Year 2006

VCE

Specialist Mathematics

Trial Examination 1



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Victorian Certificate of Education 2006

STUDENT NUMBER

						Letter	
Figures							
Words							

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of	Number of questions	Number of
questions	to be answered	marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question. In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude g m/s², where g = 9.8

Question 1

Give	en the vector $\mathbf{b} = -2\mathbf{i} + y\mathbf{j} + 4\mathbf{k}$ find the value of y if	
a.	the length of vector \underline{b} is 5.	
		2 marks
b.	the vector b makes an angle of 150° with the y-axis.	2 marks

A tank has a capacity of 300 litres and contains 200 litres of water in which 8 kilograms of salt has been dissolved. A salt solution of concentration 0.5 kilograms per litre is poured into the tank at a rate of 2 litres per minute, and the well-stirred mixture flows out at a rate of 3 litres per minute. If the amount of salt in the tank at a time t minutes is Q kilograms,

a.	set up the differential equation, for the amount of salt Q kg in the tank at a minutes.	time t
		1 mark
b.	If $Q = \frac{1}{2}(200 - t) + C(200 - t)^n$ is a solution, where C is a constant,	
	find the value of n .	

a.	Show that	$\frac{d}{dx}\left(\cos^{-1}\left(\sqrt{\frac{3}{x}}\right)\right) = \frac{\sqrt{3}}{2x\sqrt{x-3}}$	for $x > 3$

b.	Hence find the exact value of $\int_{4}^{12} \frac{1}{x\sqrt{x-3}} dx.$

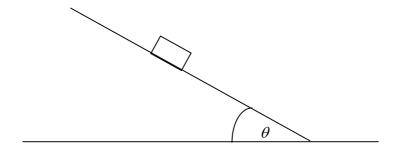
a. If $y = \frac{x}{\sqrt{x-3}}$ then gradient function can be represented as $\frac{ax+b}{\sqrt{(x-3)^3}}$

find the exact values of a and b.

2 marks

b. Find using calculus the exact area bounded by the curve $y = \frac{x}{\sqrt{x-3}}$ the x-axis and the lines x = 3 and x = 4.

A block of mass m is projected with a speed U so as to move straight up an inclined slide. The slide makes an angle of θ with the horizontal and the coefficient of friction between the block and the slide is μ .



a. On the diagram above, mark in all the forces on the block on its movement up the slide.

1 mark

b. Show that the block travels a distance of $\frac{U^2}{2g(\sin(\theta) + \mu\cos(\theta))}$ up the slide before coming to rest.

One	4.	(
C 1116	2CT17	m n

The area enclosed by the curves with the equations $y = \cos(2x)$ and $y = \sin(x)$, $0 \le x \le \frac{\pi}{6}$
is rotated about the <i>x</i> -axis to form a solid of revolution. Find the exact volume of this solid of revolution.
4 marks
Question 7
a. Solve for z if $z^4 + z^2 - 12 = 0$

Given that $z = a + bi$ where $a, b \in R$ find z^2 .	
Show that if $z^2 = -1 - 4\sqrt{3}i$ then $a^2 - b^2 = -1$ and $ab = -2\sqrt{3}$.	
Hence find in exact Cartesian form the square roots of $-1-4\sqrt{3} i$	
3 ma	rko
	11/2
Ose the quadratic formula to solve $(2 + \sqrt{3} + \sqrt{1 + \sqrt{3}}) = 0$	
	Show that if $z^2 = -1 - 4\sqrt{3}i$ then $a^2 - b^2 = -1$ and $ab = -2\sqrt{3}$.

Question	8

OABC is a trapezium. The length of OA is λ times the length of CB. P and Q are the midpoints of OC and BA respectively. Let $\overrightarrow{OA} = \underline{a}$

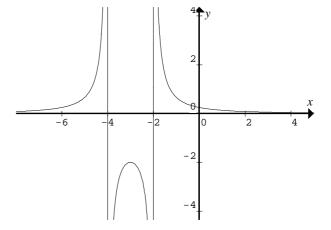
a.	Express \overrightarrow{PQ} in terms of λ and \underline{a}	
		3 marks
b.	What is the ratio of the lengths of <i>PQ</i> to <i>OA</i> ?	

a. The graph shown has the form

$$y = \frac{a}{x^2 + bx + c}$$

The graph has vertical asymptotes at x = -4 and x = -2 and has a range of $(-\infty, -2] \cup (0, \infty)$.

Prove that a = 2 b = 6 and c = 8



2 marks

b. If the area bounded by the curve $y = \frac{2}{x^2 + 6x + 8}$ the coordinates axes and x = 2 can be expressed in the form $\log_e(p)$ find using calculus the exact value of p.

EXTRA WORKING SPACE				

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

 $\pi r^2 h$ volume of a cylinder:

 $\frac{1}{3}\pi r^2 h$ volume of a cone:

volume of a pyramid: $\frac{1}{2}Ah$

 $\frac{4}{3}\pi r^3$ volume of a sphere:

 $\frac{1}{2}bc\sin(A)$ area of triangle:

 $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ sine rule:

 $c^2 = a^2 + b^2 - 2ab\cos(C)$ cosine rule:

Coordinate geometry

 $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ ellipse:

Circular (trigonometric) functions

 $\cos^2(x) + \sin^2(x) = 1$ $1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \csc^2(x)$

 $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

 $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

 $cos(2x) = cos^{2}(x) - sin^{2}(x) = 2cos^{2}(x) - 1 = 1 - 2sin^{2}(x)$

 $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$ $\sin(2x) = 2\sin(x)\cos(x)$

function	sin ⁻¹	cos ⁻¹	tan ⁻¹
domain	[-1,1]	[-1,1]	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$$

$$|z| = \sqrt{x^2 + y^2} = r \qquad -\pi < Arg \ z \le \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Vectors in two and three dimensions

$$\begin{aligned}
& \underset{\sim}{r} = x \underline{i} + y \underline{j} + z \underline{k} \\
& |\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r \\
& \underset{\sim}{\dot{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}
\end{aligned}$$

Mechanics

momentum: p = my

equation of motion: R = ma

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$$v = u + at$$
 $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u + v)t$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}(x^{n}) = nx^{n-1} \qquad \int x^{n} dx = \frac{1}{n+1}x^{n+1} + c , n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_{e}(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax) \qquad \int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax) \qquad \int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a\sec^{2}(ax) \qquad \int \sec^{2}(ax) dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^{2}}} \qquad \int \frac{dx}{\sqrt{a^{2}-x^{2}}} = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^{2}}} \qquad \int \frac{1}{\sqrt{a^{2}-x^{2}}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^{2}} \qquad \int \frac{a}{a^{2}+x^{2}} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule:
$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

END OF FORMULAE SHEET