Year 2006

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education
2006

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 32 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

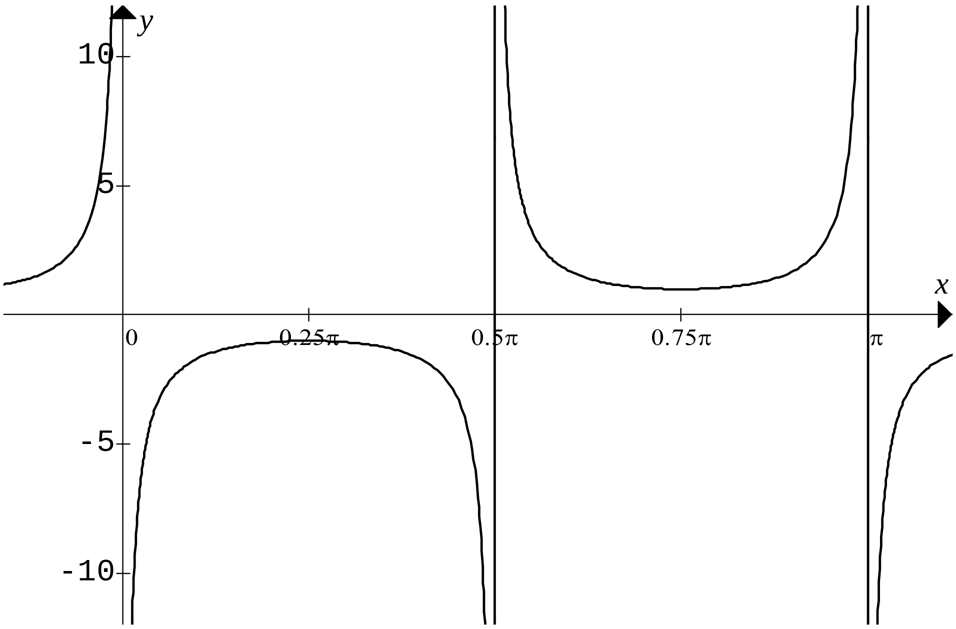
SECTION 1

Instructions for Part I

Answer **all** questions in pencil, on the answer sheet provided for multiple-choice questions. A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Given the graph below, which of the following could **NOT** be its equation



A. $y = \dfrac{-1}{x\left(x - \frac{\pi}{2}\right)(x - \pi)}$

B. $y = \text{cosec}\left(2\left(x + \frac{\pi}{2}\right)\right)$

C. $y = \text{cosec}\left(2\left(x - \frac{\pi}{2}\right)\right)$

D. $y = \text{cosec}(2x)$

E. $y = -\sec\left(2\left(x - \frac{\pi}{4}\right)\right)$

Question 2

Given that $f(x)=\dfrac{1}{x}$ and $g(x)=-x^2-(a-b)x+ab$ where $a,b\in R$ then the graph of $f(g(x))$ has

A. a vertical asymptotes at $x=0$

B. no asymptotes.

C. a turning point at $x=\dfrac{a-b}{2}$

D. vertical asymptotes at $x=-a$ and $x=b$

E. vertical asymptotes at $x=a$ and $x=-b$

Question 3

Given that $a,b\in R$ and that the graph of $y=\dfrac{ax^2+b}{x}$ has no turning points then

A. $a>0$ and $b<0$

B. $a<0$ and $b<0$

C. $a>0$ and $b>0$

D. $b>a$

E. $a=b\neq 0$

Question 4

If $\operatorname{cosec}(x)=\dfrac{4\sqrt{7}}{7}$, $\dfrac{\pi}{2}<x<\pi$, then $\sec(x)$ is equal to

A. $\dfrac{4}{3}$

B. $-\dfrac{4}{3}$

C. $\dfrac{3\sqrt{7}}{7}$

D. $-\dfrac{3\sqrt{7}}{7}$

E. $\dfrac{15\sqrt{7}}{4}$

Question 5

The ellipse $\frac{(y+1)^2}{18}+\frac{(x-4)^2}{8}=2$ has a domain and range respectively given by

A. $\left[4-2\sqrt{2},4+2\sqrt{2}\right]$ $\left[-1-3\sqrt{2},-1+3\sqrt{2}\right]$

B. $\left[-1-3\sqrt{2},-1+3\sqrt{2}\right]$ $\left[4-2\sqrt{2},4+2\sqrt{2}\right]$

C. $\left[4-3\sqrt{2},4+3\sqrt{2}\right]$ $\left[-1-2\sqrt{2},-1+2\sqrt{2}\right]$

D. $[-7,5]$ $[0,8]$

E. $[0,8]$ $[-7,5]$

Question 6

Let $u=3\operatorname{cis}\left(\frac{\pi}{4}\right)$ and $v=a\operatorname{cis}(b)$, where a and b are real constants.

If $\frac{u}{v}=-6$ then

A. $a=\frac{1}{2}$ $b=\frac{3\pi}{4}$

B. $a=\frac{1}{2}$ $b=-\frac{\pi}{4}$

C. $a=\frac{1}{2}$ $b=-\frac{3\pi}{4}$

D. $a=-\frac{1}{2}$ $b=\frac{3\pi}{4}$

E. $a=-\frac{1}{2}$ $b=-\frac{3\pi}{4}$

Question 7

If $z=\left(\cos(\theta)+i\sin(\theta)\right)^3$ and $z^2=a+bi$ then

A. $\cos^3(\theta)=\sqrt{a}$ and $\sin^3(\theta)=\sqrt{b}$

B. $\cos(3\theta)=\sqrt{a}$ and $\sin(3\theta)=\sqrt{b}$

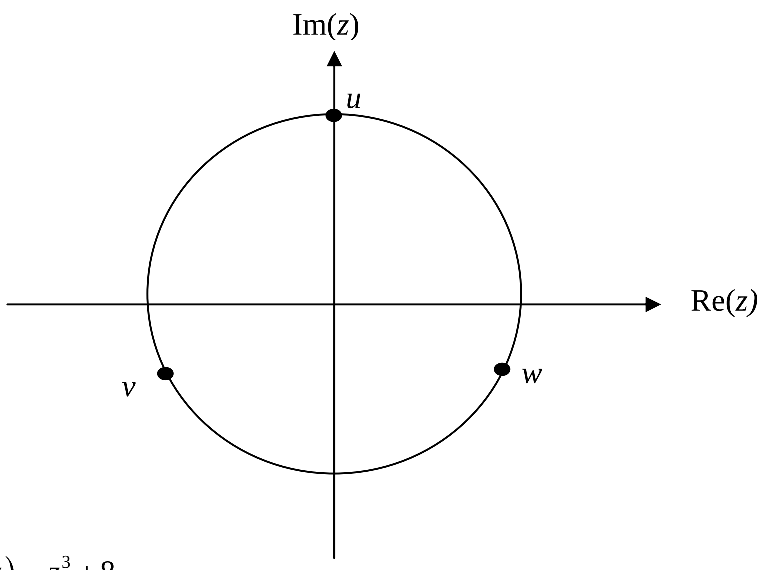
C. $\cos\left(\frac{2\theta}{3}\right)=a$ and $\sin\left(\frac{2\theta}{3}\right)=b$

D. $\cos(\theta)=a^{\frac{2}{3}}$ and $\sin(\theta)=b^{\frac{2}{3}}$

E. $\cos(6\theta)=a$ and $\sin(6\theta)=b$

Question 8

The diagram shows a circle of radius 2 on an Argand diagram. The points shown u, v and w are equally spaced around the circle and are the solutions of the equation $P(z)=0$. Then



A. $P(z)=z^3+8$

B. $P(z)=z^3-8$

C. $P(z)=z^3+8i$

D. $P(z)=z^3-8i$

E. $P(z)=z^2-4$

Question 9

If $P(z) = z^3 + bz^2 + cz + d$ and $P(-\alpha i) = 0$ and $P(\beta) = 0$

where b, c, d, α and β are all real non-zero numbers, then

A. $b = -\beta \quad c = \alpha^2 \quad d = -\alpha^2\beta$

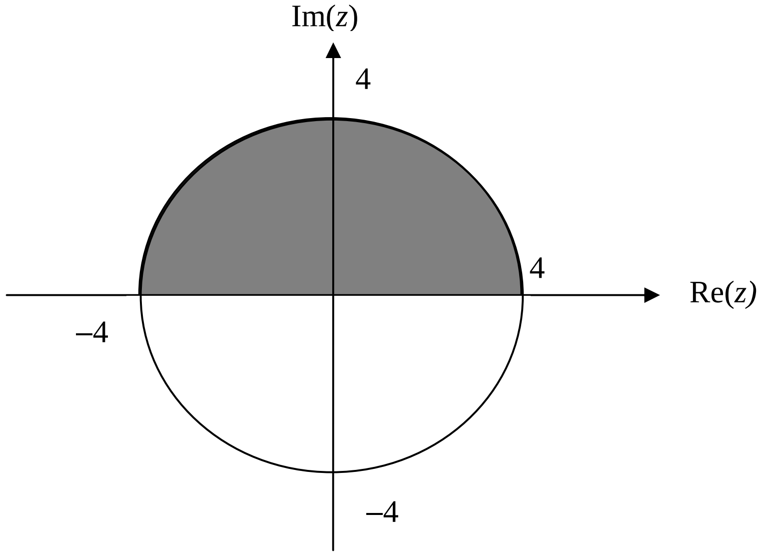
B. $b = -\beta \quad c = -\alpha^2 \quad d = \alpha^2\beta$

C. $b = -\beta \quad c = \alpha^2 \quad d = \alpha^2\beta$

D. $b = \beta \quad c = \alpha^2 \quad d = -\alpha^2\beta$

E. $b = \beta \quad c = -\alpha^2 \quad d = -\alpha^2\beta$

Question 10



Which of the following represents the shaded region above.

A. $\{z: |z| \le 16\} \cap \{z: 0 \le \text{Arg}(z) \le \pi\}$

B. $\{z: |z| \le 4\} \cap \{z: -4 \le \text{Re}(z) \le 4\}$

C. $\{z: \text{Re}^2(z) + \text{Im}^2(z) \le 16\} \cap \{z: 0 \le \text{Arg}(z) \le \pi\}$

D. $\{z: z\bar{z} \le 16\} \cap \{z: -4 \le \text{Re}(z) \le 4\}$

E. $\{z: z\bar{z} \le 4\} \cap \{z: \text{Im}(z) \ge 0\}$

Question 11

$\dfrac{3x+1}{(x+3)^2(x^2+3)}$ expressed in partial fractions has the form

A. $\dfrac{A}{x+3}+\dfrac{B}{x^2+3}$

B. $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x^2+3}$

C. $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x+\sqrt{3}}+\dfrac{D}{x-\sqrt{3}}$

D. $\dfrac{A}{(x+3)^2}+\dfrac{Bx+C}{x^2+3}$

E. $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{Cx+D}{x^2+3}$

Question 12

With a suitable substitution $\displaystyle\int_1^4 \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$ can be expressed as

A. $\displaystyle\int_1^4 \frac{\cos(u)}{u}\,du$

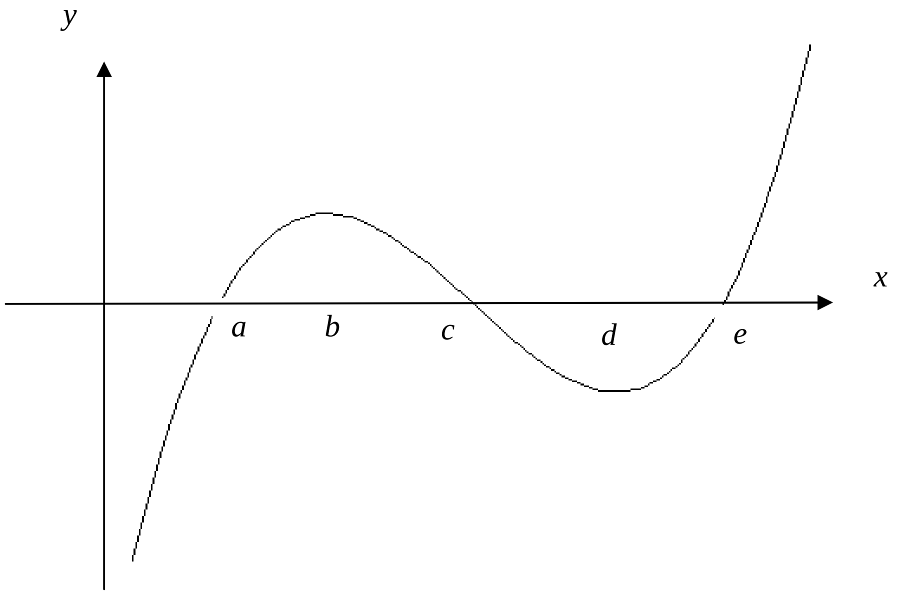
B. $\displaystyle\int_1^2 \frac{\cos(u)}{u}\,du$

C. $\displaystyle 2\int_1^2 \frac{\cos(u)}{u}\,du$

D. $\displaystyle \frac{1}{2}\int_1^2 \cos(u)\,du$

E. $\displaystyle 2\int_1^2 \cos(u)\,du$

Question 13



The graph of $y = f(x)$ is shown above. Let $F(x)$ be an antiderivative of $f(x)$
The graph of $y = F(x)$ has

A. a local maximum at $x = b$ and a local minimum at $x = d$.

B. a stationary point of inflexion at $x = c$ and local maximums at $x = a$ and $x = e$.

C. a stationary point of inflexion at $x = c$ and local minimums at $x = a$ and $x = e$.

D. two local minimums at $x = a$ and $x = e$, one local maximum at $x = c$ and two points of inflexions one somewhere between $x = a$ and $x = c$ and another somewhere between $x = c$ and $x = e$.

E. two local maximums at $x = a$ and $x = e$, one local minimum at $x = c$ and two points of inflexions one somewhere between $x = a$ and $x = c$ and another somewhere between $x = c$ and $x = e$.

Question 14

ABC is a equilateral triangle, with *P* the mid-point of $\overrightarrow{AB}$.

Which of the following statements is **FALSE**

A. $\overrightarrow{AP} = \overrightarrow{PB} = \frac{1}{2}\overrightarrow{AB}$

B. $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right| = \left|\overrightarrow{AC}\right|$

C. $\overrightarrow{AP} = \lambda \overrightarrow{PC}$ where $\lambda \in R$

D. $2\,\overrightarrow{AC}.\overrightarrow{AB} = \left|\overrightarrow{AC}\right|\left|\overrightarrow{AB}\right|$

E. $2\,\overrightarrow{CP}.\overrightarrow{CB} = \sqrt{3}\left|\overrightarrow{CP}\right|\left|\overrightarrow{CB}\right|$

Question 15

If $\underset{\sim}{a} = -2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + y\,\underset{\sim}{j} + 2\underset{\sim}{k}$ then which of the following is **FALSE**

A. If $y = -4$ the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are linearly independent.

B. The angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $\cos^{-1}\left(\dfrac{2y-10}{3\sqrt{(20+y^2)}}\right)$

C. If $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$ then $y = 5$

D. If $y > 5$ then the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is acute.

E. If $y < 5$ then the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is obtuse.

Question 16

A particle moves in such a way that its velocity vector at a time *t* is given by

$4e^{\frac{t}{2}}\underset{\sim}{i} - 2\sin\left(\frac{t}{2}\right)\underset{\sim}{j}$. Initially the position vector of the particle is $3\underset{\sim}{i} - 3\underset{\sim}{j}$

The position vector of the particle at a time *t* is given by

A. $\left(8e^{\frac{t}{2}} - 5\right)\underset{\sim}{i} - \left(4\cos\left(\frac{t}{2}\right) + 3\right)\underset{\sim}{j}$

B. $\left(8e^{\frac{t}{2}} - 5\right)\underset{\sim}{i} + \left(4\cos\left(\frac{t}{2}\right) - 7\right)\underset{\sim}{j}$

C. $\left(8e^{\frac{t}{2}} - 5\right)\underset{\sim}{i} - \left(2 + \cos\left(\frac{t}{2}\right)\right)\underset{\sim}{j}$

D. $\left(1 + 2e^{\frac{t}{2}}\right)\underset{\sim}{i} - \left(\cos\left(\frac{t}{2}\right) - 4\right)\underset{\sim}{j}$

E. $\left(1 + 2e^{\frac{t}{2}}\right)\underset{\sim}{i} + \left(4\cos\left(\frac{t}{2}\right) - 7\right)\underset{\sim}{j}$

Question 17

A particle is held in equilibrium by three concurrent coplanar forces $\underset{\sim}{P}$, $\underset{\sim}{Q}$ and $\underset{\sim}{R}$.
$\underset{\sim}{P}$ has a magnitude of P and acts in the west direction, $\underset{\sim}{Q}$ has a magnitude of Q and acts in the south direction and $\underset{\sim}{R}$ has a magnitude of R and acts in the north θ^0 east direction. Which of the following is **FALSE?**

A. If $\theta = 45$ then $P = Q = \sqrt{2}\,R$

B. $R^2 = P^2 + Q^2$

C. $P = R\sin(\theta)$ and $Q = R\cos(\theta)$

D. $\tan(\theta) = \dfrac{P}{Q}$

E. $\underset{\sim}{P} + \underset{\sim}{Q} + \underset{\sim}{R} = \underset{\sim}{0}$

Question 18

A television has a mass of 60 kg and is on a level cabinet. The co-efficient of friction between the television and the cabinet is 0.75. A horizontal force of F newtons is applied to the television, then if

A. $F = 45$ the television moves with constant acceleration.

B. $F = 441$ the television moves with constant velocity.

C. $F = 440$ the television is on the point of moving.

D. $F > 441$ the television does not move.

E. $F = 450$ the television moves with constant acceleration.

Question 19

A suitcase of mass *m* kilograms rests on a rough plane inclined at an angle of θ to the horizontal. The suitcase is just prevented from slipping down the incline by a force of P newtons acting up and parallel to the plane. If the coefficient of friction between the suitcase and the plane is μ, which of the following is correct?

A. $P = mg\sin\theta$

B. $P = mg\left(\sin\theta - \mu\cos\theta\right)$

C. $P = mg\left(\sin\theta + \mu\cos\theta\right)$

D. $P = mg\left(\cos\theta - \mu\sin\theta\right)$

E. $P = mg\left(\cos\theta + \mu\sin\theta\right)$

Question 20

Euler's method, with a step size of 0.25, is used to solve the differential equation

$\tan^{-1}\left(\frac{1}{2t}\right)\frac{dt}{dy} = 1$ with initial condition $t = 1$ $y = 2$.

When $t = 1.5$, the value obtained for *y*, correct to four decimal places, is

A. 2.2110

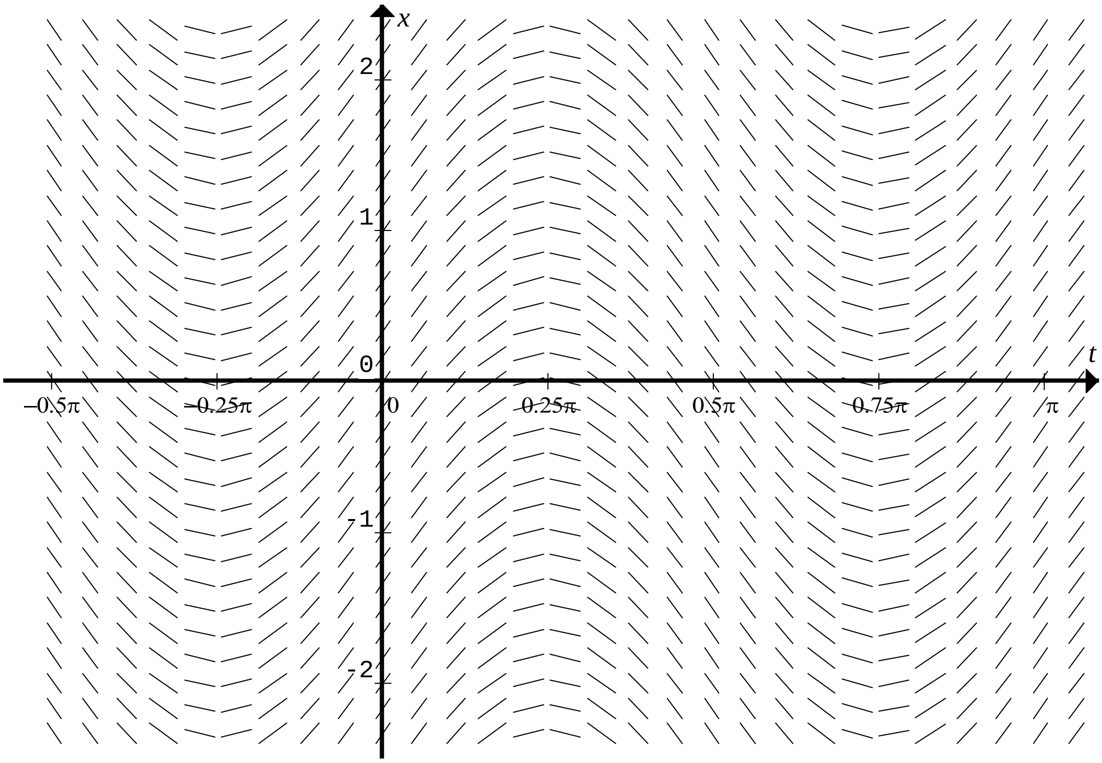
B. 2.2556

C. 3.1962

D. 2.2768

E. 2.9868

Question 21



The direction (slope) field for a certain first order differential equation is shown above. The differential equation could be

A. $\dfrac{dx}{dt} = \sin(2t)$

B. $\dfrac{dx}{dt} = 2\cos(2t)$

C. $\dfrac{dx}{dt} = \cos(2t)$

D. $\dfrac{dx}{dt} = -\dfrac{1}{2}\cos(2t)$

E. $\dfrac{dx}{dt} = 2\sin(2t)$

Question 22

A tank with vertical sides has a cross-section of area A m^2. It is initially filled with water to a height of h_0. The tank has a hole in the bottom through which the water escapes at a rate of $c\sqrt{h}$ m^3/min, where h is the height of water in the tank and c is a constant. Water is poured into the tank at a rate of Q m^3/min. The differential equation relating h at a time t minutes is given by

A. $\dfrac{dh}{dt} = \dfrac{c\sqrt{h} - Q}{A} \qquad h(0) = h_0$

B. $\dfrac{dh}{dt} = \dfrac{Q - c\sqrt{h}}{A} \qquad h(0) = h_0$

C. $\dfrac{dh}{dt} = \dfrac{\left(Q - c\sqrt{h}\right)t}{A} \qquad h(0) = h_0$

D. $\dfrac{dh}{dt} = \dfrac{Q - c\sqrt{h}}{At} \qquad h(0) = h_0$

E. $\dfrac{dh}{dt} = \dfrac{Qt - c\sqrt{h}}{At} \qquad h(0) = h_0$

SECTION 2

Instructions for Section 2

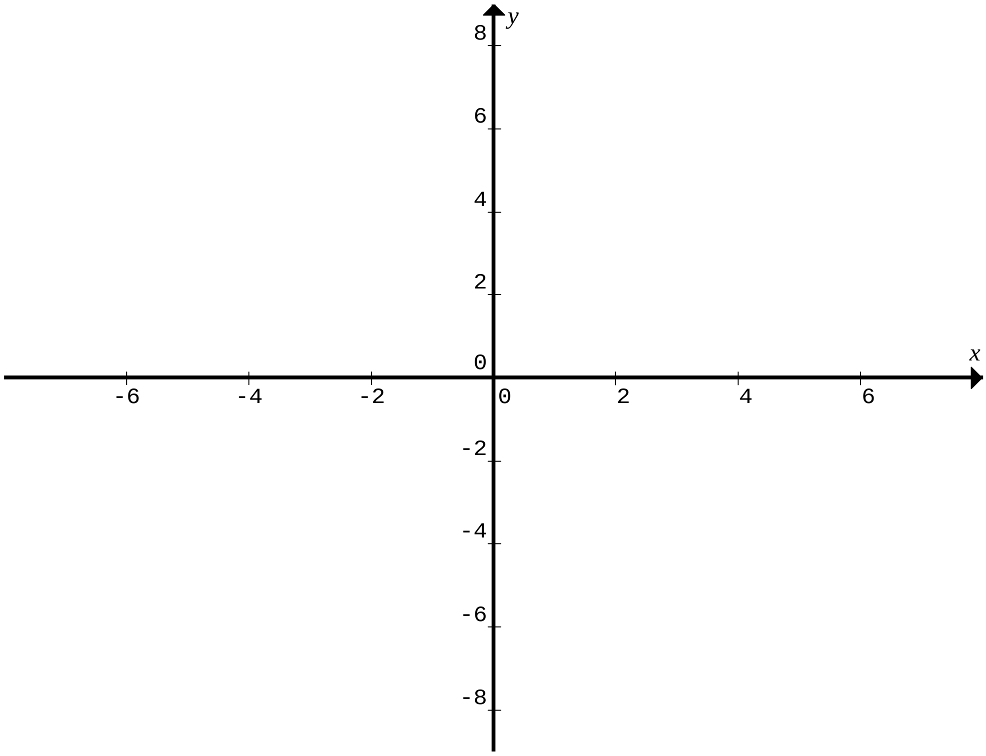
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

a.i. Express $3x^2 + 18x - y^2 + 4y + 11 = 0$ in the form $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

2 marks

ii. Hence sketch the relation $3x^2 + 18x - y^2 + 4y + 11 = 0$ on the axes below, stating the equations of any asymptotes.



2 marks

b. Consider the relation $3x^2 + 18x - y^2 + 4y + 11 = 0$

i. Find an expression for $\frac{dy}{dx}$ in terms of both *x* and *y*.

1 mark

ii. Find the coordinates where the tangent to the curve is parallel to the *y*-axis.

1 mark

c. Given the points $R\left(0,2-3\sqrt{3}\right)$, $S\left(0,3\sqrt{3}+2\right)$ and $C\left(-3,2\right)$

Find using vectors the angle between $\overrightarrow{CS}$ and $\overrightarrow{CR}$, in relation with the graph in **a.**

Explain what this angle represents.

3 marks

d. Let $m = -4 + yi$, $f = -7 + 2i$ and $z = x + yi$

Find the Cartesian equation of $\{ z : |z - f| = 2\,|z - m| \}$

3 marks

e. A particle moves so that its position vector is given by

$\underset{\sim}{r}(t) = (-3 + 2\sec(2t))\underset{\sim}{i} + (2 + 2\sqrt{3}\tan(2t))\underset{\sim}{j}$ for $t \geq 0$

Find the Cartesian equation of the path.

2 marks

Total 14 marks

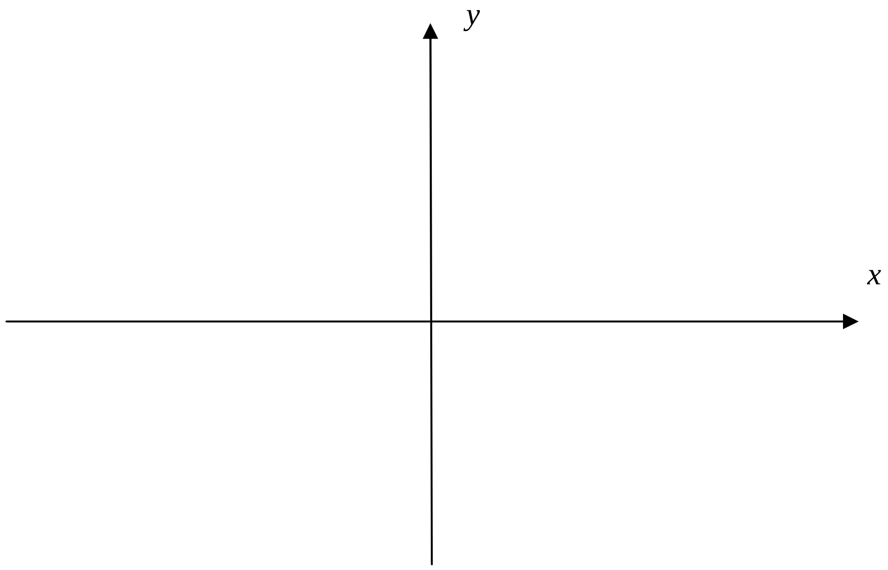
Question 2

Consider the function f with the rule $f(x) = 25 - 20\cos^{-1}\left(\frac{x}{15}\right)$

a. State the largest domain for which f is defined.

1 mark

b. Sketch the graph of $y = f(x)$ on the axes below, clearly indicating the scale.



1 mark

c. Find $f'(x)$ stating its domain.

1 mark

A vase is formed when the section of the curve $y = f(x)$ bounded by the co-ordinate axes and the line $y = 25$ is rotated about the *y*-axis. Draw the shape of the vase of the diagram above. The dimensions of the vase are in centimetres.

d. Find the diameter of the base of the vase, giving your answer correct to three decimal places.

1 mark

e. Find the angle in degrees and minutes, of the slope of the vase at the point where $x = 10$

2 marks

f. i. Find a definite integral which gives the volume V of the vase in cubic centimetres.

3 marks

ii. Find the volume V correct to two decimal places.

1 mark

Total 10 marks

Question 3

Pete is riding his bike in a straight line at a steady speed of 9 m/s along a level road. He encounters a brief head-wind and stops pedalling. The wind slows his speed down to 1 m/s. The total resistive force of the head-wind was $260\sqrt{v}$ newtons where v m/s is the speed of Pete on his bike after a time t seconds when the head-wind started. The mass of Pete and his bike is 45 kg.

a. If a m/s^2 is the acceleration of Pete during the head wind, write down the equation of motion for Pete during the head wind.

1mark

b. Find the exact duration of the head-wind.

2 marks

c.i. Express v in terms of t.

1 mark

ii. Sketch the velocity time graph of Pete during the head wind on the axes below, clearly indicating the scale.

1 mark

d. Hence write down a definite integral representing the exact distance D metres travelled by Pete during the head wind. Find the value of D.

2 marks

e. Show that a differential equation relating v and x is $\frac{dv}{dx} = -\frac{52}{9\sqrt{v}}$ where x metres is the distance Pete travels on his bike during the head-wind.

1 mark

f. Find another different definite integral representing the exact distance D metres travelled by Pete during the head-wind. Find, using calculus, the exact value of D.

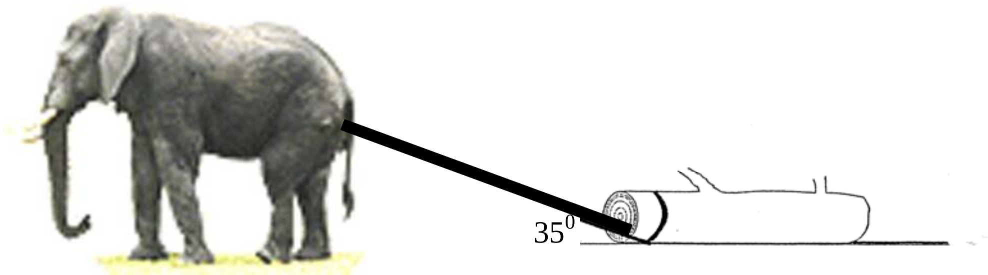
3 marks

Total 11 marks

Question 4

a. A elephant of mass 5000 kg is pulling a 400 kg log along the ground as shown. The elephant is connected to the log by a rope which makes an angle of 35^0 with the ground. The elephant exerts a horizontal constant pulling force of P newtons. The coefficient of friction between the log and the ground is 0.8.
(You may assume that there is no resistance to the motion of the elephant)

i. On the diagram below mark in all the forces acting on the log and the elephant.



1 mark

The elephant and the log are moving with an acceleration of 0.1 m/s^2.

ii. Find the tension in the rope giving your answer correct to the nearest newton.

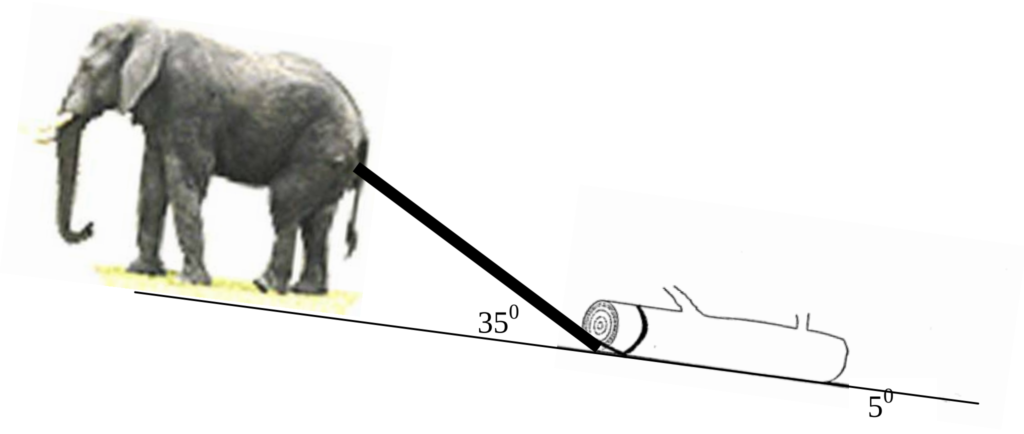
3 marks

iii. Find the value of P, giving your answer correct to the nearest newton.

1 mark

b. Sometime later, the elephant and the log are moving up a hill inclined at an angle of 5^0 to the horizontal. The elephant is still connected to the log by a rope which makes an angle of 35^0 with the hill. The elephant exerts a constant pulling force of Q newtons, up and parallel to the hill. The coefficient of friction between the log and the ground is still 0.8.
(You may assume that there is no resistance to the motion of the elephant)

i. On the diagram below, mark in all the forces acting on the log and the elephant.



1 mark

The elephant and the log are moving up the hill with a constant speed of 0.1 m/s.

ii. Find the tension in the rope giving your answer correct to the nearest newton.

3 marks

iii. Find the value of Q, giving your answer correct to the nearest newton.

1 mark

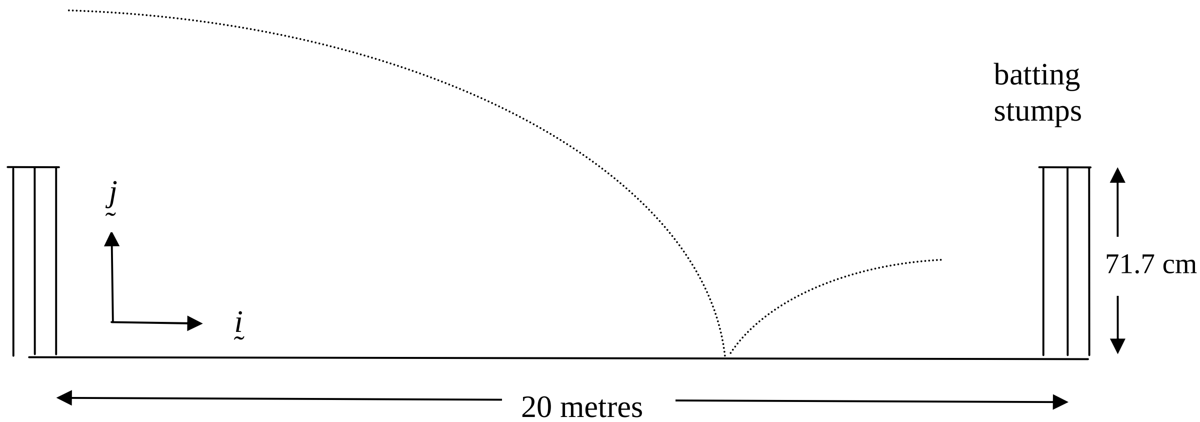
Total 10 marks

Question 5

A cricket pitch is in a west to east direction. A fast bowler in cricket bowls the ball in a vertical plane in an easterly direction. The flight of the ball can be modelled by the vector equation

$$\underset{\sim}{r}(t) = \left(-25t^2 + 52.5t\right)\underset{\sim}{i} + \left(2e^{-\frac{7t}{2}}\left|\cos\left(\frac{17\pi t}{10}\right)\right|\right)\underset{\sim}{j} \qquad t \geq 0$$

where t is the time in seconds of the cricket ball after it leaves the bowlers hand and $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors of one metre in the easterly and vertically upwards directions above ground level.



The horizontal (easterly) distance from the point where the ball leaves his hands to the stumps is 20 metres, and the cricket stumps are 71.7 cm high as shown in the diagram above.

a. How high above the ground was the ball when it left the bowlers hand?

1 mark

b. Find the velocity vector of the ball at a time t.

2 marks

c. If a cricket ball has a mass of 155 gm, find the magnitude of the momentum of the ball at the instant when it left the bowlers hand. Give your answer correct to two decimal places.

2 marks

d. During the motion of the ball it strikes the ground. Find when the ball hits the ground and the horizontal distance in metres correct to two decimal places to the batting stumps at this time.

3 marks

e. Find the angle (in degrees and minutes) at which the ball strikes the ground.

2 marks

f. After the bounce the ball strikes the batting stumps, find the total time of flight, that is from the instant the ball leaves the bowlers hand to the time when it strikes the stumps. Find the vertical distance above ground level to the nearest centimetre, that the ball strikes the batting stumps.

2 marks

g. Find in km/hr the average horizontal velocity of the ball from the instant that it leaves the bowlers hand to the time when it strikes the batting stumps.

1 mark

Total 13 marks

EXTRA WORKING SPACE

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\quad -\pi < Arg\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\quad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\quad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1} \qquad \int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax} \qquad \int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x} \qquad \int \frac{1}{x}dx=\log_e(x)+c,\ \text{for}\ \ x>0$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax) \qquad \int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax) \qquad \int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax) \qquad \int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}} \qquad \int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2} \qquad \int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b,$ then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x)$

END OF FORMULAE SHEET

ANSWER SHEET

STUDENT NUMBER

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