Year 2008

VCE

Specialist Mathematics Solutions Trial Examination 1

KILBAHA MULTIMEDIA PUBLISHING	TEL: (03) 9817 5374
PO BOX 2227	FAX: (03) 9817 4334
KEW VIC 3101	kilbaha@gmail.com
AUSTRALIA	http://kilbaha.googlepages.com

Question 1

$x^3 - 4x^2y^2 + 2y^2 = 7$ taking $\dfrac{d}{dx}$ of each term (implicit differentiation)

$\dfrac{d}{dx}\left(x^3\right) - \dfrac{d}{dx}\left(4x^2y^2\right) + \dfrac{d}{dx}\left(2y^2\right) = \dfrac{d}{dx}(7)$

product rule in the second term

$3x^2 - \left(8xy^2 + 8x^2y\dfrac{dy}{dx}\right) + 4y\dfrac{dy}{dx} = 0$ M1

$3x^2 - 8xy^2 = \left(8x^2y - 4y\right)\dfrac{dy}{dx}$

$\dfrac{dy}{dx} = \dfrac{3x^2 - 8xy^2}{8x^2y - 4y}$ A1

Question 2

$y = \dfrac{3}{\sqrt{9+4x^2}}$ $V = \pi\displaystyle\int_a^b y^2\,dx$

$V = \pi\displaystyle\int_0^{\frac{3}{2}} \dfrac{9}{9+4x^2}\,dx$ let $u = 2x$ $\dfrac{du}{dx} = 2$ terminals $x = \dfrac{3}{2}$ $u = 3$ and $x = 0$ $u = 0$

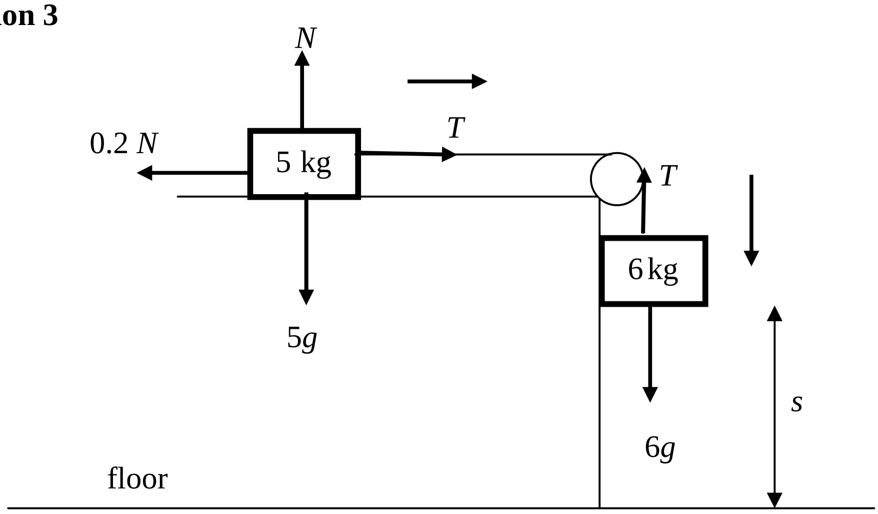
$V = \dfrac{9\pi}{2}\displaystyle\int_0^3 \dfrac{1}{9+u^2}\,du$ M1

$V = \dfrac{9\pi}{2}\left[\dfrac{1}{3}\tan^{-1}\left(\dfrac{u}{3}\right)\right]_0^3$

$V = \dfrac{3\pi}{2}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$V = \dfrac{3\pi^2}{8}$ A1

Question 3



A1

let a be the acceleration of the system, and T the tension in the string.
resolving horizontally on the table around the 5 kg block.

$(1)\quad T-0.2N=5a$

resolving vertically on the table around the 5 kg block. M1

$(2)\quad N-5g=0 \qquad \Rightarrow N=5g$ substitute into (1) gives $T-0.2\text{x}5g=5a$

or $T-g=5a$

resolving vertically downwards around the 6 kg block hanging vertically.

$(3)\quad 6g-T=6a$ adding these last two equations, to eliminate T gives, $5g=11a$

$a=\dfrac{5g}{11}=\dfrac{5\text{x}9.8}{11}=\dfrac{49}{11}$ m/s^2 $\quad s=?\quad u=0\quad t=\dfrac{1}{2}$sec M1

using $s=ut+\dfrac{1}{2}at^2$

$s=0+\dfrac{1}{2}\text{x}\dfrac{49}{11}\text{x}\left(\dfrac{1}{2}\right)^2=\dfrac{49}{88}$ metres A1

Question 4

a. $z^2+2zi-4=0$

using the quadratic formulae with $a=1\quad b=2i\quad c=-4$

$\Delta=b^2-4ac=(2i)^2+16=-4+16=12$

$z=\dfrac{-b\pm\sqrt{\Delta}}{2a}$ M1

$z=\dfrac{-2i\pm\sqrt{12}}{2}=\dfrac{-2i\pm 2\sqrt{3}}{2}$

$z=\sqrt{3}-i$ and $-\sqrt{3}-i$ A1

b. $z=-\sqrt{3}-i \qquad |z|=\sqrt{\left(-\sqrt{3}\right)^2+(-1)^2}=2$

$\text{Arg}(z)=-\pi+\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)=-\pi+\dfrac{\pi}{6}=-\dfrac{5\pi}{6}$

$z=2\text{cis}\left(-\dfrac{5\pi}{6}\right)$ A1

$z^6=\left(2\text{cis}\left(-\dfrac{5\pi}{6}\right)\right)^6=2^6\left(\text{cis}(-5\pi)\right)$ M1

$z^6=64\text{cis}(-\pi)$

$z^6=-64$ A1

Question 5

a. Let $y=\sin^{-1}\left(\frac{3}{\sqrt{x}}\right)=\sin^{-1}(u)$ where $u=\frac{3}{\sqrt{x}}=3x^{-\frac{1}{2}}$

$\frac{dy}{du}=\frac{1}{\sqrt{1-u^2}}$ $\frac{du}{dx}=-\frac{3}{2}x^{-\frac{3}{2}}=\frac{-3}{2\sqrt{x^3}}$ chain rule M1

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=\frac{-3}{2\sqrt{x^3}\sqrt{1-\frac{9}{x}}}$$

$\frac{dy}{dx}=\frac{-3}{2\sqrt{x^3}\sqrt{\frac{x-9}{x}}}$ since $x>9$

$$\frac{dy}{dx}=\frac{-3}{2x\sqrt{x-9}}$$

so shown $\frac{d}{dx}\left(\sin^{-1}\left(\frac{3}{\sqrt{x}}\right)\right)=\frac{-3}{2x\sqrt{x-9}}$ for $x>9$ A1

b. $\int_{12}^{18}\frac{1}{x\sqrt{x-9}}\,dx.$

$-\frac{2}{3}\left[\sin^{-1}\left(\frac{3}{\sqrt{x}}\right)\right]_{12}^{18}$ M1

$=-\frac{2}{3}\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

M1

$=-\frac{2}{3}\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$

$=\frac{\pi}{18}$ A1

Question 6

a. Using Euler's method $\frac{dy}{dx} = \log_e(2x-3)$ $y(2)=1$ $h=0.5$

$x_0 = 2$ $y_0 = 1$ $h = \frac{1}{2}$ $f(x) = \log_e(2x-3)$

$y_1 = y_0 + h\,f(x_0)$

$y_1 = 1 + \frac{1}{2}\log_e(1) = 1$ $x_1 = x_0 + h = \frac{5}{2}$ M1

$y_2 = y_1 + h\,f(x_1)$

$y_2 = 1 + \frac{1}{2}\log_e(2) = \log_e(e) + \log_e\left(\sqrt{2}\right) = \log_e\left(\sqrt{2}\,e\right)$

$p = \sqrt{2}\,e$ A1

b. $\frac{d}{dx}\left[(2x-3)\log_e(2x-3)\right] = (2x-3) \times \frac{2}{(2x-3)} + 2\log_e(2x-3)$

$\frac{d}{dx}\left[(2x-3)\log_e(2x-3)\right] = 2 + 2\log_e(2x-3)$ A1

Hence $\int\left(2 + 2\log_e(2x-3)\right)dx = 2x + 2\int\log_e(2x-3)\,dx = (2x-3)\log_e(2x-3)$

$\int\log_e(2x-3)\,dx = \frac{1}{2}\left[(2x-3)\log_e(2x-3) - 2x\right]$

$\frac{dy}{dx} = \log_e(2x-3)$ $y = \int\log_e(2x-3)\,dx$

$y = \frac{1}{2}\left[(2x-3)\log_e(2x-3)\right] - x + C$ to find C use $x=2$ when $y=1$ M1

$1 = \frac{1}{2}\left[(4-3)\log_e(4-3)\right] - 2 + C$ $\Rightarrow C = 3$

$y(x) = \frac{1}{2}\left[(2x-3)\log_e(2x-3)\right] - x + 3$ A1

$y(3) = \frac{3}{2}\log_e(3)$

$y(3) = \log_e\left(\sqrt{27}\right)$ $q = \sqrt{27}$ A1

Question 7

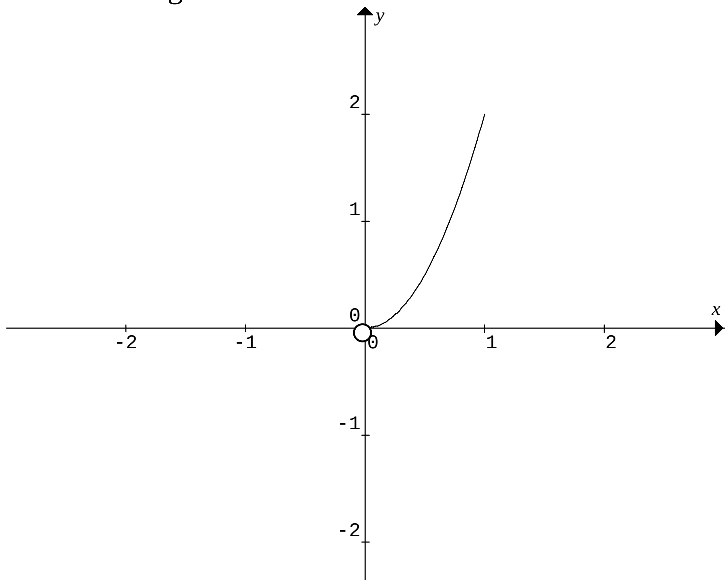
a. $\underset{\sim}{r}(t) = e^{-t}\underset{\sim}{i} + 2e^{-2t}\underset{\sim}{j}$ for $t \geq 0$ vector equation,

the parametric equations are $x = e^{-t}$ and $y = 2e^{-2t}$

now $y = 2e^{-2t} = 2\left(e^{-t}\right)^2 = 2x^2$

$y = 2x^2$ is the Cartesian equation of the path, but! A1

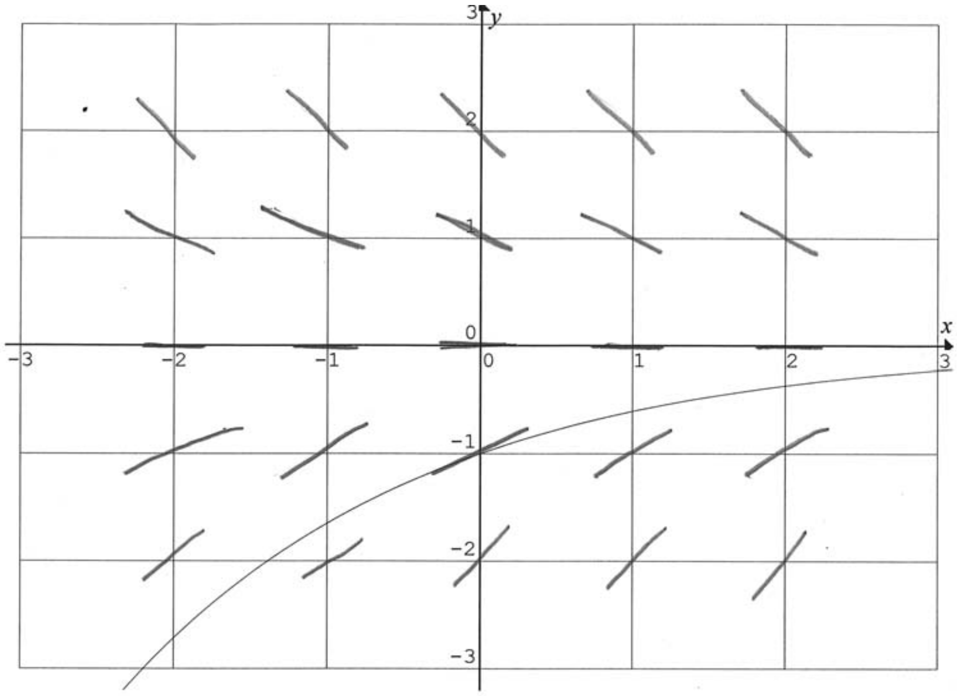
b. since $t \geq 0 \Rightarrow 0 < x \leq 1$ and $0 < y \leq 2$, the graph is not the whole parabola it has a hole at the origin. A1



Question 8

a.

$\frac{dy}{dx} = -\frac{y}{2}$	y	-2	-1	0	1	2
	slope	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1



A2

b. $2\dfrac{dy}{dx}+y=0 \quad \Rightarrow \quad \dfrac{dy}{dx}=-\dfrac{y}{2} \qquad y(0)=-1$

$-\dfrac{1}{2}\int dx=\int \dfrac{1}{y}\,dy=\log_e(|y|)$ M1

$-\dfrac{x}{2}+C=\log_e(|y|) \qquad \Rightarrow y=\exp\left(-\dfrac{x}{2}+C\right)=Ae^{-\frac{x}{2}}$

$x=0 \quad y=-1 \quad \Rightarrow A=-1$

$y=-e^{-\frac{x}{2}}$ A1

c. The graph of $y=-e^{-\frac{x}{2}}$ on the above diagram passing through $(0,-1)$ A1

Question 9

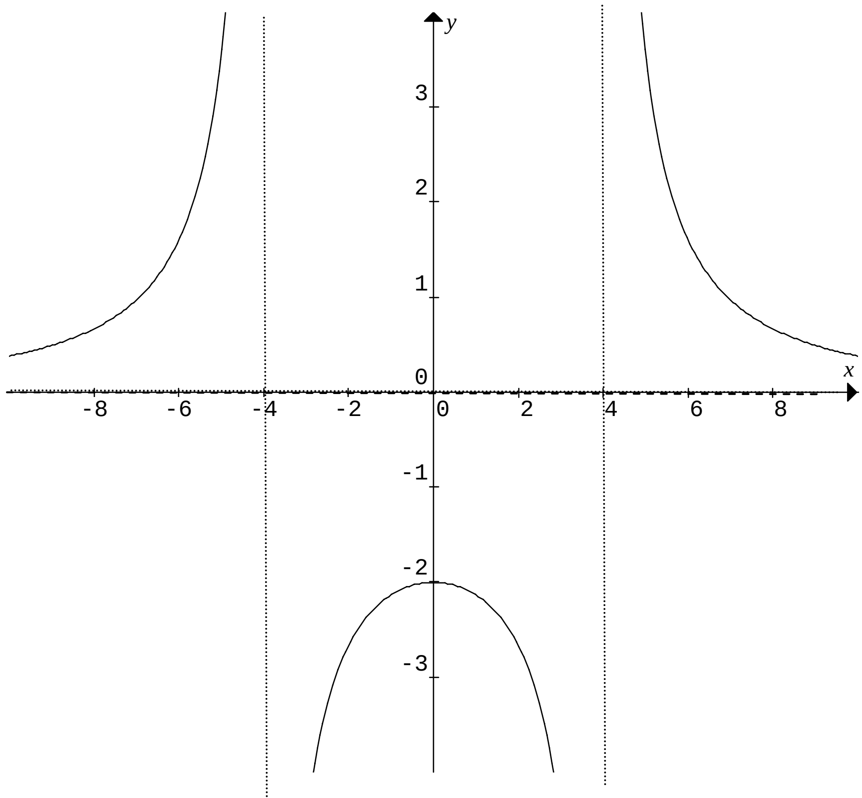
a. $y=\dfrac{32}{x^2-16}$

vertical asymptotes at $x=4$ and $x=-4$

horizontal asymptotes at $y=0$ (the *x*-axis) A1

the turning point is a maximum turning point at $(0,-2)$ also the *y*-intercept

correct graph and turning point A1



b. the area is $\int_{-2}^{0} \frac{32}{x^2-16}\,dx$ but this is below the x-axis and negative, so the area is

$A=\int_{-2}^{0} \frac{32}{16-x^2}\,dx$ A1

by partial fractions $\frac{32}{16-x^2}=\frac{B}{4+x}+\frac{C}{4-x}$ adding the partial fractions

$=\frac{B(4-x)+C(4+x)}{(4+x)(4-x)}=\frac{x(C-B)+4B+4C}{16-x^2}$ M1

$(1)\ \ 4(B+C)=32$ and $(2)\ \ C-B=0$ so that $B=C=4$

$A=\int_{-2}^{0} \frac{32}{16-x^2}\,dx=4\int_{-2}^{0}\left(\frac{1}{4+x}+\frac{1}{4-x}\right)dx$

$A=4\left[\log_e(4+x)-\log_e(4-x)\right]_{-2}^{0}=4\left[\log_e\left(\frac{4+x}{4-x}\right)\right]_{-2}^{0}$ M1

$A=4\left[\log_e(1)-\log_e\left(\frac{1}{3}\right)\right]=4\log_e(3)$

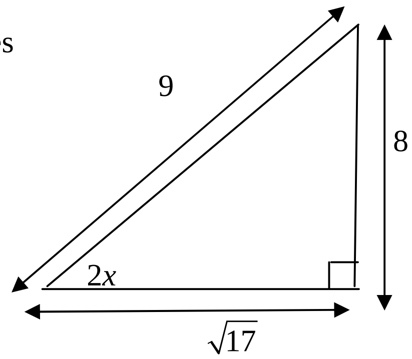
$A=\log_e(81)$ $a=81$ A1

Question 10

$\cos(x)-\sin(x)=\frac{1}{3}$ squaring both sides gives

$\cos^2(x)-2\sin(x)\cos(x)+\sin^2(x)=\frac{1}{9}$

$2\sin(x)\cos(x)=\sin(2x)=1-\frac{1}{9}=\frac{8}{9}$



M2

since $0<2x<\frac{\pi}{2}$ in the first quadrant

$\cot(2x)=\frac{\sqrt{17}}{8}$ A1

END OF SUGGESTED SOLUTIONS