Student Name: _____

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

2008 Trial Examination

Reading Time: 15 minutes
Writing Time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 11 pages.
- Working space is provided throughout the book.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

This page is blank

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The position vectors of points A and B are $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$

a. Find the unit vectors of $\mathbf{a}$ and $\mathbf{b}$.

2 marks

b. Find the unit vector which bisects $\angle AOB$.

3 marks

TURN OVER

Question 2

a. Express $z = 1$ in polar form.

_____

_____

_____

1 mark

b. If $u^6 = 1$, write an expression for u over the set of complex numbers.

_____

_____

1 mark

c. Show that the product of any sixth root of 1 and any twelfth root of 1 is also a fourth root of 1.

_____

_____

_____

_____

_____

3 marks

Question 3

a. Prove that $\frac{1}{4}\cos 3\theta = \cos^3 \theta - \frac{3}{4}\cos\theta$.

3 marks

b. Let $x = \frac{2}{3}\cos\theta$, where $\theta \in [0, \pi]$.

Use the result from **a.** to find the solutions to the equation $27x^3 - 9x = 1$ in terms of π.

3 marks

TURN OVER

Question 4

If $x = y\ln(xy)$, show that $\dfrac{dy}{dx} = \dfrac{y(x-y)}{x(x+y)}$.

3 marks

Question 5

A particle is projected vertically upwards from the ground with an initial velocity of $80\,ms^{-1}$. Its acceleration is given by $a = -g - 0.1v^2$. The maximum height reached by the particle is $x = a\ln\left(1 + \frac{b}{g}\right)$. Find the values of a and b.

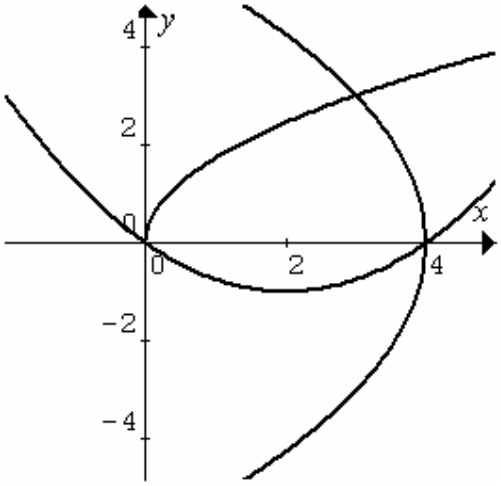
4 marks

TURN OVER

Question 6

The diagram below shows three curves defined by the equations:

$y^2 = 36 - 9x$, $y = \frac{1}{4}x(x-4)$ and $y = \sqrt{3x}$.



a. Label each graph with a corresponding equation and shade the region bounded by the three curves.

1 mark

b. Write down the integrals that give the area of the shaded region.

_____

_____

_____

_____

2 marks

c. Calculate the shaded area.

_____

_____

_____

_____

_____

_____

2 marks

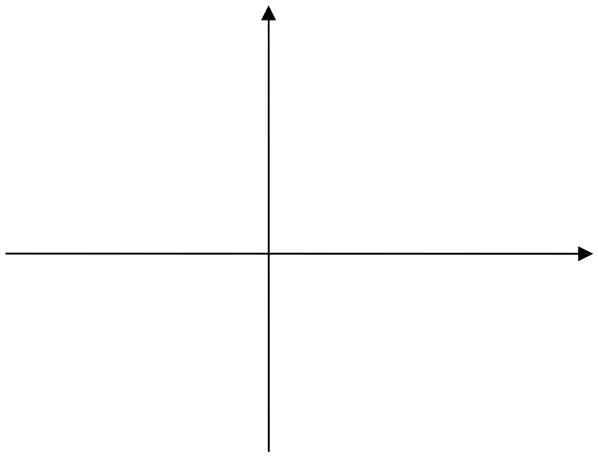
Question 7

The position vector of a moving particle is given by $\mathbf{r}(t) = (2 + 3\cos 2t)\mathbf{i} + (-3 + \sin 2t)\mathbf{j}$, $t \geq 0$.

a. Find the Cartesian equation of the path followed by the particle.

2 marks

b. Sketch the path of the particle**.**



1 mark

c. Determine the direction of the motion and the time required for the particle to return to its initial position.

2 marks

TURN OVER

Question 8

Find the value of a such that $\int_{-2}^{a} \frac{3}{x^2 + 4x + 20} dx = \frac{3\pi}{16}$

3 marks

Question 9

a. A box of mass 4 *kg* slides down a smooth plane inclined θ^0 to the horizontal. Find the acceleration of the box in terms of θ.

1 mark

b. The same box slides down a rough plane inclined θ^0 to the horizontal. The acceleration of the box is now $\frac{1}{4}$ of that found in **a.** Show that the coefficient of friction is $\mu = \frac{3}{4}\tan\theta$.

3 marks

END OF QUESTION AND ANSWER BOOK