

# **SPECIALIST MATHEMATICS**

## **Written examination 1**

### **FORMULA SHEET**

#### **Directions to students**

Remove this formula sheet during reading time.

This formula sheet is provided as a reference.

## Specialist Mathematics Formulas

### Mensuration

|                                    |  |
|------------------------------------|--|
| area of a trapezium:               | $\frac{1}{2}(a+b)h$                                      |
| curved surface area of a cylinder: | $2\pi rh$  |
| volume of a cylinder:              | $\pi r^2 h$  |
| volume of a cone:                  | $\frac{1}{3}\pi r^2 h$                                   |
| volume of a pyramid:               | $\frac{1}{3}Ah$  |
| volume of a sphere:                | $\frac{4}{3}\pi r^3$                                     |
| area of a triangle:                | $\frac{1}{2}bc \sin A$                                   |
| sine rule:                         | $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ |
| cosine rule:                       | $c^2 = a^2 + b^2 - 2ab \cos C$                           |

### Coordinate geometry

|          |   |            |   |
|----------|---|------------|---|
| ellipse: | $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ | hyperbola: | $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ |
|----------|---|------------|---|

### Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

| function | $\sin^{-1}$                                  | $\cos^{-1}$ | $\tan^{-1}$                                  |
|----------|--|-------------|--|
| domain   | $[-1, 1]$                                    | $[-1, 1]$   | $R$  |
| range    | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $[0, \pi]$  | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |

**Algebra (Complex numbers)**

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

**Calculus**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: 
$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule: 
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule: 
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method: If  $\frac{dy}{dx} = f(x)$ ,  $x_0 = a$  and  $y_0 = b$ , then  $x_{n+1} = x_n + h$  and  $y_{n+1} = y_n + hf(x_n)$

acceleration: 
$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration: 
$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

**TURN OVER**

## Vectors in two and three dimensions

$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

## Mechanics

momentum:

$$\underline{p} = m\underline{v}$$

equation of motion:

$$\underline{R} = m\underline{a}$$

friction:

$$F \leq \mu N$$

**END OF FORMULA SHEET**