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Specialist Mathematics

2006

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

The graph of $y = \dfrac{2x^2 + x - 3}{x^2 + 7x - 8}$ has

A. no straight line asymptotes.

B. only one straight line asymptote.

C. only two straight line asymptotes.

D. only three straight line asymptotes.

E. $x = 1$ and $x = -8$ as its vertical asymptotes.

Question 2

Which one of the following hyperbolas has straight line asymptotes with equations $y = \dfrac{x}{2}$ and $y = -\dfrac{x}{2} - 2$?

A. $\dfrac{(x-2)^2}{4} - (y-1)^2 = 1$

B. $\dfrac{(x-2)^2}{8} - \dfrac{(y-1)^2}{2} = 1$

C. $\dfrac{(x-2)^2}{12} - \dfrac{(y-1)^2}{3} = 1$

D. $\dfrac{(x+2)^2}{8} - \dfrac{(y+1)^2}{2} = 1$

E. $\dfrac{(x+2)^2}{3} - \dfrac{(y+1)^2}{12} = 1$

Question 3
The roots of $z^4 + 1$ are

A. $z^2 - i$, $z^2 + i$

B. -1, 1, $-i$, i

C. $z-1$, $z+1$, $z-i$, $z+i$

D. $\frac{\sqrt{2}}{2}(1+i)$, $\frac{\sqrt{2}}{2}(1-i)$, $-\frac{\sqrt{2}}{2}(1+i)$, $-\frac{\sqrt{2}}{2}(1-i)$

E. $z+\frac{\sqrt{2}}{2}(1+i)$, $z+\frac{\sqrt{2}}{2}(1-i)$, $z-\frac{\sqrt{2}}{2}(1+i)$, $z-\frac{\sqrt{2}}{2}(1-i)$

Question 4
Given $z = \sqrt{2} - 3i$ and $w = 3 + i\sqrt{2}$, $(z\overline{w})^{-1}$ is equal to

A. $\frac{6\sqrt{2}+7i}{121}$

B. $\frac{6\sqrt{2}-7i}{121}$

C. $\frac{1}{6\sqrt{2}+7i}$

D. $\frac{i}{7}$

E. $\frac{i}{11}$

Question 5
Which one of the following statements is **false** for $z = -1.82 + 0.91i$?

A. $z \in \left\{z : Arg(z) \geq -\frac{5\pi}{6}\right\} \cap \{z : 2 < |z| < 4\}$

B. $z \in \left\{z : Arg(z) \geq \frac{5\pi}{6}\right\} \cap \{z : 2 < |z| < 4\}$

C. $z \in \left\{z : Arg(z) \geq \frac{5\pi}{6}\right\} \cup \{z : 3 < |z| < 5\}$

D. $z \in \left\{z : Arg(z) \leq \frac{5\pi}{6}\right\} \cup \{z : 2 < |z| < 4\}$

E. $z \in \left\{z : Arg(z) \leq -\frac{5\pi}{6}\right\} \cup \{z : 3 < |z| < 5\}$

Question 6

The simplified form of $\dfrac{\cos x - \sin x}{\cos x + \sin x}$ is

A. $\sec(2x) - \tan(2x)$

B. $\sec(2x) + \tan(2x)$

C. $\operatorname{cosec}(2x) - \cot(2x)$

D. $\operatorname{cosec}(2x) + \cot(2x)$

E. $(\sec(2x) - \tan(2x))^{-1}$

Question 7

Given $f(x) = 3\sec\left(\dfrac{x-\pi}{2}\right) + 1,\ 0 < x \le \pi$, then

A. $f^{-1}(x) = 2\cos^{-1}\left(\dfrac{3}{x-1}\right) + \pi,\ 0 < x \le \pi$

B. $f^{-1}(x) = 3\cos^{-1}\left(\dfrac{2}{x-1}\right) + \pi,\ 0 < x \le \pi$

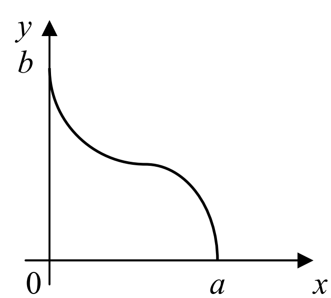
C. $f^{-1}(x) = 3\cos^{-1}\left(\dfrac{2}{x-1}\right) + \pi,\ x \ge 4$

D. $f^{-1}(x) = 2\cos^{-1}\left(\dfrac{3}{x-1}\right) + \pi,\ x \ge 4$

E. $f^{-1}(x) = \cos^{-1}\left(\dfrac{6}{x-1} + \pi\right),\ x \ge 4$

Question 8

The graph of $y = \dfrac{b}{\pi}\cos^{-1}\left(\dfrac{2x-a}{a}\right)$ is shown below,



the value of $\int_0^b x\,dy$ is

A. $\dfrac{ab\pi}{6}$ **B.** $\dfrac{3ab}{2\pi}$ **C.** $\dfrac{ab}{2}$ **D.** $\dfrac{a^2+b^2}{2\pi}$ **E.** $\dfrac{(a+b)^2}{2\pi}$

Question 9

The points on the curve $x^2 - y^2 = \frac{3}{4}$ where the gradient is 2 are

A. $\left(-\frac{1}{2},-1\right)$, $\left(\frac{1}{2},1\right)$

B. $(-2,-1)$, $(2,1)$

C. $\left(-1,\frac{1}{2}\right)$, $\left(1,\frac{1}{2}\right)$

D. $(-2,1)$, $(2,1)$

E. $\left(-1,-\frac{1}{2}\right)$, $\left(1,\frac{1}{2}\right)$

Question 10

The gradient(s) of the curve $y = \log_e|x+1|$ where $y = 1$ is/are

A. $-e$, e

B. $-e-1$, $e-1$

C. $e+1$, $e-1$

D. $-e^{-1}$, e^{-1}

E. $1-e^{-1}$, $1+e^{-1}$

Question 11

If $a < b < -1$, then $\int_a^b \log_e|x|dx$ is equal to

A. $\int_{-b}^{-a} \log_e|x|dx$

B. $\int_b^a \log_e|x|dx$

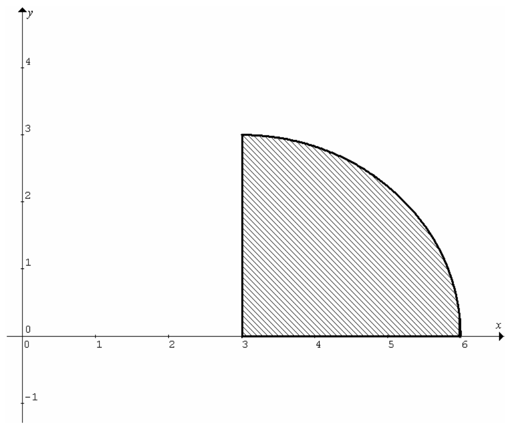
C. $-\int_{-b}^{-a} \log_e|x|dx$

D. $\int_b^a \log_e(x)dx$

E. $\int_{-b}^{-a} \log_e(-x)dx$

Question 12

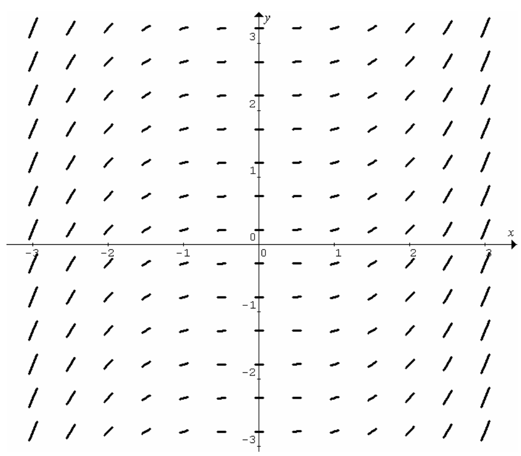
The graph of $f:[3,6]\rightarrow R, f(x)=\sqrt{9-(x-3)^2}$ is shown below. Length measure is in cm.



The shaded region is rotated about the x-axis to form a solid of revolution. The volume of this solid is given by

A. 18 cm^3 **B.** 36 cm^3 **C.** 18π cm^3 **D.** 36π cm^3 **E.** $\dfrac{36}{\pi}$cm^3

Question 13



The direction field for a certain first order differential equation is shown above. The first order differential equation could be

A. $\dfrac{dy}{dx}=\dfrac{k}{x}$, where k is a real constant

B. $\dfrac{dy}{dx}=k\log_e x$, where k is a real constant

C. $\dfrac{dy}{dx}=e^{kx}$, where k is a real constant

D. $\dfrac{dy}{dx}=kx^3$, where k is a real constant

E. $\dfrac{dy}{dx}=kx^2$, where k is a real constant

Question 14

The definite integral $\int_0^{\frac{\pi}{3}} \cot\left(\frac{\pi}{2} - x\right)dx$ can be expressed as

A. $\int_{\frac{1}{2}}^{1} \frac{dx}{x}$

B. $\int_{\frac{1}{2}}^{1} \left(-\frac{1}{x}\right)dx$

C. $\left[-\log_e\left|\frac{\pi}{2} - x\right|\right]_0^{\frac{\pi}{3}}$

D. $\left[\sec^2\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{3}}$

E. $\left[\cos ec^2\left(\frac{\pi}{2} - x\right)\right]_0^{\frac{\pi}{3}}$

Question 15

Euler's method, with a step size of 0.2, is used to solve the differential equation $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$, with initial condition $y = -2$ when $x = 1$. When $x = 1.4$ the approximate value for y is

A. $-2 + \frac{0.2}{\sqrt{2.44}}$

B. $-2 + \frac{0.2}{\sqrt{2}} + \frac{0.2}{\sqrt{2.44}}$

C. $-2 + \frac{0.2}{\sqrt{2}} + \frac{0.4}{\sqrt{2.44}}$

D. $-2 + \frac{0.2}{\sqrt{2.44}} + \frac{0.2}{\sqrt{2.96}}$

E. $-2 + \frac{0.2}{\sqrt{2.44}} + \frac{0.4}{\sqrt{2.96}}$

Question 16

The position vector of a particle at time $t \geq 0$ is given by $\boldsymbol{r}(t) = 3\sin(t)\boldsymbol{i} + \sqrt{3}\cos(t)\boldsymbol{j}$, where $\boldsymbol{i}$ points to east and $\boldsymbol{j}$ points to north. At $t = \frac{\pi}{3}$, the particle is moving in the direction

A. SE **B.** SW **C.** NE **D.** NW **E.** $N30^oW$

Question 17

p, ***q*** and ***r*** are the position vectors of three collinear points P, Q and R respectively. Point Q divides the line segment PR into the ratio 1 : 4.

q is equal to

A. $\frac{1}{3}(\boldsymbol{p} + 4\boldsymbol{r})$

B. $\frac{1}{4}(\boldsymbol{p} + 4\boldsymbol{r})$

C. $\frac{1}{5}(\boldsymbol{p} + 4\boldsymbol{r})$

D. $\frac{1}{5}(4\boldsymbol{p} + \boldsymbol{r})$

E. $\frac{1}{4}(4\boldsymbol{p} + \boldsymbol{r})$

Question 18

A particle of mass m kg slides along a rough horizontal surface. The reaction force of the surface on the particle makes an angle θ° with the direction of motion of the particle. Which one of the following statements is true?

A. $\theta = 0$

B. $0 < \theta < 90$

C. $\theta = 90$

D. $90 < \theta < 180$

E. $\theta = 180$

Question 19

A body of mass 2 kg moves with velocity $\boldsymbol{v}(t) = \cos(2t)\boldsymbol{i} - 5\boldsymbol{j}$ ms^{-1} at time t (in seconds). The magnitude of the rate of change of its momentum with respect to time (in kgms^{-2}) at $t = \frac{\pi}{4}$ is

A. -4 B. -2 C. 2 D. 4 E. 0

Question 20

A body of mass 5 kg falls vertically from rest. Assuming that air resistance is negligible, the distance fallen by the body in the third second is

A. 24.5 m **B.** 44.1 m **C.** 19.6 m **D.** 34.3 m **E.** 78.4 m

Question 21

Vectors $\boldsymbol{a} = 3\boldsymbol{i} + p\boldsymbol{j}$, $\boldsymbol{b} = 2\boldsymbol{i} - 5\boldsymbol{j}$ and $\boldsymbol{c} = 5\boldsymbol{i} + 2\boldsymbol{j}$ are coplanar. They are linearly dependent when

A. $p = \frac{3}{2}$ only

B. $p = \frac{3}{2}$ or $-\frac{7}{2}$ only

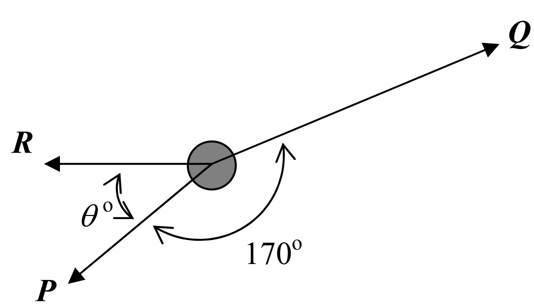
C. $p \in R \setminus \left\{-\frac{15}{2}\right\}$

D. $p \in R \setminus \left\{\frac{6}{5}\right\}$

E. $p \in R \setminus \left\{-\frac{15}{2}, \frac{6}{5}\right\}$

Question 22

A body of mass 5 kg is in equilibrium when it is acted upon by three concurrent coplanar forces $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{R}$ as shown in the diagram below. $\boldsymbol{P}$ = 10 newtons and $\boldsymbol{Q}$ = 20 newtons. The angle between $\boldsymbol{P}$ and $\boldsymbol{Q}$ is 170° and the angle between $\boldsymbol{P}$ and $\boldsymbol{R}$ is θ°.



The value of θ is closest to

A. 15 B. 20 C. 25 D. 30 E. 35

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Consider the set of complex numbers $S = \{z : |z| = \arg z\}$. Let $r = |z|$ and $\theta = \arg z$.

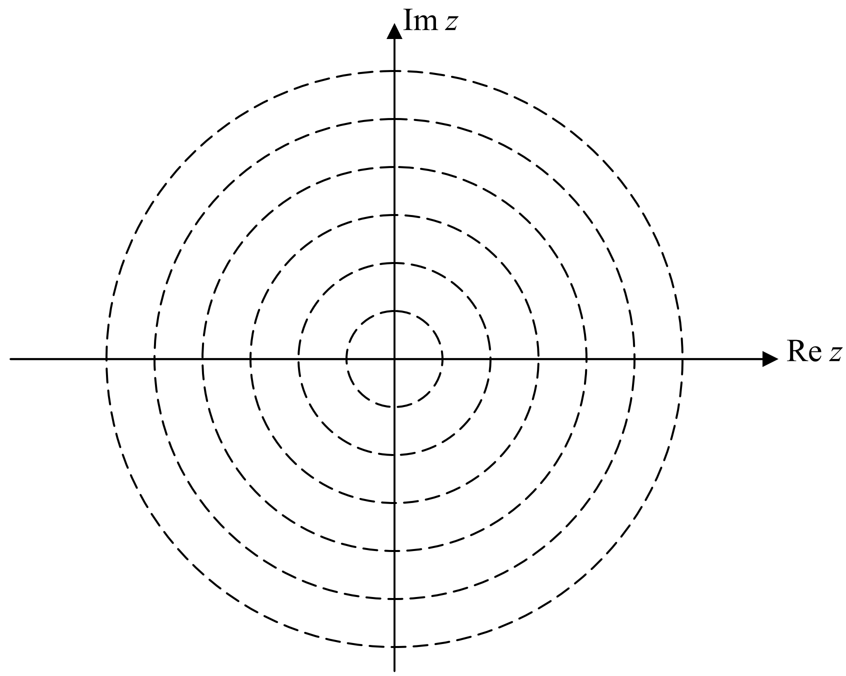
a. Complete the following table. 1 mark

θ	0		2		4		6
r		1		3		5	

b. Find the complex number $z = x + yi$ in S such that $\arg z = \frac{\pi}{3}$. 2 marks

c. Given $w = \frac{\pi}{2} i$, and $0 < \arg w < \pi$, determine whether or not $w \in S$. Explain. 2 marks

d. Plot the graph of the seven complex numbers shown in the table in part **a**. 3 marks



e. Each member of $S = \{z : |z| = \arg z\}$ has a complex conjugate.
Find T, the set of complex conjugates of $z \in S$. Specify the values of $\arg z$ for $z \in T$.

3 marks
Total 11 marks

Question 2

Two particles P and Q are in motion. Their position vectors at time t, $\boldsymbol{p}$ and $\boldsymbol{q}$, are given by

$$\boldsymbol{p} = \left(\log_e(t+0.5)\right)\boldsymbol{i} + (t+0.5)\boldsymbol{j} \text{ and } \boldsymbol{q} = \left(e^{t-0.5}\right)\boldsymbol{i} + (t-0.5)\boldsymbol{j}, \text{ where } 0 \le t \le 1.$$

a. Find the distance between the two particles at time t. 3 marks

b. Hence find

i. the closest approach of the two particles and the time that it occurs, 2 marks

ii. the greatest distance between the two particles and the time it occurs. 2 marks

c. Find the time when the two particles move in the same direction. 3 marks

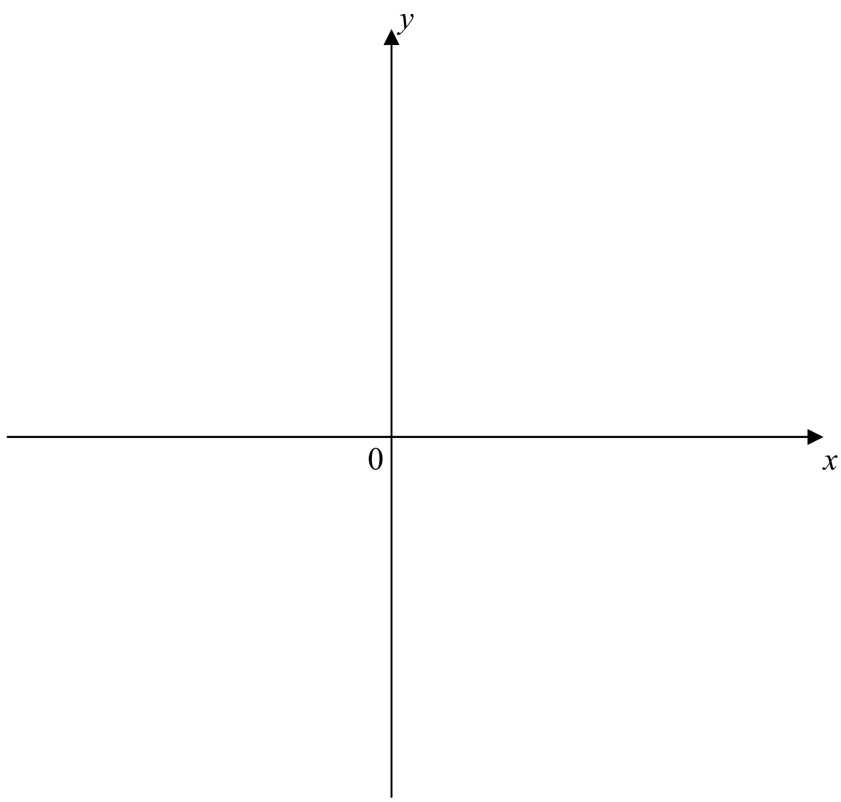
d. Find the cartesian equation, in terms of x and y, of the path of particle P, and the same of particle Q.

2 marks

e. Hence sketch the two paths on the same set of axes (use the same scale for both axes), and explain your answers for t to parts **bi** and **c**.

4 marks

Total 16 marks

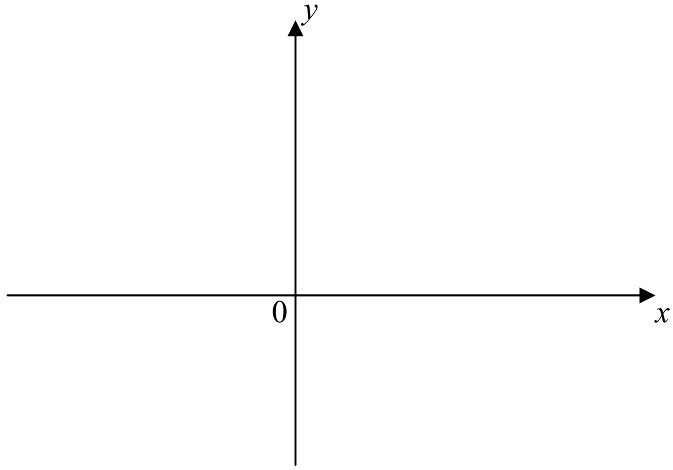


Question 3

Consider the function $f:[0,2] \to R, f(x) = \frac{1}{3}(x-1)^2(x+1)^2$. Lengths are measured in metres.

a. Find the range of the function. 1 mark

b. Sketch the graph of f showing the coordinates of intercepts, stationary point and end points. 2 marks



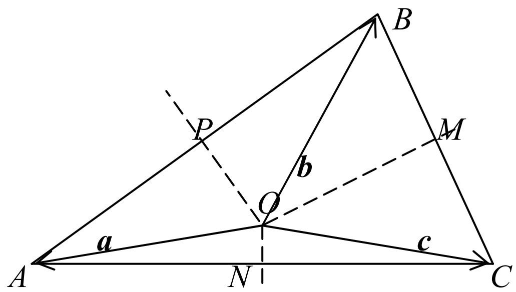
c. Use calculus to find the exact area of the region enclosed by the graph of the function, the y-axis and the line $y = 3$.

3 marks

d. The region described in part **c** is rotated about the y-axis. Find the exact volume of the solid of revolution.

5 marks
Total 11 marks

Question 4
Consider ΔABC in the following diagram.
$\overline{OM}$ and $\overline{ON}$ are perpendicular bisectors of $\overline{BC}$ and $\overline{AC}$ respectively, and $\overline{OP}$ bisects $\overline{AB}$.



Let $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{c}$.

a. Express $\overrightarrow{AC}$, $\overrightarrow{BC}$ and $\overrightarrow{BA}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. 1 mark

b. Express $\overrightarrow{OM}$, $\overrightarrow{ON}$ and $\overrightarrow{OP}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. 2 marks

c. Hence show that

i. $|\boldsymbol{a}| = |\boldsymbol{b}| = |\boldsymbol{c}|$, 3 marks

ii. $\overline{OP}$ is perpendicular to $\overline{BA}$. 1 mark

d. Show that $\left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{BA}\right|^2 = 2d^2\left[3-(\cos\alpha+\cos\beta+\cos\gamma)\right]$, where $d = |\boldsymbol{a}| = |\boldsymbol{b}| = |\boldsymbol{c}|$ and α, β and γ are angles between ***a*** and ***b***, ***b*** and ***c***, and ***c*** and ***a*** respectively.

3 marks

Total 10 marks

Question 5

A 5-kg particle, moving at 10 ms^{-1}, experiences a force of magnitude $\frac{500}{25-t^2}$ newtons at time $t \geq 0$ s until it comes to a stop.

a. Show that a differential equation relating v and t is $\frac{dv}{dt} = -\frac{100}{25-t^2}$. 1 mark

b. Show that the velocity of the particle at time t seconds is $10\left(1-\log_e\left|\frac{5+t}{5-t}\right|\right)$ ms^{-1}. 4 marks

c. Find the exact time when the particle comes to a stop. 2 marks

d. **i.** Write down a definite integral for the stopping distance. 1 mark

ii. Hence determine the stopping distance to the nearest metre. 2 marks

Total 10 marks

End of Exam 2