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SPECIALIST MATHS TRIAL EXAMINATION 2 SOLUTIONS 2006

Section 1 – Multiple-choice answers

1.	**E**	**7.**	**B**	**13.**	**D**	**19.**	**E**
2.	**D**	**8.**	**C**	**14.**	**D**	**20.**	**C**
3.	**E**	**9.**	**A**	**15.**	**E**	**21.**	**B**
4.	**D**	**10.**	**B**	**16.**	**C**	**22.**	**C**
5.	**B**	**11.**	**C**	**17.**	**D**		
6.	**A**	**12.**	**A**	**18.**	**E**		

Section 1- Multiple-choice solutions

Question 1

The equation of the asymptote through $(-3,0)$ is $y=\frac{2}{3}x+2$.

For the hyperbola $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ the asymptotes are given by $y-k=\pm\frac{b}{a}(x-h)$.

We require $b=2$ and $a=3$.

The required equation is $\frac{x^2}{9}-\frac{(y-2)^2}{4}=1$

The answer is E.

Question 2

	function	asymptotes
A.	$y=\frac{1}{(x-1)(x+1)}$	$y=0,\ x=1,\ x=-1$
B.	$y=\frac{x^2+1}{x}$	
	$=x+\frac{1}{x}$	$y=x,\ x=0$
C.	$y=\frac{x^3+1}{x^2}$	
	$=x+\frac{1}{x^2}$	$y=x,\ x=0$
D.	$y=\frac{x^4+1}{x^2}$	
	$=x^2+\frac{1}{x^2}$	$y=x^2,\ x=0$
E.	$y=x^2+\frac{1}{x^2+1}$	$y=x^2$

The answer is D.

Question 3

$$f(x)=\sec(ax)$$
$$=\frac{1}{\cos(ax)}$$

The graph has asymptotes for

$\cos(ax)=0$

$$ax=\ldots-\frac{3\pi}{2},-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{2},\ldots$$
$$x=\ldots-\frac{3\pi}{2a},-\frac{\pi}{2a},\frac{\pi}{2a},\frac{3\pi}{2a},\ldots$$

The answer is E.

Question 4

$$\sin^2(\theta)=\frac{1}{2}(1-\cos(2\theta))$$
$$\sin(\theta)=\sqrt{\frac{1}{2}(1-\cos(2\theta))}\quad \text{(option A)}$$
$$\sin^2(\theta)+\cos^2(\theta)=1$$
$$\sin(\theta)=\sqrt{1-\cos^2(\theta)}\quad \text{(option B)}$$
$$\sin(\theta)=2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\ \text{(option C, double angle formula)}$$
$$\sin(\theta)=\sin(3\theta-2\theta)$$
$$=\sin(3\theta)(\cos(2\theta)-\cos(3\theta)\sin(2\theta))\ \text{(option D) is incorrect}$$

$$\sin(\theta)=\sin\left(\frac{\theta}{3}+\frac{2\theta}{3}\right)$$
$$=\sin\left(\frac{\theta}{3}\right)\cos\left(\frac{2\theta}{3}\right)+\cos\left(\frac{\theta}{3}\right)\sin\left(\frac{2\theta}{3}\right)\ \text{(option E)}$$

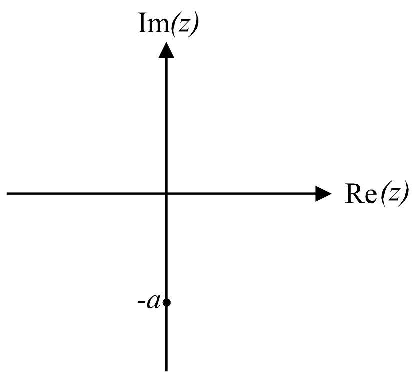
The answer is D.

Question 5

$$w=a\text{cis}(60^\circ)$$
$$=a(\cos(60^\circ)+i\sin(60^\circ))$$
$$=a\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$
$$=\frac{a}{2}\left(1+\sqrt{3}\,i\right)$$

The answer is B.

Question 6



$$-i(\overline{w}+w)=-i\left(\frac{a}{2}\left(1-\sqrt{3}\,i\right)+\frac{a}{2}\left(1+\sqrt{3}\,i\right)\right)$$
$$=-i\left(\frac{a}{2}+\frac{a}{2}\right)$$
$$=-ai$$

This complex number is located on the negative branch of the imaginary axis, a units from the origin.
Hence in polar form it can be expressed as $a\,\text{cis}(-90^\circ)$.
The answer is A.

Question 7

Let $z^3 = w$

$(r\text{cis}(\theta))^3 = w$

$r^3\text{cis}(3\theta) = a\text{cis}(60^\circ)$ (De Moivre)

$r^3 = a \qquad 3\theta = 60^\circ + 360^\circ k$

$r = \sqrt[3]{a} \qquad \theta = 20^\circ + 120^\circ k \qquad k \in Z$

$\theta = 20^\circ, 140^\circ, 260^\circ ...$

The three cube roots are $\sqrt[3]{a}\text{cis}(20^\circ)$, $\sqrt[3]{a}\text{cis}(140^\circ)$ and $\sqrt[3]{a}\text{cis}(260^\circ)$.
If $a = 1$, the cube roots lie on the circle with radius *a* units. If $a > 1$, the cube roots lie on a circle with radius $\sqrt[3]{a}$ units which is inside the circle with radius *a* units. So option A is correct.
Option B is incorrect.
Options C, D and E are correct.
The answer is B.

Question 8

$$y = \sin^{-1}\left(\frac{x}{\sqrt{a}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a - x^2}}$$

$$= (a - x^2)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}(a - x^2)^{-\frac{3}{2}} \times -2x$$

$$= \frac{x}{(a - x^2)^{\frac{3}{2}}}$$

The answer is C.

Question 9

$$xy^2 = 1$$

$$\frac{d}{dx}\left(xy^2\right) = \frac{d}{dx}(1)$$

$$1 \times y^2 + x \times 2y\frac{dy}{dx} = 0$$

$$2xy\frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = \frac{-y}{2x} \qquad x \neq 0$$

At $x = 4,\ y^2 = \frac{1}{4},\ y = \pm\frac{1}{2}$

In the fourth quadrant $y = -\frac{1}{2}$

So $\frac{dy}{dx} = \frac{1}{2} \div 8$

$= \frac{1}{16}$

The answer is A.

Question 10

$$\int_0^{\pi} (\sin(x)\cos(x))^3\, dx$$

$$= \int_0^{\pi} \sin^3(x)\cos^3(x)dx$$

$$= \int_0^{\pi} \sin(x)\left(1 - \cos^2(x)\right)\left(\cos^3(x)dx\right)$$

$$= \int_1^{-1} -\frac{du}{dx}\left(1 - u^2\right)u^3\, dx$$

$$= \int_{-1}^{1} \left(u^3 - u^5\right)du$$

where $u = \cos(x)$

$\frac{du}{dx} = -\sin(x)$

$x = \pi,\ u = -1$

$x = 0,\ u = 1$

The answer is B.

Question 11

Stationary points on the graph of $y = F(x)$ occur at $x = -5,\ x = 0,\ x = 3$ and $x = 7$.
The gradient (i.e. the value of $y = f(x)$) of the graph of $y = F(x)$ is different i.e. positive or negative on either side of each of these stationary points and hence there is no stationary point of inflection.
Since $f(x) > 0$ for $x \in (-5, 0) \cup (3, 7)$, the gradient of the graph of $y = F(x)$ is positive for these values.
At $x = 0$, there is a maximum turning point since the gradient is positive to the left of $x = 0$ and negative to the right. Therefore, option C is incorrect.
Options D and E are both correct.
The answer is C.

Question 12

$$\begin{aligned}\text{volume} &= \pi\int_0^{\frac{\pi}{2}} x^2\,dy\\ &= \pi\int_0^{\frac{\pi}{2}} \cos^2(y)\,dy\\ &= \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}(\cos(2y)+1)\,dy\\ &= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} (\cos(2y)+1)\,dy\end{aligned}$$

Now,

$$\begin{aligned}y &= \cos^{-1}(x)\\ x &= \cos(y)\\ x^2 &= \cos^2(y)\end{aligned}$$

The answer is A.

Question 13

A. $\underset{\sim}{a} \bullet \underset{\sim}{a} = \underset{\sim}{c} \bullet \underset{\sim}{c}$

$|\underset{\sim}{a}||\underset{\sim}{a}|\cos(0) = |\underset{\sim}{c}||\underset{\sim}{c}|\cos(0)$

$|\underset{\sim}{a}|^2 = |\underset{\sim}{c}|^2$ may or may not be true

B. $\underset{\sim}{a} \bullet \underset{\sim}{b} = \underset{\sim}{b} \bullet \underset{\sim}{c}$ may or may not be true

C.

$(\underset{\sim}{a} \bullet \underset{\sim}{c})\underset{\sim}{b} = 0 \times \underset{\sim}{b}$

$= \underset{\sim}{0}$

$\neq \underset{\sim}{b}$

D.

$\underset{\sim}{a} \bullet (\underset{\sim}{a} + \underset{\sim}{c})$

$= \underset{\sim}{a} \bullet \underset{\sim}{a} + \underset{\sim}{a} \bullet \underset{\sim}{c}$

$= \underset{\sim}{a} \bullet \underset{\sim}{a}$ since $\underset{\sim}{a} \bullet \underset{\sim}{c} = 0$

$= |\underset{\sim}{a}|^2$

E.

$(\underset{\sim}{a} + \underset{\sim}{c}) \bullet (\underset{\sim}{a} - \underset{\sim}{c}) = \underset{\sim}{a} \bullet \underset{\sim}{a} - \underset{\sim}{a} \bullet \underset{\sim}{c} + \underset{\sim}{a} \bullet \underset{\sim}{c} - \underset{\sim}{c} \bullet \underset{\sim}{c}$

$= \underset{\sim}{a} \bullet \underset{\sim}{a} - \underset{\sim}{c} \bullet \underset{\sim}{c}$

$= |\underset{\sim}{a}||\underset{\sim}{a}|\cos(0) - |\underset{\sim}{c}||\underset{\sim}{c}|\cos(0)$

$= |\underset{\sim}{a}|^2 - |\underset{\sim}{c}|^2$

Now $|\underset{\sim}{a}| - |\underset{\sim}{c}| = |\underset{\sim}{a}|^2 - |\underset{\sim}{c}|^2$ may or may not be true.

The answer is D.

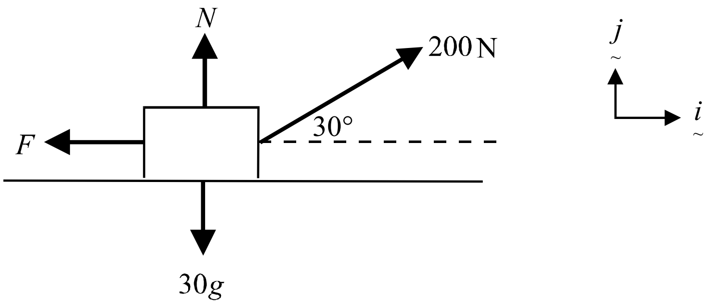
Question 14

$\underset{\sim}{c}$ is the component of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$.

Hence $\underset{\sim}{c} = (\underset{\sim}{a} \bullet \hat{\underset{\sim}{b}})\hat{\underset{\sim}{b}}$.

The answer is D.

Question 15
Draw a diagram.



$$\underset{\sim}{R} = m\,\underset{\sim}{a}$$

$$\left(200\cos(30^\circ) - F\right)\underset{\sim}{i} + \left(N + 200\sin(30^\circ) - 30g\right)\underset{\sim}{j} = 30a\,\underset{\sim}{i}$$

$$100\sqrt{3} - F = 30a \quad \text{and} \quad N + 100 - 30g = 0$$

$$100\sqrt{3} - 0\cdot 5(30g - 100) = 30a \qquad\qquad N = 30g - 100$$

$$100\sqrt{3} - 15g + 50 = 30a$$

$$a = \frac{10\sqrt{3} - 1\cdot 5g + 5}{3}$$

The answer is E.

Question 16
Resolving

$1 = Q\sin(\theta) \quad -(1) \qquad$ and $\sqrt{3} = Q\cos(\theta) \quad -(2)$

$(1) \div (2)$ gives $\dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{1}{\sqrt{3}}$

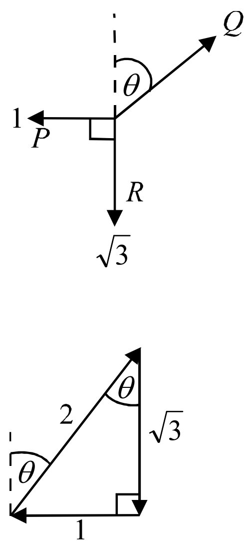
$$\tan(\theta) = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

$$\sin(\theta) = \frac{1}{2}$$

So $Q = 2$

$\left(\text{Check } \cos(\theta) = \dfrac{\sqrt{3}}{2},\ Q = 2\right)$.



Q has a magnitude of $2N$ and acts in a direction of $N30°E$.
The answer is C.

Question 17
Let $\underset{\sim}{v}$ be the velocity of the particle. So $\underset{\sim}{v} = 6\underset{\sim}{i} + 8\underset{\sim}{j}$ where $\underset{\sim}{i}$ runs in the horizontal direction and $\underset{\sim}{j}$ runs in the vertical direction.

$$\underset{\sim}{P} = m\,\underset{\sim}{v}$$

$$= 3\left(6\underset{\sim}{i} + 8\underset{\sim}{j}\right)$$

$$\left|\underset{\sim}{P}\right| = 3\sqrt{36 + 64}$$

$$= 30\,\text{kg m/s}$$

The answer is D.

Question 18

$\frac{dy}{dx} = a + by$ where a and b are constants

$\frac{dx}{dy} = \frac{1}{a + by}$

$x = \int \frac{1}{a + by}\, dy$ (option A)

$= \frac{1}{b} \int \frac{b}{a + by}\, dy$

$x = \frac{1}{b} \log_e |a + by| + c$

$y(0) = 0$

$0 = \frac{1}{b} \log_e |a| + c$

$c = -\frac{1}{b} \log_e |a|$

$= \log_e |a|^{-\frac{1}{b}}$ (option B)

$x = \frac{1}{b} \log_e |a + by| - \frac{1}{b} \log_e |a|$

$= \frac{1}{b} \log_e \left| \frac{a + by}{a} \right|$ (option C)

$bx = \log_e \left| \frac{a + by}{a} \right|$

$e^{bx} = \frac{a + by}{a}$ (option D)

$ae^{bx} = a + by$

$by = a\left(e^{bx} - 1\right)$

$y = \frac{a}{b}\left(e^{bx} - 1\right)$ (making option E not true)

The answer is E.

Question 19

$$\frac{dy}{dx}=\frac{1}{x+1}, \quad x_0=0, \quad y_0=0, \quad h=0.1$$

$$x_{n+1}=x_n+h \qquad y_{n+1}=y_n+h\,f(x_n)$$

So, $x_1=0+0\cdot 1$ $\qquad y_1=0+0\cdot 1\times\frac{1}{1}$

$=0\cdot 1$ $\qquad =0\cdot 1$

$x_2=0\cdot 1+0\cdot 1$ $\qquad y_2=0\cdot 1+0\cdot 1\times\frac{1}{1\cdot 1}$

$=0\cdot 2$ $\qquad =\frac{1}{10}+\frac{1}{11}$

$\qquad =\frac{21}{110}$

The answer is E.

Question 20

$$\begin{aligned}\frac{dS}{dt}&=\text{rate of inflow}-\text{rate of outflow}\\&=\frac{dS}{dl}\cdot\frac{dl}{dt}(\text{inflow})-\frac{dS}{dl}\cdot\frac{dl}{dt}(\text{ outflow})\\&=0\times 2-\frac{S}{300-3t}\times 5\\&=\frac{-5S}{3(100-t)}\end{aligned}$$

Note that the amount of sugar per litre in the tank is $\frac{S}{300-3t}$ since every second the volume of solution decreases by 3L.

The answer is C.

Question 21

For $y=\sqrt{x},\ x>0$ therefore the slope diagram is not appropriate to this function since gradients are assigned for $x\le 0$.

A similar argument is true for $y=\log_e(x)$.

Note that for $x=2,\ \frac{dy}{dx}=0$.

This is true for $y=-x(x-4)$ since $\frac{dy}{dx}=-2x+4$ and when $x=2,\ \frac{dy}{dx}=0$.

For $y=2x-x^2,\ \frac{dy}{dx}=2-2x$, so at $x=2,\ \frac{dy}{dx}=-2$ so the slope diagram is not suitable for this function.

Similarly for $y=1-e^{-x},\ \frac{dy}{dx}=e^{-x}$.

At $x=2\ \frac{dy}{dx}=e^{-2}=0\cdot 135$

$\neq 0$

The answer is B.

Question 22

$$v^2 = -2\int 5\,dx$$

$$\frac{1}{2}v^2 = -\int 5\,dx$$

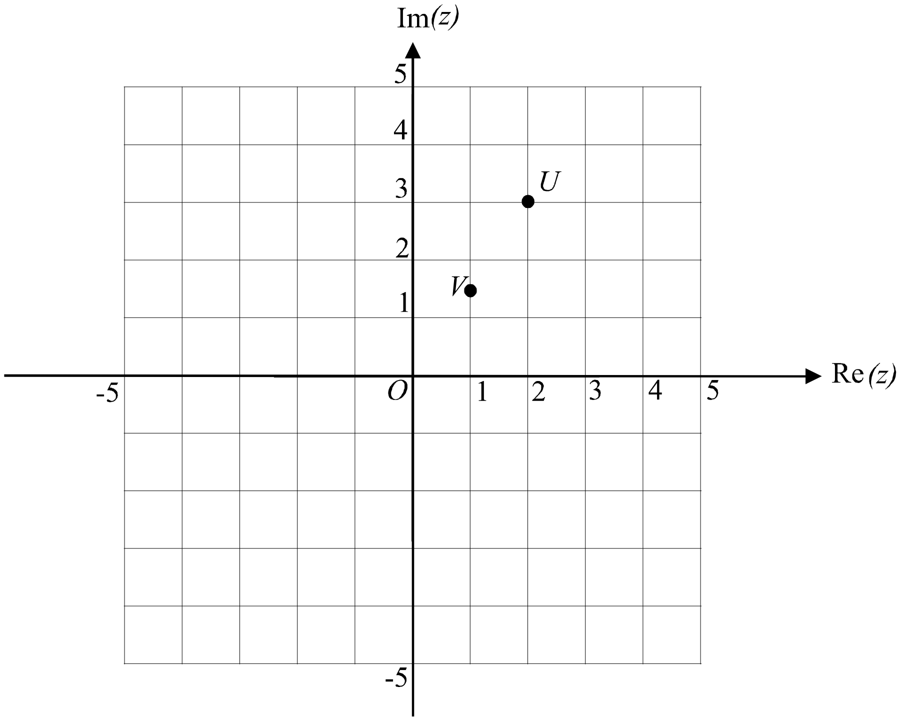
$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -5$$

Since $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ we have $a = -5$.

Since the particle is moving with constant, negative acceleration, the velocity will be decreasing.
The answer is C.

Section 2

Question 1

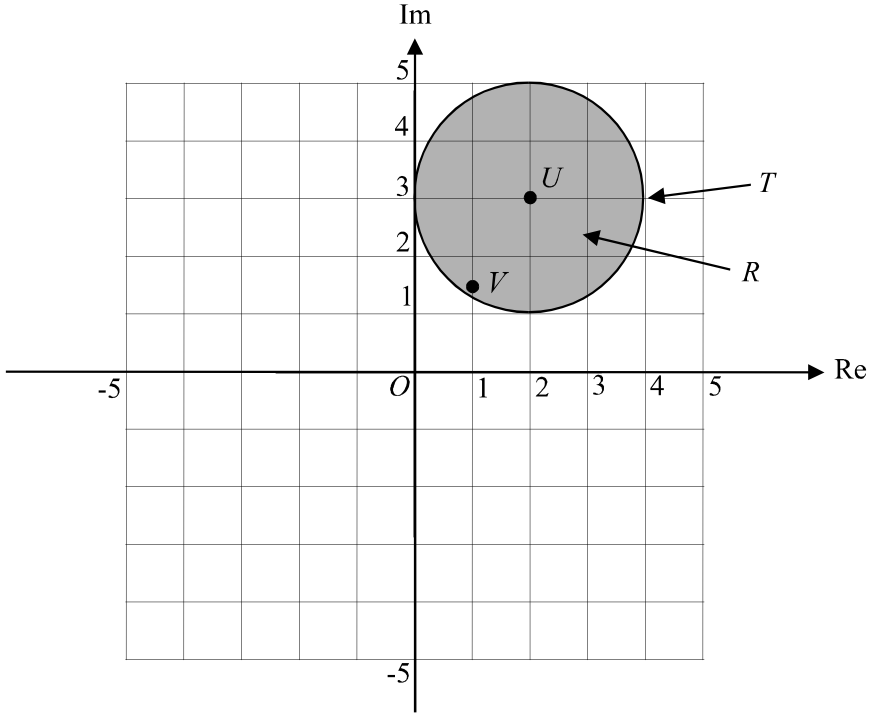


a. **(1 mark)** correctly showing *U* and *V*.

b. *T* is a circle with centre at *U* and radius of 2 units.
Since *w* lies on the imaginary axis, the circle touches that axis at $0+3i$ only.
So $b=3$.

(1 mark)

c. **i.**



(1 mark)

ii. $v \in R$ if $|v - u| \le 2$

$$LS = \left|1 + \frac{3i}{2} - 2 - 3i\right|$$

$$= \left|-1 - \frac{3}{2}i\right|$$

$$= \sqrt{1 + \frac{9}{4}}$$ **(1 mark)**

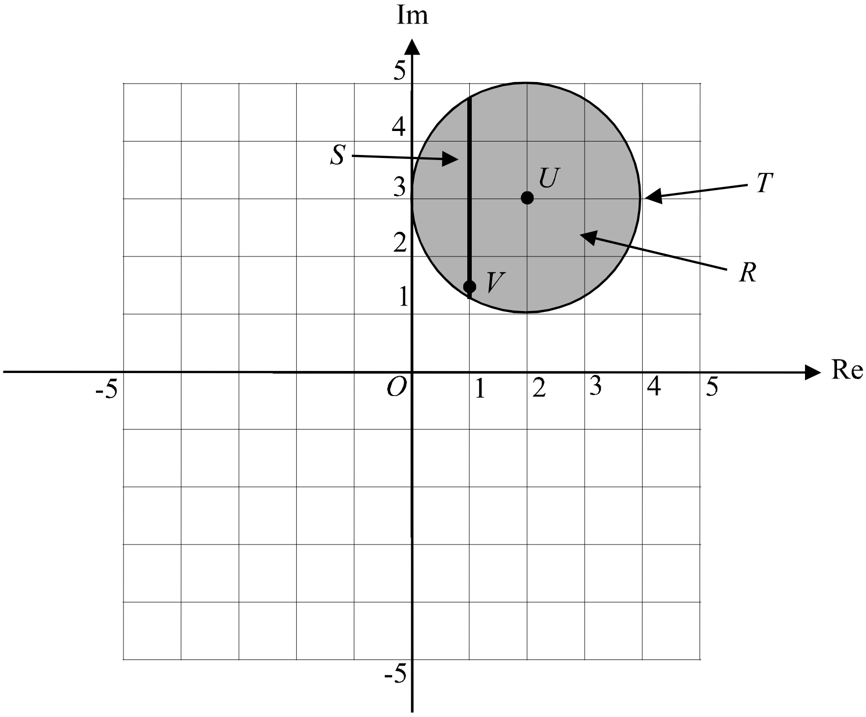
$$= \frac{\sqrt{13}}{2}$$

$$< 2$$

So $v \in R$

(1 mark)

d. **i.**



(1 mark)

ii. Since $p \in S$ and $p = x + yi$, then $x = 1$ and the maximum and minimum values of y lie on the circle defined by

$$|z - u| = 2$$
$$|1 + yi - 2 - 3i| = 2$$
$$|-1 + (y-3)i| = 2$$
$$\sqrt{1 + (y-3)^2} = 2$$
$$1 + (y-3)^2 = 4$$
$$y^2 - 6y + 6 = 0$$
$$y = \frac{6 \pm \sqrt{36 - 4 \times 1 \times 6}}{2}$$ **(1 mark)**
$$= \frac{6 \pm \sqrt{12}}{2}$$

So the maximum and minimum values of y are given respectively by
$y = 3 + \sqrt{3}$ and $y = 3 - \sqrt{3}$ **(1 mark)**

e. Method 1

The vectors $\overrightarrow{OV}$ and $\overrightarrow{VU}$ are parallel if

$\overrightarrow{OV} = k\overrightarrow{VU}$ where k is a constant.

Now, $\overrightarrow{OV} = \underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j}$

$$\begin{aligned}\overrightarrow{VU} &= \overrightarrow{VO} + \overrightarrow{OU}\\ &= -\overrightarrow{OV} + \overrightarrow{OU}\\ &= -\underset{\sim}{i} - \frac{3}{2}\underset{\sim}{j} + 2\underset{\sim}{i} + 3\underset{\sim}{j}\\ &= \underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j}\end{aligned}$$ **(1 mark)**

So $\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j} = k\left(\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j}\right)$ where $k = 1$ **(1 mark)**

So $\overrightarrow{OV}$ and $\overrightarrow{VU}$ are parallel and since they share the point V, they must be collinear.

(1 mark)

Method 2

$$\begin{aligned}\overrightarrow{OU} &= 2\underset{\sim}{i} + 3\underset{\sim}{j}\\ &= 2(\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j}) \quad \textbf{(1 mark)}\\ &= 2\overrightarrow{OV} \quad \textbf{(1 mark)}\end{aligned}$$

So $\overrightarrow{OV}$ and $\overrightarrow{VU}$ are parallel and since they share the point O, they must be collinear.

(1 mark)

Total 11 marks

Question 2

a. **i.** $\underset{\sim}{a} = \sqrt{2}\sin(t)\underset{\sim}{i} + \sqrt{2}\cos(t)\underset{\sim}{j}$

$$x = \sqrt{2}\sin(t) \quad y = \sqrt{2}\cos(t)$$
$$x^2 = 2\sin^2(t) \quad y^2 = 2\cos^2(t)$$
$$x^2 + y^2 = 2\sin^2(t) + 2\cos^2(t)$$
$$x^2 + y^2 = 2\left(\sin^2(t) + \cos^2(t)\right)$$
$$x^2 + y^2 = 2$$

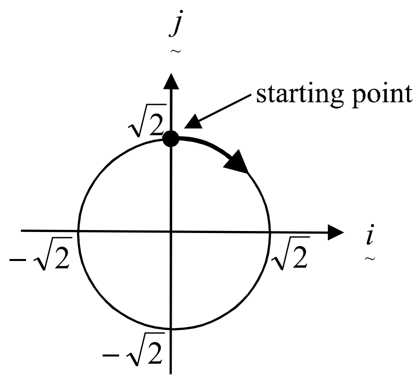
(1 mark)

ii. At $t = 0$, $\underset{\sim}{a} = 0\underset{\sim}{i} + \sqrt{2}\underset{\sim}{j}$

Particle *A* starts at the point $\left(0, \sqrt{2}\right)$ in the $\underset{\sim}{i} - \underset{\sim}{j}$ plane.

It moves in a clockwise direction on a circular path with centre at $(0, 0)$ and radius $\sqrt{2}$.

(We know it moves in a clockwise direction since as *t* increases from $t = 0$, $\sin(t)$ increases and $\cos(t)$ decreases so the $\underset{\sim}{i}$ component increases and the $\underset{\sim}{j}$ component decreases.)



(1 mark) stating starting point
(1 mark) clockwise direction around a circle with correct centre and radius

b. **i.** $\underset{\sim}{b} = \sqrt{2}\cos(t)\underset{\sim}{i} + \underset{\sim}{j}$

The Cartesian equation of the path of particle *B* is $y = 1$.

(1 mark)

ii. Since $x = \sqrt{2}\cos(t)$, the domain of the Cartesian equation that describes the path of particle *B* is $x \in \left[-\sqrt{2}, \sqrt{2}\right]$.

(1 mark)

c. $\underset{\sim}{a} \bullet \underset{\sim}{b}$

$= \left(\sqrt{2}\sin(t)\underset{\sim}{i} + \sqrt{2}\cos(t)\underset{\sim}{j}\right) \bullet \left(\sqrt{2}\cos(t)\underset{\sim}{i} + \underset{\sim}{j}\right)$

$= 2\sin(t)\cos(t) + \sqrt{2}\cos(t)$ **(1 mark)**

When *A* and *B* are at right angles, $\underset{\sim}{a} \bullet \underset{\sim}{b} = 0$ **(1 mark)**

$2\sin(t)\cos(t) + \sqrt{2}\cos(t) = 0$

$\sqrt{2}\cos(t)\left(\sqrt{2}\sin(t) + 1\right) = 0$

$\sqrt{2}\cos(t) = 0$ or $\sqrt{2}\sin(t) + 1 = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2} \ldots$ $\sin(t) = -\frac{1}{\sqrt{2}}$

$t = \frac{5\pi}{4}, \frac{7\pi}{4} \ldots$ **(1 mark)**

S	A
T	C

A and *B* are first at right angles at $t = \frac{\pi}{2}$ seconds.

(1 mark)

d.

$\underset{\sim}{a} = \sqrt{2}\sin(t)\underset{\sim}{i} + \sqrt{2}\cos(t)\underset{\sim}{j}$

$\dot{\underset{\sim}{a}} = \sqrt{2}\cos(t)\underset{\sim}{i} - \sqrt{2}\sin(t)\underset{\sim}{j}$

$|\dot{\underset{\sim}{a}}| = \sqrt{2\cos^2(t) + 2\sin^2(t)}$ **(1 mark)**

$= \sqrt{2\left(\cos^2(t) + \sin^2(t)\right)}$

$= \sqrt{2}$

So particle *A* moves with a constant speed of $\sqrt{2}$ units.

(1 mark)

e.

i. $\underset{\sim}{a} = \sqrt{2}\sin(t)\underset{\sim}{i} + \sqrt{2}\cos(t)\underset{\sim}{j}$

$\underset{\sim}{b} = \sqrt{2}\cos(t)\underset{\sim}{i} + \underset{\sim}{j}$

A and *B* will collide iff **(1 mark)**

$\sqrt{2}\sin(t) = \sqrt{2}\cos(t)$ AND $\sqrt{2}\cos(t) = 1$

$\frac{\sin(t)}{\cos(t)} = 1$ $\cos(t) = \frac{1}{\sqrt{2}}$

$\tan(t) = 1$ $t = \frac{\pi}{4}, \frac{7\pi}{4}, \ldots$

$t = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots$

S	A
T	C

Since *A* and *B* will be at the same spot at the same time, that is at $t = \frac{\pi}{4}$, they will collide. **(1 mark)**

ii. At $t=0$, A is at the point $\left(0,\sqrt{2}\right)$ on the $\underset{\sim}{i}-\underset{\sim}{j}$ plane.

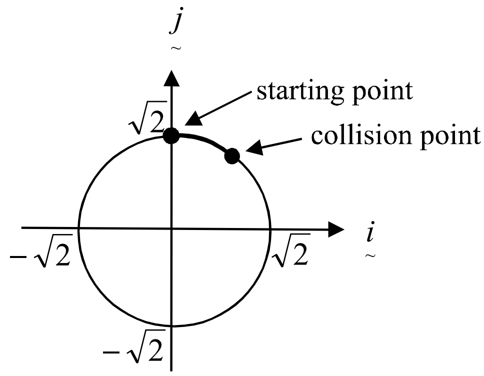
At $t=\frac{\pi}{4}$, when A collides with B,

$$\underset{\sim}{a}=\sqrt{2}\sin\left(\frac{\pi}{4}\right)\underset{\sim}{i}+\sqrt{2}\cos\left(\frac{\pi}{4}\right)\underset{\sim}{j}$$
$$=\sqrt{2}\cdot\frac{1}{\sqrt{2}}\underset{\sim}{i}+\sqrt{2}\cdot\frac{1}{\sqrt{2}}\underset{\sim}{j}$$
$$=\underset{\sim}{i}+\underset{\sim}{j}$$

So A is at the point $(1,1)$ when the collision occurs.

Particle A has travelled $\frac{1}{8}$ of the way around the circle with perimeter $2\sqrt{2}\pi$.

So particle A has travelled $\frac{\sqrt{2}}{4}\pi$ units.



(1 mark)
Total 14 marks

Question 3

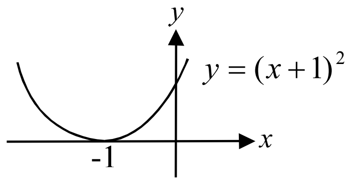
a. For $g(x)=\log_e\left(x^2+2x+1\right)$,

$x^2+2x+1>0$

$(x+1)^2>0$

$x\in R\backslash\{-1\}$

$d_g=R\backslash\{-1\}$ or $(-\infty,-1)\cup(-1,\infty)$



(1 mark)

b. **i.** Let $y=\log_e\left(x^2+2x+1\right)$

Swap x and y

$x=\log_e\left(y^2+2y+1\right)$

$e^x=(y+1)^2$

$\pm\sqrt{e^x}=y+1$

These steps must be shown

$y=-1\pm\sqrt{e^x}$

(1 mark)

ii. $r_{g^{-1}}=d_g$

$=R\backslash\{-1\}$ or $(-\infty,-1)\cup(-1,\infty)$

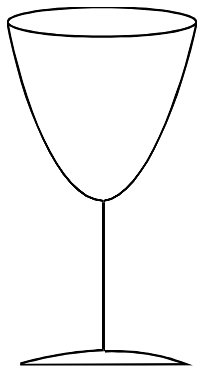
(1 mark)

c. First find $g(10)$.

$g(10)=\log_e\left(10^2+20+1\right)$

$=\log_e(121)$

The diameter of the rim is $2\log_e(121)$ cm.



(1 mark)

d. **i.** $$\text{volume} = \pi\int_0^{10} y^2\,dx$$
$$= \pi\int_0^{10} \left(\log_e\left(x^2 + 2x + 1\right)\right)^2 dx$$

(1 mark) correct integrand
(1 mark) correct terminals

ii. Using a calculator,

$$\pi\int_0^{10} \left(\log_e\left(x^2 + 2x + 1\right)\right)^2 dx = 121\cdot 98089\pi\ \text{cm}^3$$ **(1 mark)**

$$= 1\cdot 2198089\pi \times 100\text{cm}^3$$
$$= 1\cdot 2198089\pi \text{ standard drinks}$$
$$= 3\cdot 8 \text{ standard drinks (correct to 1 decimal place)}$$

(1 mark)

e. Using a calculator, try different values of a where a is an integer. We require the largest value of a so that the value of y when $x = 10$ is less than 5.

For $y = \ln\left(x^2 + ax + 1\right)$
when $x = 10$ and $a = 2$, $y = 4\cdot 795...$
For $y = \ln\left(x^2 + ax + 1\right)$ **(1 mark)** for trying some values of a
when $x = 10$, and $a = 3$, $y = 4\cdot 875...$
For $y = \ln\left(x^2 + ax + 1\right)$
when $x = 10$, and $a = 4$, $y = 4\cdot 948...$
For $y = \ln\left(x^2 + ax + 1\right)$
when $x = 10$, and $a = 5$, $y = 5\cdot 017...$
So the maximum value of a is 4.

(1 mark)

f. For the function

$$f(x) = \log_e\left(x^2 + ax + 1\right),\ a > 0 \quad \text{and} \quad 0 \le x \le 10$$
$$f'(x) = \frac{2x + a}{x^2 + ax + 1},\ a > 0 \text{ and } 0 \le x \le 10$$

so $f'(x) > 0$ since $a > 0$ and $x \ge 0$ **(1 mark)**
so $f'(x) \ne 0$ for all $0 \le x \le 10$

So for $x \in [0,10]$ there is no horizontal slope. Hence when the mould is placed upright there is no point where the slope is vertical.

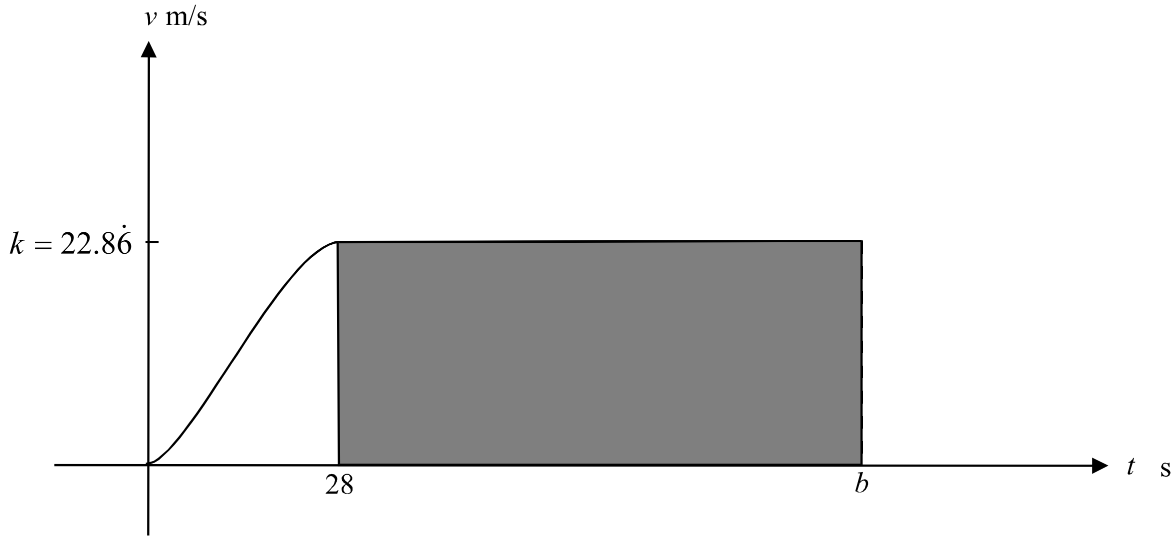
(1 mark)
Total 12 marks

Question 4

a. The maximum speed of the train is given by $\dfrac{-28^3}{480}+\dfrac{28\times 28^2}{320}=22\cdot 8\dot{6}\text{m/s}$.

(1 mark)

b.



The shaded area on the graph above represents the distance travelled by the train whilst it is travelling at maximum speed.

So $(b-28)\times 22\cdot 8\dot{6}=2\ 058$

$b=118$

(1 mark)

c. i. $\text{average acceleration}=\dfrac{v(50)-v(10)}{50-10}$

$=\left(22\cdot 8\dot{6}-\left(\dfrac{-10^3}{480}+\dfrac{28\times 100}{320}\right)\right)\div 40$

$=0\cdot 41\text{m/s}^2$ (to 2 decimal places)

(1 mark)

ii. For $t=10$, $v(t)=\dfrac{-t^3}{480}+\dfrac{28t^2}{320}$

$a(t)=\dfrac{-3t^2}{480}+\dfrac{56t}{320}$

$a(10)=\dfrac{-300}{480}+\dfrac{560}{320}$

$=1\cdot 125\text{m/s}^2$

(1 mark)

d. Method 1

Use a graphics calculator to solve

$$0 = \frac{-23 \cdot 6}{\pi} \tan^{-1}(t - 130) + 11 \cdot 691$$

$t = 198 \cdot 91$ seconds (to 2 decimal places)

So $c = 198.91$ (to 2decimal places).

(1 mark)

Method 2

By hand.

$$\begin{aligned} 0 &= \frac{-23 \cdot 6}{\pi} \tan^{-1}(t - 130) + 11 \cdot 691 \\ \frac{-11 \cdot 691 \times \pi}{-23 \cdot 6} &= \tan^{-1}(t - 130) \\ 1 \cdot 556... &= \tan^{-1}(t - 130) \\ \tan(1 \cdot 556...) &= t - 130 \\ t &= 198 \cdot 9136... \\ &= 198 \cdot 91 \text{ seconds (to two decimal places)} \end{aligned}$$

So $c = 198.91$ (to 2decimal places). **(1 mark)**

e. $$\text{distance} = \int_0^{28} \left(\frac{-t^3}{480} + \frac{28t^2}{320} \right) dt + \int_{118}^{c} g(t)\,dt + 2\,058$$

(1 mark) 1 correct integral

(1 mark) 2nd correct integral and including 2 058

f. Sketch the function $y = g(t)$ for $118 \le t \le 198 \cdot 91$.

There is a rapid decrease in the velocity of the train between approximately $t = 120$ and $t = 140$ and then there is virtually zero velocity between $t = 140$ and $t = 198 \cdot 91$ when the train finally comes to rest.

This suggests that the train decelerates quickly but then crawls along for approximately one minute; maybe having to wait for another train already in the station.

(1 mark)

g. $$g'(t) = \frac{-23 \cdot 6}{\pi\left(1 + (t - 130)^2\right)}$$

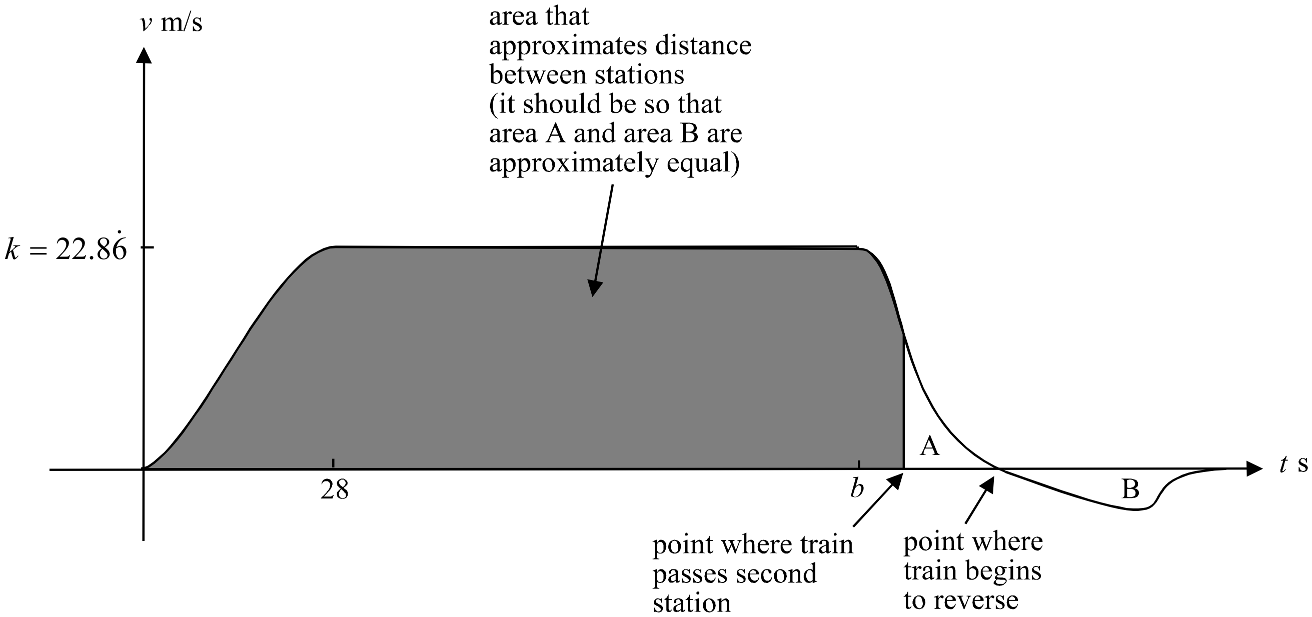
The velocity is constant when $g'(t) = 0$.

Now, $\dfrac{-23 \cdot 6}{\pi\left(1 + (t - 130)^2\right)} = 0$ has no real solution since $-23 \cdot 6 \ne 0$.

So the velocity of the train is never constant for $b < t \le c$.

(1 mark)

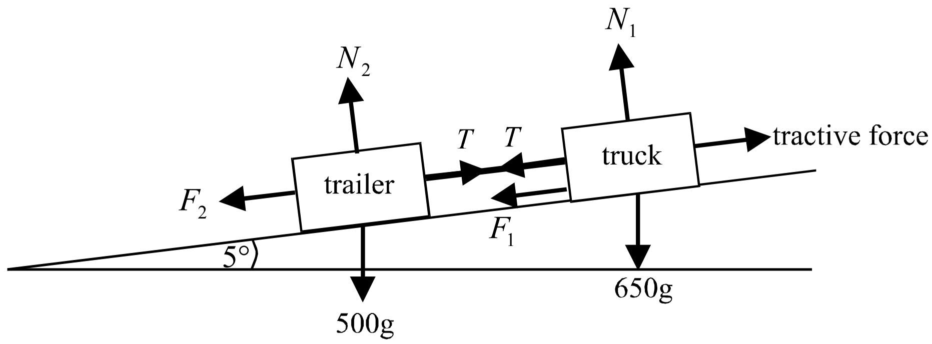
h.



(1 mark) correct graph
(1 mark) showing area
(1 mark) showing reverse point
Total 12 marks

Question 5

a. Draw a force diagram.



(1 mark) diagram showing forces around the trailer

Around the trailer

$T - 500g\sin(5^\circ) - F_2 = ma$ **(1 mark)** and $N_2 = 500g\cos(5°)$ **(1 mark)**

$2\,000 - 427 \cdot 0631 - \mu N_2 = 500 \times 0 \cdot 15$

$\mu \times 500g\cos(5^\circ) = 1497 \cdot 93...$

$\mu = 0 \cdot 307$ (to 3 decimal places)

(1 mark)

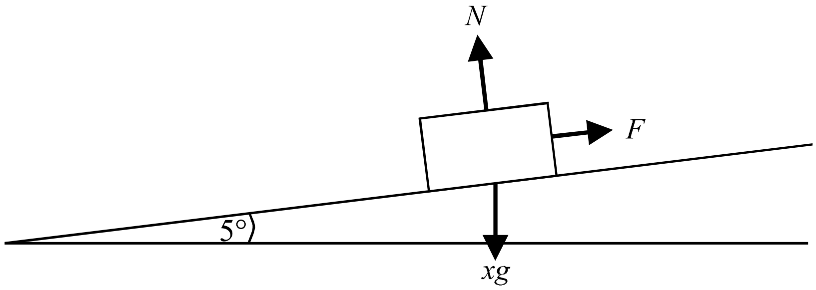
b.

$s = \frac{1}{2}(u + v)t$ **(1 mark)**

$150 = \frac{1}{2}(4 + 0)t$

$t = 75$ secs **(1 mark)**

c. **i.**



Draw a force diagram.
The mass of the trailer is unknown since it is being loaded with timber. Let the mass be x kg.

So, $F = xg\sin(5^\circ)$

$= 0\cdot 0872xg$ (correct to 4 decimal places)

(1 mark)

Now $F \le \mu N$

When $F = \mu N$, the trailer is on the point of rolling down the track.

Now, $\mu N = 0\cdot 307 \times xg\cos(5^\circ)$

$= 0\cdot 3058xg$ (correct to 4 decimal places)

Since $0\cdot 0872xg < 0\cdot 3058xg$, the trailer is not at the point of rolling down the track.

(1 mark)

ii. $F \le \mu N$

$\mu \ge \frac{F}{N}$

$\ge \frac{xg\sin(5^\circ)}{xg\cos(5^\circ)}$

$\ge 0\cdot 0874...$

So $\mu = 0\cdot 087$ (to 3 decimal places).

(1 mark)
Total 9 marks

Total for Section 2 – 58 marks

