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Students Name:……………………………………

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2010

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the ten questions are indicated throughout.
Students may not bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 12-14 of this exam.

Question 1

Given that $z = 1 - 2i$ is a solution to the equation $z^3 + 2z^2 - 3z + 20 = 0$, find the other solutions.

2 marks

Question 2

Three points *A*, *B* and *O* are given by $A\left(2, 1, \sqrt{15}\right)$, $B(2, -4, 0)$ and $O(0, 0, 0)$.

a. Find the vector $\overrightarrow{AB}$ expressed in the form $x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.

1 mark

b. Prove that triangle ABO is a right-angled, isosceles triangle.

2 marks

Question 3

Consider the relation $3y^2 + 4x - 2x^2 y = 5$.

a. Find an expression for $\frac{dy}{dx}$.

2 marks

b. Find the gradient of the curve at the point where $x = 1$ in the first quadrant.

2 marks

Question 4

A crate of mass 10kg is sitting on a rough, horizontal floor. A pulling force of 100 newtons, acting at an angle of 30° to the horizontal, is applied to the crate and it accelerates at 8ms^{-2} across the floor.

a. Show all the forces acting on the crate on the diagram below.

1 mark

b. Find the value of μ, the coefficient of friction between the crate and the floor. Given that $g = 9.8\text{ms}^{-2}$, express your answer in the form $\dfrac{a\sqrt{3}-b}{c}$ where a, b and c are all positive integers.

2 marks

(*Question 4 is continued over the page.)*

Question 4 (cont'd)

c. The crate is moved to a different horizontal surface where the coefficient of friction between the crate and the surface is $\frac{1}{10}$. A pulling force of 8 newtons acting at an angle of 30° to the horizontal is applied to the crate.

Explain whether or not the crate is about to move.

2 marks

Question 5

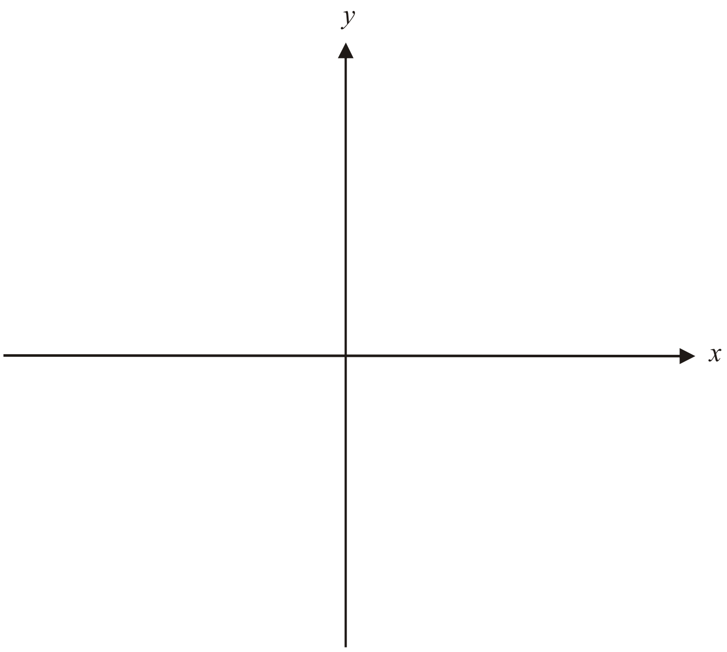
Solve the differential equation $\frac{dy}{dx} = 3\sqrt{4 - y^2}$ to find y in terms of x given that $y(0) = 2$.

3 marks

Question 6

Consider the function *f* with the rule $f(x) = 1 + \cos^{-1}\left(\frac{x}{2}\right)$. The function has its maximal domain.

a. Sketch the graph of $y = f(x)$ on the set of axes below.



2 marks

b. Find

i. the domain of *f*.

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ii. the range of *f*.

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$1+1=2$ marks

c. Find the equation of the normal to *f* at the point where $x = \sqrt{3}$.

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2 marks

Question 7

Find the volume generated when the region enclosed by the curve $y=\dfrac{2}{\sqrt{x^2-4}}$, the x-axis and the lines $x=3$ and $x=4$ is rotated about the x-axis to form a solid of revolution.

4 marks

Question 8

A particle moves in a straight line and at time t seconds it has acceleration a ms^{-2}, velocity v ms^{-1} and position x m from a fixed point. Its acceleration is given by $a = v - 5$.
Find x in terms of v given that $v = 6$ when $x = 2$.

3 marks

Question 9

a. Express the complex number $2+2\sqrt{3}\,i$ in polar form.

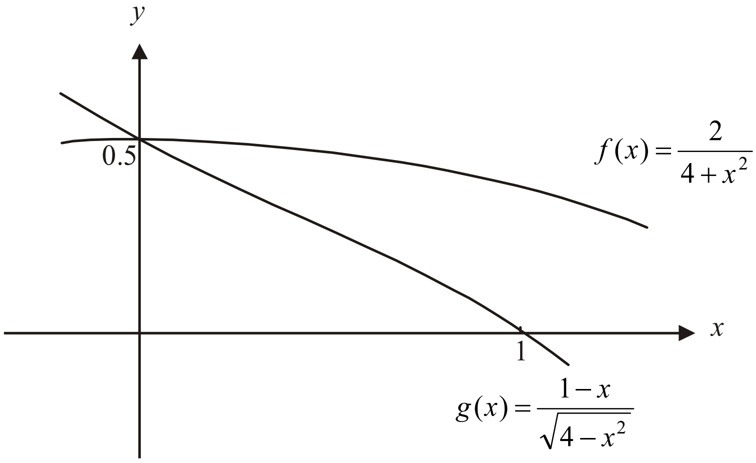
1 mark

b. Hence find the solutions to the equation $\sqrt{3}z^2+\sqrt{2}z-\frac{i}{2}=0$.

3 marks

Question 10

Part of the graph of the function $f(x)=\dfrac{2}{4+x^2}$ and of the function $g(x)=\dfrac{1-x}{\sqrt{4-x^2}}$ is shown below.



Find the area enclosed by the graphs of $y=f(x)$ and $y=g(x)$ and of the line $x=1$.

6 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\qquad$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\qquad$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\qquad$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\qquad$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\qquad$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\int \cos\ (ax)\, dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$

$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad$ $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum:	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion:	$\underset{\sim}{R} = m\underset{\sim}{a}$
friction:	$F \le \mu N$