Year 2006

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2006

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Given the vector $\underset{\sim}{b} = -2\underset{\sim}{i} + y\underset{\sim}{j} + 4\underset{\sim}{k}$ find the value of y if

a. the length of vector $\underset{\sim}{b}$ is 5.

2 marks

b. the vector $\underset{\sim}{b}$ makes an angle of 150° with the *y*-axis.

2 marks

Question 2

A tank has a capacity of 300 litres and contains 200 litres of water in which 8 kilograms of salt has been dissolved. A salt solution of concentration 0.5 kilograms per litre is poured into the tank at a rate of 2 litres per minute, and the well-stirred mixture flows out at a rate of 3 litres per minute. If the amount of salt in the tank at a time *t* minutes is *Q* kilograms,

a. set up the differential equation, for the amount of salt *Q* kg in the tank at a time *t* minutes.

1 mark

b. If $Q = \frac{1}{2}(200-t) + C(200-t)^n$ is a solution, where C is a constant, find the value of *n*.

2 marks

Question 3

a. Show that $\dfrac{d}{dx}\left(\cos^{-1}\left(\sqrt{\dfrac{3}{x}}\right)\right) = \dfrac{\sqrt{3}}{2x\sqrt{x-3}}$ for $x > 3$

2 marks

b. Hence find the exact value of $\displaystyle\int_4^{12} \frac{1}{x\sqrt{x-3}}\,dx.$

2 marks

Question 4

a. If $y = \dfrac{x}{\sqrt{x-3}}$ then gradient function can be represented as $\dfrac{ax+b}{\sqrt{(x-3)^3}}$

find the exact values of a and b.

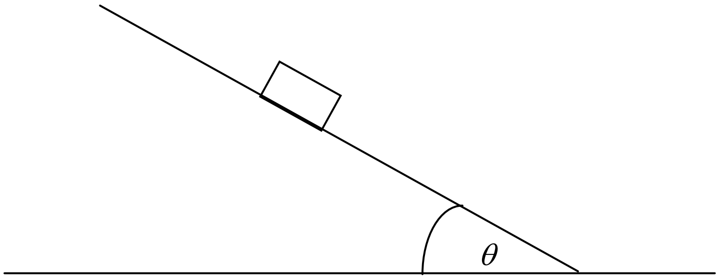
2 marks

b. Find using calculus the exact area bounded by the curve $y = \dfrac{x}{\sqrt{x-3}}$ the x-axis and the lines $x = 3$ and $x = 4$.

2 marks

Question 5

A block of mass m is projected with a speed U so as to move straight up an inclined slide. The slide makes an angle of θ with the horizontal and the coefficient of friction between the block and the slide is μ.



a. On the diagram above, mark in all the forces on the block on its movement up the slide.

1 mark

b. Show that the block travels a distance of $\dfrac{U^2}{2g\left(\sin(\theta)+\mu\cos(\theta)\right)}$ up the slide before coming to rest.

3 marks

Question 6

The area enclosed by the curves with the equations $y = \cos(2x)$ and $y = \sin(x)$, $0 \le x \le \frac{\pi}{6}$ is rotated about the *x*-axis to form a solid of revolution. Find the exact volume of this solid of revolution.

4 marks

Question 7

a. Solve for *z* if $z^4 + z^2 - 12 = 0$

2 marks

b. Given that $z = a + bi$ where $a, b \in R$ find z^2.

Show that if $z^2 = -1 - 4\sqrt{3}i$ then $a^2 - b^2 = -1$ and $ab = -2\sqrt{3}$.

Hence find in exact Cartesian form the square roots of $-1 - 4\sqrt{3}\, i$

3 marks

c. Use the quadratic formula to solve $z^2 + \sqrt{3}\, z + \left(1 + \sqrt{3}\, i\right) = 0$

3 marks

Question 8

OABC is a trapezium. The length of *OA* is λ times the length of *CB*.
P and *Q* are the midpoints of *OC* and *BA* respectively.
Let $\overrightarrow{OA} = \underset{\sim}{a}$

a. Express $\overrightarrow{PQ}$ in terms of λ and $\underset{\sim}{a}$

3 marks

b. What is the ratio of the lengths of *PQ* to *OA* ?

1 mark

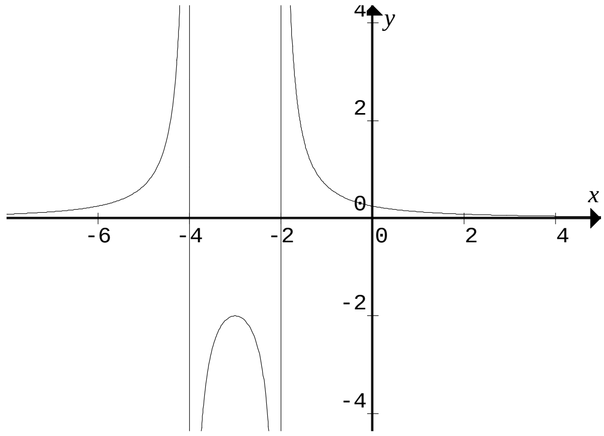
Question 9

a. The graph shown has the form

$$y = \frac{a}{x^2 + bx + c}$$

The graph has vertical asymptotes at $x = -4$ and $x = -2$ and has a range of $(-\infty, -2] \cup (0, \infty)$.

Prove that $a = 2$ $b = 6$ and $c = 8$



2 marks

b. If the area bounded by the curve $y = \frac{2}{x^2 + 6x + 8}$ the coordinates axes and $x = 2$ can be expressed in the form $\log_e(p)$ find using calculus the exact value of p.

3 marks

EXTRA WORKING SPACE

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc\sin(A)$

sine rule: $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Coordinate geometry

ellipse: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < Arg\, z \le \pi$

$z_1 z_2 = r_1 r_2\, \text{cis}(\theta_1 + \theta_2)$ $\qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad s = ut + \frac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \frac{1}{2}(u+v)t$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$ $$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$ $$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$$ $$\int \frac{1}{x}dx=\log_e\left(x\right)+c,\ \text{ for }\ x>0$$

$$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$$ $$\int \sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$$

$$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$$ $$\int \cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$$

$$\frac{d}{dx}\left(\tan\left(ax\right)\right)=a\sec^2\left(ax\right)$$ $$\int \sec^2\left(ax\right)dx=\frac{1}{a}\tan\left(ax\right)+c$$

$$\frac{d}{dx}\left(\sin^{-1}\left(x\right)\right)=\frac{1}{\sqrt{1-x^2}}$$ $$\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\frac{d}{dx}\left(\cos^{-1}\left(x\right)\right)=\frac{-1}{\sqrt{1-x^2}}$$ $$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\frac{d}{dx}\left(\tan^{-1}\left(x\right)\right)=\frac{1}{1+x^2}$$ $$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$$

product rule: $\frac{d}{dx}\left(uv\right)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x\right)$

END OF FORMULAE SHEET