Neap:

Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 8 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this book during reading time.
Write **your name** and your **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2007 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Consider the relation $xy - y^2 - 4 = 0$, where $y > -4$.

Find the gradient of the curve when $x = -5$.

4 marks

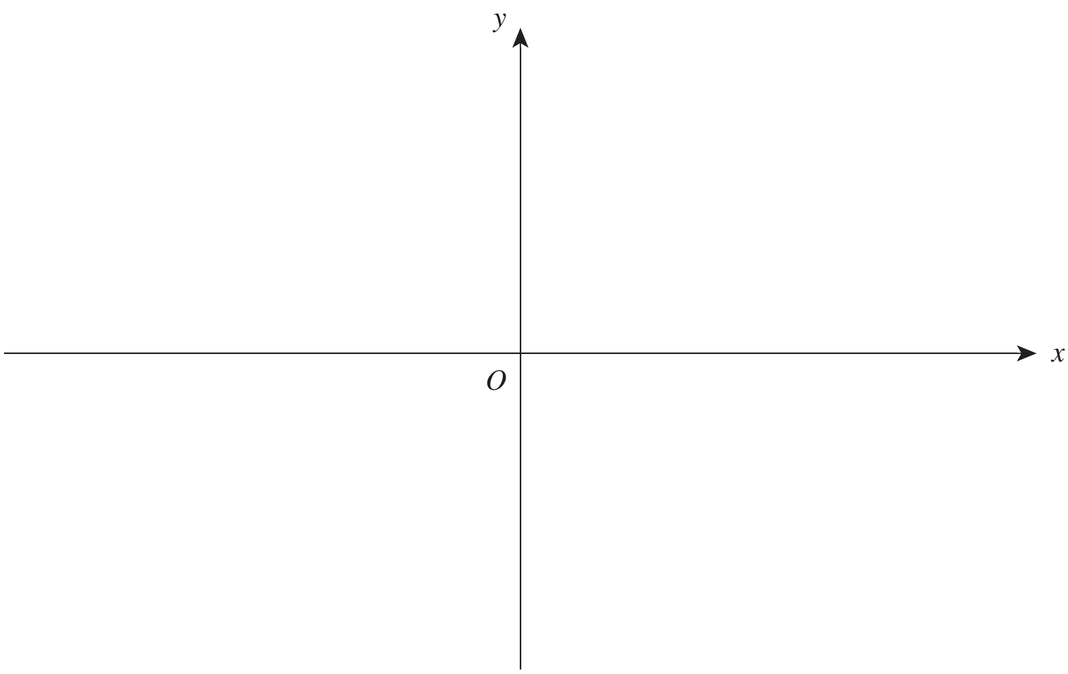
Question 2

Solve the differential equation $\frac{dy}{dx} - \cos(2x)e^{-\sin(2x)} = 0$ given that $y = 1$ when $x = 0$.

4 marks

Question 3

a. Sketch the graph of the function $y = \frac{1}{x^2 + 4x + 8}$, showing all asymptotes, stationary points and intercepts with the axes.



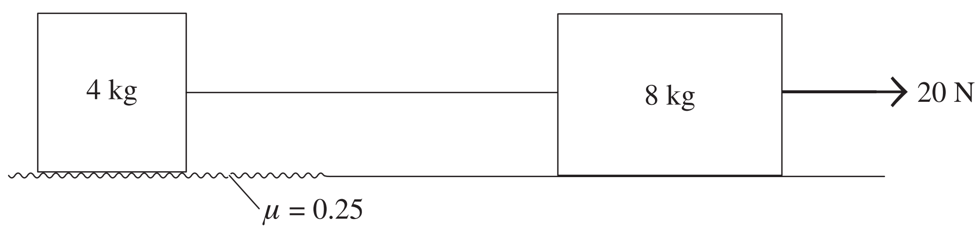
3 marks

b. Find the exact value of the area bounded by the curve $y = \dfrac{1}{x^2 + 4x + 8}$, the x-axis and the lines $x = -4$ and $x = 0$.

3 marks
Total 6 marks

Question 4

The diagram below shows a horizontal force of 20 N being applied to an 8 kg mass which sits on a frictionless section of a horizontal surface. The 8 kg mass is joined by a light, inextensible string to a 4 kg mass which sits on the same horizontal surface but on a section which has a coefficient of friction of 0.25.



a. On the diagram above, label all the forces acting on each mass.

1 mark

TEVSMU34EX1_QA_07.FM

Neap:

Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Formula Sheet

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \mathrm{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\text{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \ne -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$, for $x > 0$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$

$\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\tilde{r} = x\tilde{i} + y\tilde{j} + z\tilde{k}$

$|\tilde{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\dot{\tilde{r}} = \frac{d\tilde{r}}{dt} = \frac{dx}{dt}\tilde{i} + \frac{dy}{dt}\tilde{j} + \frac{dz}{dt}\tilde{k}$

$\tilde{r}_1 \cdot \tilde{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum:	$\tilde{p} = m\tilde{v}$
equation of motion:	$\tilde{R} = m\tilde{a}$
sliding friction:	$F \leq \mu N$

END OF FORMULA SHEET

b. Let a m/s^2 be the acceleration of the system for the length of time that the 4 kg mass remains on the section for which friction applies. Find the value of a, giving your answer in terms of g.

3 marks
Total 4 marks

Question 5

a. Find the values of the integers a and b such that $\tan(15°) = a + b\sqrt{3}$.

3 marks

b. Prove that $\frac{\cos(2A)}{1+\sin(2A)} = \frac{\cos(A)-\sin(A)}{\cos(A)+\sin(A)}$.

3 marks
Total 6 marks

Question 6

Find the volume of the solid created when the curve with equation $y = 4 - x^2$,
where $x \in [0, 2]$, is rotated around the y-axis.

4 marks

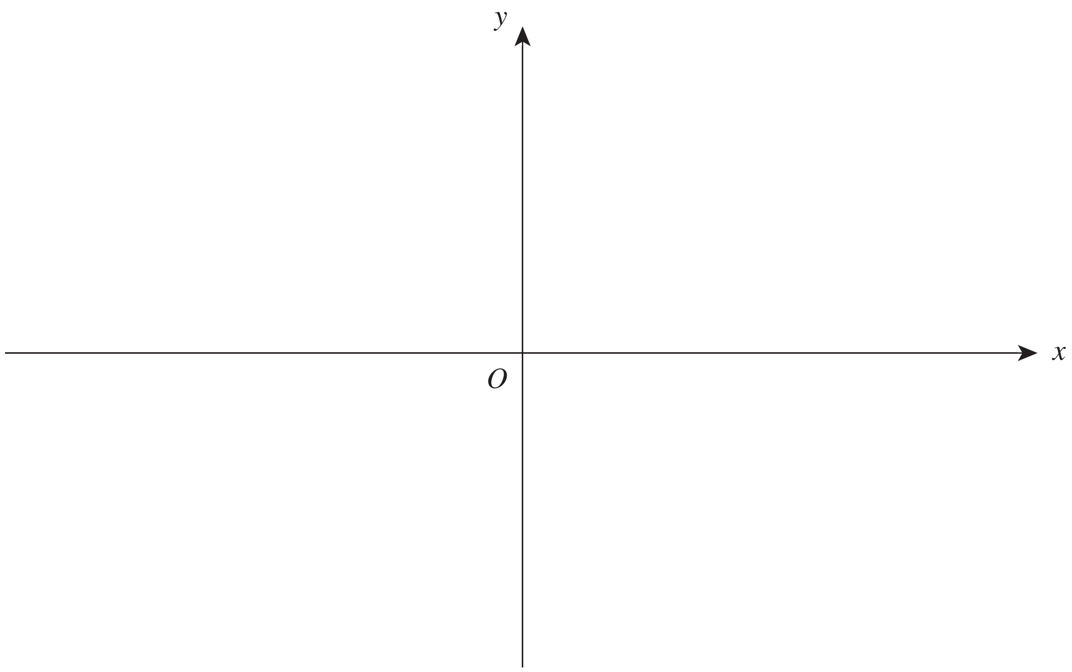
Question 7

The position vector of a moving particle is given by $\underset{\sim}{r} = \cos^2(t)\underset{\sim}{i} + 4\sin^2(t)\underset{\sim}{j},\ t \geq 0$.

a. Find the Cartesian equation of the path followed by the particle, expressing your answer in the form $y = f(x)$.

2 marks

b. Sketch the path of the particle on the axes provided.



2 marks
Total 4 marks

Question 8

Consider $P(z) = z^2 + bz + (1 + i)$, where $b \in C$.

a. If $z = i$ is a root of the equation $P(z) = 0$, find the value of *b*.

1 mark

b. Hence solve the equation $P(z) = 0$.

3 marks

Total 4 marks

Question 9

Find an antiderivative of $\frac{x^2}{x^2 - 4}$.

4 marks

END OF QUESTION AND ANSWER BOOKLET