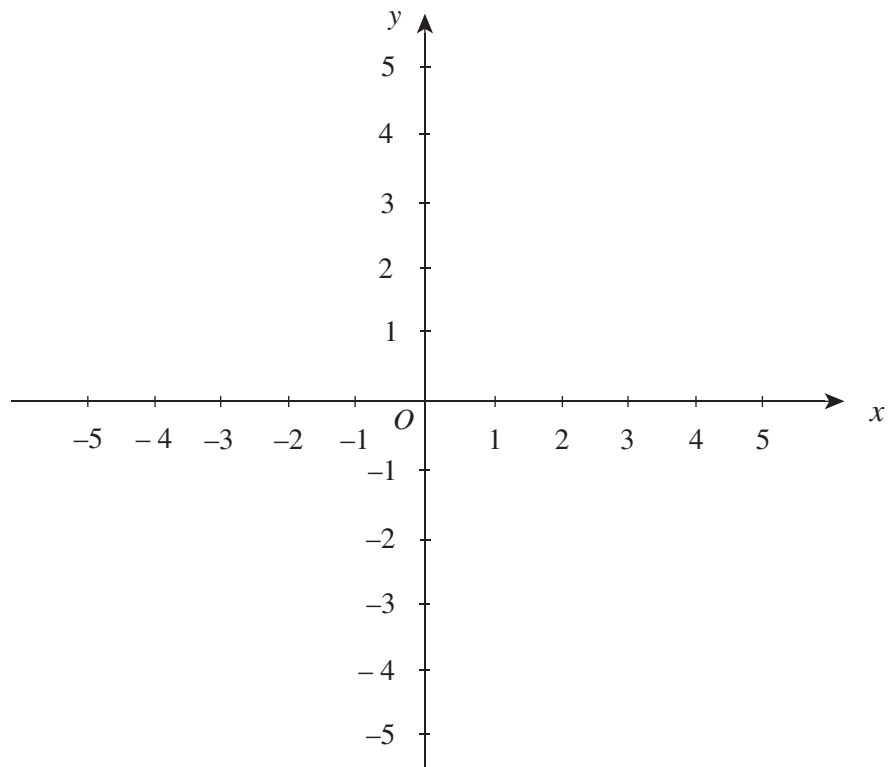
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3 marks

- b. Sketch the path followed by the particle on the axes below. Label the coordinates of any end points.  
**Note:** You are not expected to find any  $x$ -axis intercepts.



2 marks

**Question 3**

Evaluate  $15 \int_{-1}^0 x\sqrt{1+x} \, dx$ .

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3 marks

**Question 4**

Show that the three vectors  $\underline{u} = \underline{i} + \underline{j} - \underline{k}$ ,  $\underline{v} = 2\underline{i} + \underline{j} - 2\underline{k}$  and  $\underline{w} = \underline{i} + 2\underline{j} + \underline{k}$  are linearly independent.

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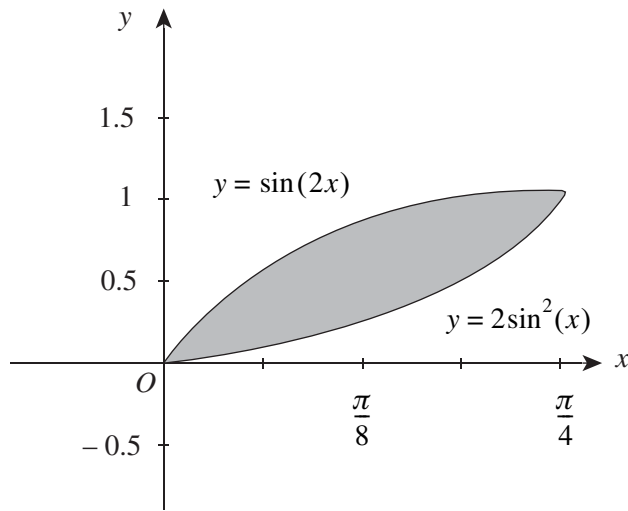
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4 marks

**Question 5**

The graphs of  $y = \sin(2x)$  and  $y = 2\sin^2(x)$  for  $0 \leq x \leq \frac{\pi}{4}$  are shown below.



Find the area of the shaded region.

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4 marks

**Question 6**

A mass of 2 kg is attached to the end of a light string  $OM$  of length 4 metres.

The string is fixed at  $O$  and the tension in the string is  $T$  newtons.

Find  $F$ , the magnitude of the horizontal force required to hold the mass 2 metres from the vertical through  $O$ .

Give your answer in the form  $\frac{a\sqrt{b}g}{b}$  where  $a$  and  $b$  are integers and  $g$  ( $\text{m/s}^2$ ) is the acceleration due to gravity.

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3 marks

**Question 7**

The acceleration of a particle moving in a horizontal straight line relative to a fixed origin  $O$  is given by  $a = 3x^2 - 12x$  where  $x$  metres is the displacement from  $O$ . Initially the particle is at  $O$  and its velocity,  $v$ , is  $4\sqrt{2}$  m/s.

- a. Show that  $v^2 = 2(x^3 - 6x^2 + 16)$ .

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2 marks

- b. Calculate the particle's velocity and acceleration at  $x = 2$  and hence state the direction of the particle's motion as it moves from  $x = 2$ .

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2 marks

**Question 8**

Suppose Euler's method is used to solve the differential equation  $\frac{dy}{dx} = e^{-x}$ , with a step size of 0.1 and initial condition  $y = 1$  at  $x = 2$ .

Use Euler's method to express  $y_2$  in terms of  $y_1$ .

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2 marks

**Question 9**

Show that the equation of the tangent line to the graph of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  at the point  $P(x_1, y_1)$  is given by  $\frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} = 1$ .

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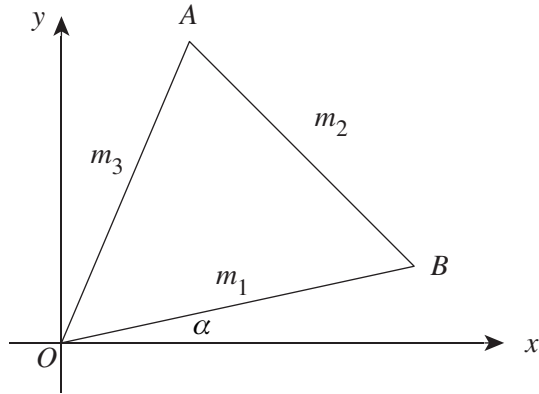
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5 marks



**Question 10**

Let  $m_1$ ,  $m_2$  and  $m_3$  be the gradients of the three sides of an equilateral triangle  $OAB$  as shown below.



No side of triangle  $OAB$  is parallel to the  $y$ -axis.  $OB$  makes an angle  $\alpha$  with the positive direction of the  $x$ -axis.

a. Show that  $m_2 = \frac{\tan(\alpha) - \sqrt{3}}{1 + \sqrt{3}\tan(\alpha)}$ .

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4 marks

b. Given that  $m_1 = \tan(\alpha)$  and  $m_3 = \frac{\tan(\alpha) + \sqrt{3}}{1 - \sqrt{3}\tan(\alpha)}$ , find the value of  $m_1m_2 + m_2m_3 + m_3m_1$ .

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4 marks

**END OF QUESTION AND ANSWER BOOKLET**