2007 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The structure of examination 1 in 2007 included 10 short answer questions worth a total of 40 marks. Students were not allowed to bring calculators or any notes into the examination. The number of students who sat for the 2007 Specialist Maths examination 1 was 4760, which was 448 fewer than in 2006.

The mean score for the 2007 was examination 23.2 out of 40 (58.0%), slightly higher than the performance on the 2006 paper where the mean score was 21.8 out of 40 (54.4%). Seven out of 17 question parts had a mean score of less than 50% of the maximum possible, which was a little better than eight out of 17 in 2006.

The overall mean and median scores were 23.2 out of 40 (58.0%) and 25 out of 40 (62.5%), compared with 21.8 (54.4%) and 22 (55%) in 2006. About 14 per cent of students scored less than 25% of the available marks, compared to 18 per cent in 2006. At the lower end of scores, 41 students (0.86% of the cohort) scored zero marks and 233 students (4.9%) scored fewer than four marks out of 40. At the top end, 56 students (1.18%) scored full marks and 413 students (8.7%) scored more than 36 marks out of 40. This indicates that fewer students scored close to no marks than in 2006 and fewer students scored close to full marks than in 2006. The examination provided all students with accessible questions and also provided parts which were found by most to be quite challenging.

In the comments on specific questions in the next section, many common mistakes that are made year after year are highlighted. These mistakes should be brought to the attention of students so that they can try to avoid them. A particular concern is the need for students to read the questions carefully.

Areas of weakness included:

- poor algebraic skills. This was evident in several questions, and the inability to simplify expressions often prevented students from completing a question
- showing a given result, which was required in Question 2a. The onus is on students to include sufficient correct and relevant working to convince the assessors that they do know how to derive the result. Just as importantly, students should be reminded that they can use a given value in the remaining part(s) of the question, whether or not they were able to derive it
- recognising the need to use the chain rule when differentiating implicitly (Question 3)
- recognising the need to use the product rule when differentiating (Question 3)
- recognising the method of integration required (Questions 4 and 8b.)
- recognising the need for an arbitrary constant when integrating (Questions 6a., 7b. and 8b.)
- recognising the need to use a double angle formula (Question 10a.)
- knowing the exact values for circular functions (Questions 1, 6a. and 8b.)
- giving answers in the required form (Questions 1, 5b., 7a., 7b. and 9).

Students need to be reminded that the instruction ‘sketch’ (Questions 6c. and 8c.) does **not** mean that a rough and careless attempt is acceptable or that details such as a reasonable scale, correct domain, asymptotes and asymptotic behaviour can be ignored.

As no calculators are permitted for this examination, it is expected that students will be able to evaluate simple arithmetic expressions correctly and efficiently. Many students were unable to do this and lost marks as a consequence.

SPECIFIC INFORMATION

Question 1

Marks	0	1	2	3	4	Average
%	13	6	9	15	56	3.1

$$2\text{cis}\left(\frac{\pi}{2}\right)$$

This question was quite well done. Two methods were widely used; rationalising using the complex conjugate and converting the numerator and denominator to polar form before proceeding. In both cases it was clear that several students had not learned the relevant exact values for circular functions.

Most students used a rationalisation approach. The typical errors for this approach included converting $2i$ to 2cis0 or $\sqrt{2}\ \text{cis}\frac{\pi}{2}$ or similar and simplifying the denominator as $1-3=-2$. A small number of students did strange things with the fraction, such as using incorrect cancellation and writing the complex number in the form $\frac{2\sqrt{3}}{1+\sqrt{3}i}+\frac{2}{1+\sqrt{3}i}i$ and then attempting to treat the fractions as real coefficients. Writing the fraction as $2\sqrt{3}-\frac{2}{\sqrt{3}i}i$ was also seen. Several students correctly found $z=2i$ and then stopped. Presumably most of these had not read the question carefully enough.

Those who converted the numerator and denominator to polar form before proceeding often had difficulty with the quadrant for the denominator. Students should be encouraged to draw small diagrams to indicate in which quadrant the complex number lies. The typical errors for the polar approach included: finding the arguments to be $\frac{\pi}{3}$ in the numerator and $-\frac{\pi}{6}$ in the denominator and hence fortuitously getting the correct answer; and simplifying incorrectly, such as $\frac{\pi}{6}-\frac{-\pi}{3}=\frac{2\pi}{3}+\frac{\pi}{3}=\pi$.

Question 2a.

Marks	0	1	Average
%	17	83	**0.9**

* Answer given.

Most students did this question very well. The vast majority recognised that the value of $z=\sqrt{5}-i$ could be substituted and the expression evaluated to obtain zero without expanding the square or cube. Those who attempted to expand often ran into difficulties. Some students grouped the expression two by two to come up with the expression $\left(z^2+4\right)\left(z-\left(\sqrt{5}-i\right)\right)=0$, which proved that $\sqrt{5}-i$ is a root and also assisted them with part b.

Question 2b.

Marks	0	1	2	Average
%	49	9	42	**1.0**

$z=\pm 2i$

This question polarised the cohort; those who understood what was required did it very well, while many others performed poorly and received no marks. Far too many students decided that the complex conjugate $\sqrt{5}+i$ was another solution despite the coefficients of the cubic polynomial not being real.

The most efficient method was to factorise by grouping terms in pairs as shown above in part a. Many students tried long division with varied success. Some tried to divide by $\sqrt{5}-i$ rather than $z-\left(\sqrt{5}-i\right)$. Of those who found z^2+4 as the quadratic factor, too many solved this to give $\pm 4i$ or $\pm\sqrt{2i}$ as the other two solutions.

A few students continued to confuse solutions of an equation with factors of an expression, giving the answer as $z+2i,\ z-2i$.

Question 3

Marks	0	1	2	3	Average
%	18	15	24	43	**2.0**

$y=3x-3$

This question was reasonably well done, although several students did not realise that differentiating the middle term required use of the product rule, or that the chain rule was required for the third term. Many students correctly found the four terms, but then made basic algebraic errors with respect to negative signs, often failing to use brackets correctly. A few students had everything correct except that when they differentiated, the right-hand side remained as 2 rather than 0. Some students correctly found the gradient to be 3 but then stopped, suggesting that they had forgotten what the question had asked. A few students found the gradient or the equation of the normal.

Question 4

Marks	0	1	2	3	4	Average
%	20	24	3	18	35	**2.4**

$\frac{\pi}{2}\log_e(3)$

$-\frac{\pi}{2}\log_e\frac{1}{3}$ and $\pi\log_e\sqrt{3}$ were also accepted. Unfortunately, a few students were concerned about the negative sign, thinking that they had a negative value, and so changed their answer to $\frac{\pi}{2}\log_e\frac{1}{3}$.

Most students were able to find the correct partial fractions, but a number then lost the negative sign in the integration, writing $\int \frac{1}{1+x}+\frac{1}{1-x}\,dx = \log_e(1+x)+\log_e(1-x)$. Some reversed the order in the denominator and then continued with $\int \frac{1}{x+1}-\frac{1}{x-1}\,dx = \log_e(x+1)-\log_e(x-1)$. The lack of modulus signs in this case leads to logarithms of negative numbers, so a correct answer cannot be properly obtained.

As in the past, too many students used the incorrect 'log rule' $\int \frac{1}{f(x)}\,dx = \log_e\left(f(x)\right)$ to obtain $\int \frac{1}{1-x^2}\,dx = \log_e(1-x^2)$ or similar. Others made errors such as $\int \left(\frac{1}{\sqrt{1-x^2}}\right)^2 dx = \left(\sin^{-1}(x)\right)^2$. A small number of students had the wrong expression for the volume by either not squaring *y*, writing the integrand as $y^2-\frac{1}{2}$ or similar, forgetting π, or having incorrect terminals.

Question 5a.

Marks	0	1	2	Average
%	17	13	70	**1.6**

$a = -\frac{8}{3}$ m/s^2

Most students answered this question very well, though some did not seem to realise that constant acceleration formulas were available on the formula sheet. Most of those that did use these obtained the correct answer, though some decided that they must have made an error and dropped the negative sign. Some used $v = 4$ and $u = 0$, giving $a = \frac{8}{3}$. A few students incorrectly solved the correct equation used fractions to get $a = -\frac{3}{8}$. A number of students included a pulling force *P* to the right during motion, indicating that they had not read the question carefully enough.

Question 5b.

Marks	0	1	2	Average
%	26	23	51	**1.3**

$\mu = \frac{8}{3g}$

This question was generally well done by students who got part a. correct, though some students 'fudged' their answer with negative signs (for example, $F = 6 \times -\frac{8}{3} = -16 = 16$). Typical incorrect answers for μ were the result of poor fraction skills and included $-\frac{8}{3g}$, $\frac{3g}{8}$ and $\frac{8g}{3}$. The negative value for μ was of particular concern.

Question 6a.

Marks	0	1	2	Average
%	13	13	74	**1.7**

$\mathbf{r} = 2\cos(2t)\mathbf{i} + 3\sin(2t)\mathbf{j}$

This question was well done by most students, with the most common error being to omit a constant (vector) of integration **c**. This fortuitously happened to be the zero vector, but students who did not show a constant of integration could not get full marks. The vast majority of students who did include **c** were able to successfully show that it was the zero vector, but some found a non-zero **c** (often by taking 2cos(0) = 0), which lead to difficulties in the remaining parts of the question.

Occasionally sign problems appeared (in the first term –2cos(2*t*)**i** and occasionally the second term –3sin(2*t*)**j**); and, surprisingly, some students differentiated the velocity vector rather than antidifferentiating. Many students wrote the constant vector of integration as $c_1\mathbf{i} + c_2\mathbf{j}$ rather than just **c**, which led them to solve two equations rather than one. It is recommended that teachers point out that this unnecessarily complicates the solution.

Notation was often very poor, with vectors often written without tildes and some students simply dropping **i** and **j** altogether, yet they were often able to identify *x* and *y* for part b. A few students took **i** to be *i* (dropping **j**) and performed some interesting tricks with complex numbers.

Question 6b.

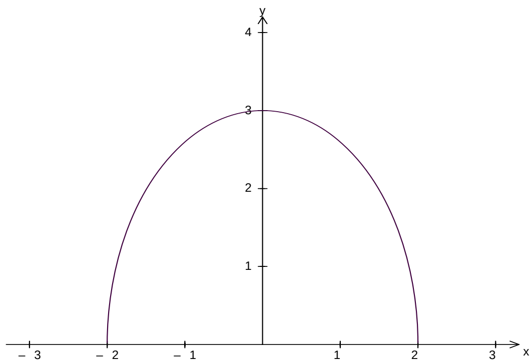
Marks	0	1	2	Average
%	21	16	63	**1.5**

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

This was a very standard conversion from parametric to cartesian form and most students handled it reasonably well, having identified *x* and *y* from part a. Some students gave the cartesian equation as $y = 3\sin\left(\cos^{-1}\left(\frac{x}{2}\right)\right)$, which was correct but unhelpful for part c. The most common errors in writing the equation in standard form for an ellipse included $\frac{x^2}{2} + \frac{y^2}{3} = 1$ and $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Many students used the *x* and *y* components of **v** rather than **r** to get $\frac{x^2}{16} + \frac{y^2}{36} = 1$. A few students with sign problems in part a. managed to obtain the equation of a hyperbola here.

Question 6c.

Marks	0	1	2	Average
%	34	44	22	**0.9**



Most students were unable to apply the domain restrictions correctly, which was also a problem with a similar question on last year's paper. Usually, a full ellipse was drawn, occasionally a quarter-ellipse. Many of the attempts at drawing ellipses were not very elliptical and there were some very strange shapes provided.

Question 7a.

Marks	0	1	2	Average
%	38	27	36	**1.1**

$\frac{131}{110}$

Many students were able to start this question but some had very little idea of how to apply the given rule. Typical errors in the application of Euler's method included $y_1 = y_0 + h \times \frac{1}{x_1}$; $y_1 = y_0 + h \times \frac{1}{x_0^2}$; and $y_1 = y_0 + h \times \log_e(x_0)$.

Quite a few of the students who started well were unable to complete the question successfully, often due to fraction errors which were again prevalent. Very often the lowest common denominator chosen for 10 and 11 was 121. Answers such as $\frac{131}{121}$ were not uncommon and working such as $1.1 + \frac{0.1}{1.1} = \frac{131}{111}$ and worse were seen.

Question 7b.

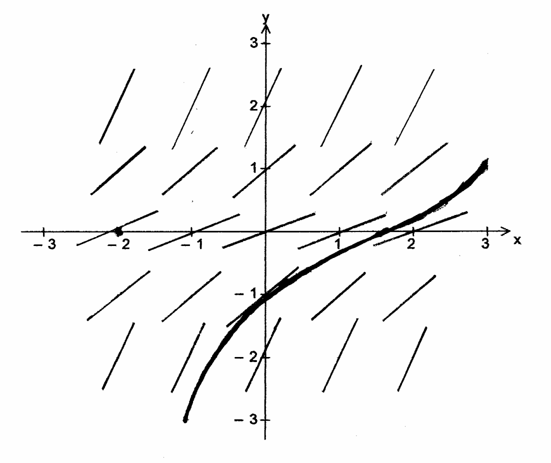
Marks	0	1	2	Average
%	40	27	33	**1.0**

$\log_e(1.2) + 1$

Most students were able to obtain $y = \log_e |x| + c$, although some forgot to include a constant of integration and others found an incorrect value for the constant by using (1.1, 1.1) as the initial condition rather than (1, 1). The other common errors were stopping after finding c, perhaps not comprehending what the question asked, or giving the answer as $\log_e(1.1) + 1$. Several students did not know that $\log_e(1) = 0$.

Question 8a.

Marks	0	1	2	Average
%	53	18	29	**0.8**



Overall this question was not well done, with a clear difference in responses between those students who knew how to approach this type of question and those who did not. It was disappointing that quite a few students had no idea how to tackle this part, in some cases suggesting that they did not know what a slope field was. Others drew a large number of random 'tick' marks across the page, rather than at the 25 points required by the question. A large number of students drew the constant gradients vertically rather than horizontally (mixing up the *x* and *y* variables). Many students did not attempt the question, and there were some attempts in which students drew slope lines which were vertical, horizontal, in various directions and, in some cases, curved. Many students drew line segments that did not pass through the points of intersection of the gridlines.

Question 8b.

Marks	0	1	2	3	Average
%	30	6	17	46	**1.9**

$y = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right)$

Most students were able to correctly integrate to obtain $x = 2\tan^{-1}(y) + c$ (although some again omitted the constant of integration) and the majority of these were able to make a reasonable attempt at finding *c*. Typical errors involved taking $\tan^{-1}(-1)$ to be 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$ or *π*. Algebraic errors were common, with the most significant errors including writing $y = \tan\left(\frac{1}{2}x\right) - \frac{1}{2}c$ or $\tan\left(\frac{1}{2}x\right) = y + c$ before attempting to find *c*, rather than the correct $y = \tan\left(\frac{1}{2}x - \frac{1}{2}c\right)$ or similar.

As usual, some students incorrectly antidifferentiated to get a 'log' expression (as mentioned in Question 4). Some did not 'flip' the fraction and ended up integrating a quadratic in *y* to get a cubic.

Question 8c.

Marks	0	1	Average
%	87	13	**0.2**

See graph given in part a. above.

Students found this question difficult. The majority of students who correctly expressed *y* in terms of *x* in part b. were unable to sketch it on the slope field produced in part a. Typical errors included some or all of: graph not through (0, –1); graph not through $\left(\frac{\pi}{2}, 0\right) \approx$ (1.6, 0); graph cutting through segments in the slope field (sometimes at right angles); and graph inconsistent with the slope field. Many different shapes were seen, with some students attempting to draw an inverse tan graph and others drawing a variety of graphs from straight lines to parabolas to various other shapes.

Question 9

Marks	0	1	2	3	Average
%	56	21	5	18	**0.9**

$\mathbf{a} = -\mathbf{r}$

Answers to this question were poor, and many students appeared to have difficulty in understanding what was being asked. Some students used a 'pattern' approach to obtain the correct expression for **a**, but without any indication of differentiation or understanding of the situation. A large proportion of students differentiated with respect to *x* (or some other variable rather than *t*), with most using $a = \frac{dv}{dx}$, but $a = v\frac{dv}{dx}$ and $a = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ were also seen. These attempts still occasionally allowed the student to reach the correct answer but credit for this could not be given. Several students assumed that $x = \cos(t)$ and $y = \sin(t)$, from which it can be deduced that $\mathbf{a} = -\mathbf{r}$, but this assumption is unjustified and oversimplifies the problem as it only considers a particular case rather than a general one.

Only a few students carefully showed that the given information translated to $\frac{dx}{dt} = -y$ and $\frac{dy}{dt} = x$, from which *a* as the derivative of *v* with respect to time could be shown to be $-x\mathbf{i} - y\mathbf{j}$ and hence equal to $-\mathbf{r}$. A few students who got $\mathbf{a} = -x\mathbf{i} - y\mathbf{j}$ either forgot what the question asked, did not read the question carefully enough, or were unable to recognise that $-x\mathbf{i} - y\mathbf{j} = -\mathbf{r}$.

Question 10

Marks	0	1	2	3	Average
%	33	35	11	20	**1.2**

$\sin(x) = \frac{1}{3}$

There were many approaches used for this question, some of which led to extremely complicated algebra which almost always resulted in errors. Some students arrived at the correct answer by circuitous and convoluted pathways but most of these got lost in the algebra. The most successful method involved first finding $\cos(2x)$, either by using the $\sec^2(\theta)$ formula or, more commonly, by drawing a right-angled triangle and using Pythagoras' theorem, then using the appropriate double angle formula.

Another common approach, though not as successful, was to use the double angle formula for $\tan(2x)$ to try to find $\tan(x)$ first. Many of these students were unable to solve the resulting quadratic in $\tan(x)$. Those who solved for $\tan(x)$ then used either a right-angled triangle together with Pythagoras' theorem or, more rarely, the $\operatorname{cosec}^2(\theta)$ formula. Some of these students successfully found $\tan(x) = \frac{1}{2\sqrt{2}}$ and then stopped, either having forgotten what the question asked, not knowing what to do next, or running out of time. A not uncommon error was to initially write $\tan(2x) = \frac{4\sqrt{2}}{7} \Rightarrow \tan(x) = \frac{2\sqrt{2}}{7}$.

Some students tried to use $\frac{\sin(2x)}{\cos(2x)} = \frac{4\sqrt{2}}{7}$, but this was rarely successful. Several of these attempts included equating the numerators and denominators, thereby stating the impossible equations $\sin(2x) = 4\sqrt{2}$ and $\cos(2x) = 7$.