VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2005

SPECIALIST MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 31 October 2005

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I
MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten), one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 15 pages, with a detachable sheet of miscellaneous formulas in the centrefold and a blank page for rough working.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The maximum value of y reached by the ellipse with equation

$$\frac{3(x+3)^2}{5} + \frac{(y-4)^2}{6} = 3$$

is

A. $-4 + 3\sqrt{2}$

B. $4 + \sqrt{5}$

C. $3\sqrt{2}$

D. $4 + \sqrt{6}$

E. $4 + 3\sqrt{2}$

Question 2

The graph of $f(x) = \dfrac{1}{x^2 + mx - n}$, where m and n are real constants, has no vertical asymptotes if

A. $m^2 < 4n$

B. $m^2 > 4n$

C. $m^2 = -4n$

D. $m^2 < -4n$

E. $m^2 > -4n$

Question 3

The number of real solutions to $x^4 - x^3 = \text{cosec}^2(x) - \cot^2(x)$ is

A. 0

B. 1

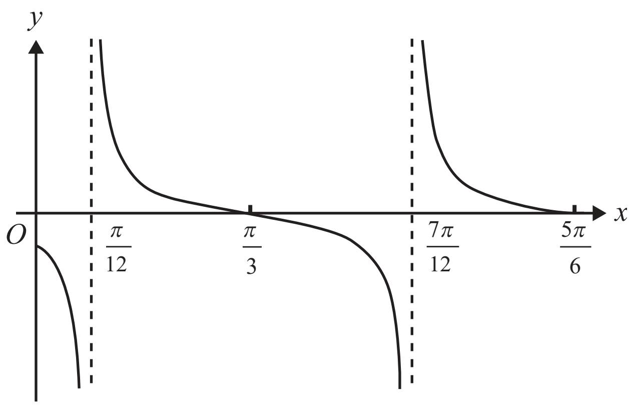
C. 2

D. 3

E. 4

Question 4

Part of the graph of $y = f(x)$ is shown below.



$f(x)$ could be

A. $y = -\tan\left(2x - \frac{\pi}{6}\right)$

B. $y = -\tan\left(2x - \frac{\pi}{3}\right)$

C. $y = \cot\left(2x - \frac{\pi}{12}\right)$

D. $y = \cot\left(2x - \frac{\pi}{6}\right)$

E. $y = \cot\left(2x + \frac{\pi}{6}\right)$

Question 5

If $y = \mathrm{Tan}^{-1}\left(\sqrt{3x}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{1+3x}$

B. $\frac{\sqrt{3}}{1+3x}$

C. $\frac{\sqrt{3}}{1+3x^2}$

D. $\frac{1}{2\sqrt{3x}(1+3x)}$

E. $\frac{\sqrt{3}}{2\sqrt{x}(1+3x)}$

Question 6

If $z = \dfrac{3-6i}{2+i}$, then $|z|$ and Arg(z) are, respectively

A. -3 and $\dfrac{\pi}{2}$

B. 3 and $\dfrac{\pi}{2}$

C. -3 and $\dfrac{3\pi}{2}$

D. 3 and $-\dfrac{\pi}{2}$

E. -3 and $-\dfrac{\pi}{2}$

Question 7

Let $u = 7\,\text{cis}\left(\dfrac{\pi}{4}\right)$ and $v = a$ cis(b), where a and b are real constants.

If $uv = 42\,\text{cis}\left(\dfrac{\pi}{20}\right)$, then

A. $a = 6$ and $b = -\dfrac{\pi}{5}$

B. $a = 35$ and $b = -\dfrac{\pi}{5}$

C. $a = 6$ and $b = \dfrac{\pi}{5}$

D. $a = 35$ and $b = \dfrac{1}{5}$

E. $a = 6$ and $b = \dfrac{1}{5}$

Question 8

The value of the discriminant for the quadratic equation $(1+i)z^2 + 4iz - 2(1-i) = 0$ is

A. –32

B. –16

C. 0

D. 16

E. 32

Question 9

$\sqrt{2}\ \text{cis}\left(\frac{\pi}{16}\right)$ is one of the fourth roots of a complex number, z.

z^{-1} is equal to

A. $\frac{1}{4}\text{cis}\left(\frac{4}{\pi}\right)$

B. $\frac{1}{4}\text{cis}\left(-\frac{\pi}{4}\right)$

C. $\frac{1}{4\sqrt{2}}\text{cis}\left(-\frac{\pi}{4}\right)$

D. $2^8\text{cis}\left(\frac{64}{\pi}\right)$

E. $2^8\text{cis}\left(-\frac{\pi}{64}\right)$

Question 10

If $z \in C$, which one of the following relations does **not** represent a circle on an Argand diagram?

A. $z\bar{z} = 4$

B. $|z-1| + |z+1| = 3$

C. $(z-3+i)(\bar{z}-3-i) = 5$

D. $3|z-2+i| = 7$

E. $|z-3| = 2$

Question 11

Which one of the following is an antiderivative of $\frac{6}{\sqrt{1-4x^2}}$ for $-\frac{1}{2} < x < \frac{1}{2}$?

A. $3\ \text{Sin}^{-1}(2x)$

B. $6\ \text{Sin}^{-1}(2x)$

C. $3\ \text{Sin}^{-1}\left(\frac{x}{2}\right)$

D. $6\ \text{Sin}^{-1}\left(\frac{x}{2}\right)$

E. $12\ \text{Sin}^{-1}\left(\frac{x}{2}\right)$

Question 12

With a suitable substitution, $\int_{\frac{\pi}{2}}^{\pi} \sin^2(2x)\sin(2x)dx$ can be expressed as

A. $\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}\left(1-u^2\right)du$

B. $\frac{1}{2}\int_{-1}^{1}\left(1-u^2\right)du$

C. $-\frac{1}{2}\int_{-1}^{1}\left(1-u^2\right)du$

D. $-2\int_{-1}^{1}\left(1-u^2\right)du$

E. $-\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi}\left(1-u^2\right)du$

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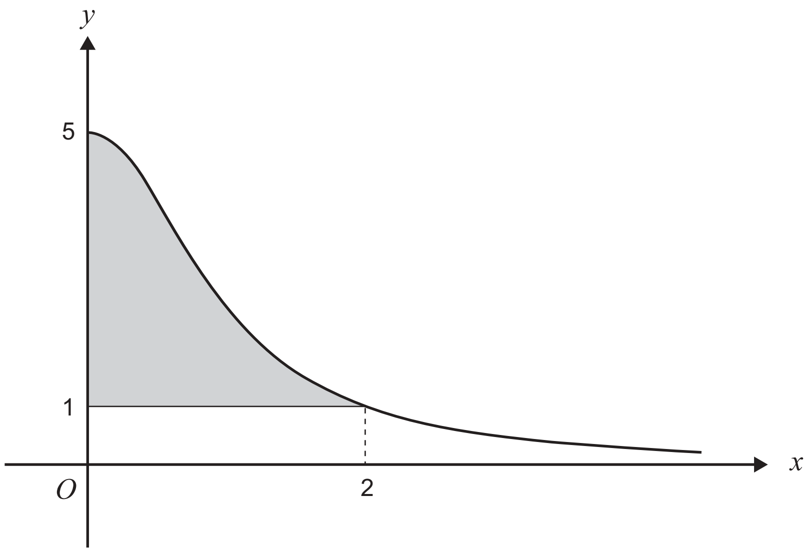
Question 13

The graph of $f\colon [0,\infty) \to R$, where $f(x) = \dfrac{5}{x^2+1}$, is shown below. The shaded region is bounded by the graph of f, the y-axis and the line with equation $y = 1$.



The shaded region is rotated about the x-axis to form a solid of revolution.
The volume of the solid, in cubic units, is given by

A. $\pi \int_0^2 \left(\left(\dfrac{5}{x^2+1} \right)^2 - 1 \right) dx$

B. $\pi \int_0^2 \left(\dfrac{5}{x^2+1} - 1 \right)^2 dx$

C. $\pi \int_0^2 \left(\dfrac{5}{x^2+1} - 1 \right) dx$

D. $\pi \int_0^2 \left(\dfrac{5}{x^2+1} \right)^2 dx$

E. $\pi \int_0^2 \left(\dfrac{4}{x^2+1} \right)^2 dx$

Question 14

The value, correct to three decimal places, of $\int_4^5 \left(\dfrac{x+3}{2\sin(x)} \right) dx$ is

A. –4.014
B. –3.523
C. 3.094
D. 3.523
E. 4.014

Question 15

An antiderivative of $x\sqrt{3-x}$, for $x < 3$, is

A. $-\frac{2x}{3}(3-x)^{\frac{3}{2}}$

B. $-\frac{x^2(3-x)^{\frac{3}{2}}}{3}$

C. $2(3-x)^{\frac{3}{2}}-\frac{2}{5}(3-x)^{\frac{5}{2}}$

D. $-2(3-x)^{\frac{3}{2}}+\frac{2}{5}(3-x)^{\frac{5}{2}}$

E. $-2(3-x)^{\frac{3}{2}}-\frac{2}{5}(3-x)^{\frac{5}{2}}$

Question 16

$\int_0^{\frac{\pi}{8}} \sec^2(2x)e^{2\tan(2x)}dx$ is equal to

A. $\frac{1}{4}\left(e^{\frac{\pi}{4}}-1\right)$

B. $\frac{1}{2}\left(e^{\frac{\pi}{4}}-1\right)$

C. $\frac{1}{4}\left(e^{\frac{\pi}{8}}-1\right)$

D. $\frac{1}{2}\left(e^2-1\right)$

E. $\frac{1}{4}\left(e^2-1\right)$

Question 17

Euler's method, with a step size of 0.1, is used to solve the differential equation

$$\frac{dy}{dx}=e^{-x}, \text{ with initial condition } y = 1 \text{ at } x = 2.$$

When $x = 2.2$, the value obtained for y, correct to four decimal places, is

A. 1.0122
B. 1.0222
C. 1.0233
D. 1.0258
E. 1.0271

Question 18

An oil slick is in the shape of a circle. Its surface area is increasing at a rate of 10 m^2/s.

Let r metres be the radius of the oil slick at time t seconds.

The rate of increase of r, in m/s, is given by

A. $\dfrac{5}{\pi r}$

B. $\dfrac{20}{\pi r}$

C. $\dfrac{10}{\pi r^2}$

D. $\dfrac{1}{20\pi r}$

E. $20\pi r$

Question 19

Given that $\dfrac{dy}{dx} = y^2 + 1$, and that $y = 1$ at $x = 0$, then

A. $y = \tan\left(x - \dfrac{\pi}{4}\right)$

B. $y = \tan\left(x + \dfrac{\pi}{4}\right)$

C. $x = \log_e\left(\dfrac{y^2 + 1}{2}\right)$

D. $y = \dfrac{1}{3}y^3 + y - \dfrac{1}{3}$

E. $y = y^2x + x + 1$

Question 20

A particle travels in a straight line with velocity v at time t.

If the velocity of the particle is given by $v = \dfrac{2}{\sqrt{1-x^2}}$, for $0 < x < 1$, then the acceleration is given by

A. $2\,\text{Sin}^{-1}(x)$

B. $\dfrac{4\,\text{Sin}^{-1}(x)}{\sqrt{1-x^2}}$

C. $\dfrac{4x}{\left(1-x^2\right)^2}$

D. $\dfrac{2x}{\left(1-x^2\right)^2}$

E. $\dfrac{2x}{\left(1-x^2\right)^{\frac{3}{2}}}$

Question 21

A particle travelling in a straight line has velocity v m/s at time t s. Its acceleration is given by $\dfrac{dv}{dt} = \dfrac{3}{v^2-9}$.
The time taken, in seconds, for the velocity to decrease from 2 m/s to 1 m/s is given by

A. $\displaystyle\int_1^2 \frac{v^2-9}{3}\,dv$

B. $\displaystyle\int_2^1 \frac{v^2-9}{3}\,dv$

C. $\displaystyle\int_2^1 \frac{3}{v^2-9}\,dt$

D. $\displaystyle\int_2^1 \frac{3}{v^2-9}\,dv$

E. $\displaystyle\int_1^2 \frac{3}{v^2-9}\,dv$

Question 22

PQRS is a parallelogram. The position vectors of *P*, *Q*, *R* and *S* are, respectively,

$$\underset{\sim}{p} = -3\underset{\sim}{k},\ \underset{\sim}{q} = \underset{\sim}{i} + y\underset{\sim}{j},\ \underset{\sim}{r} = 5\underset{\sim}{i} + 2x\underset{\sim}{j} + \underset{\sim}{k} \text{ and } \underset{\sim}{s} = y\underset{\sim}{i} - 2\underset{\sim}{k}.$$

The values of x and y are

A. $x = 0,\quad y = 5$

B. $x = 2,\quad y = 4$

C. $x = 3,\quad y = 6$

D. $x = 8,\quad y = 4$

E. $x = 12,\quad y = 6$

Question 23

Point *P* has coordinates (2, 2, –1) and point *Q* has coordinates (–4, 0, –3).

The cosine of angle *POQ* is equal to

A. $-\frac{1}{5}$

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. $\frac{11}{15}$

E. $-\frac{11}{15}$

Question 24

The path of a particle satisfies the Cartesian equation $(x+1)^2 + y^2 = 1$.

The position vector of the particle at time t, $t \geq 0$, could be given by

A. $\cos(t)\underset{\sim}{i} + \sin(t)\underset{\sim}{j}$

B. $\cos(t)\underset{\sim}{i} + (\sin(t) - 1)\underset{\sim}{j}$

C. $(\cos(t) + 1)\underset{\sim}{i} + \sin(t)\underset{\sim}{j}$

D. $(\sin(t) + 1)\underset{\sim}{i} + \cos(t)\underset{\sim}{j}$

E. $(\sin(t) - 1)\underset{\sim}{i} + \cos(t)\underset{\sim}{j}$

Question 25

The position vector of a particle at time t is given by

$$\underset{\sim}{r}(t) = (3t^2 - 2)\underset{\sim}{i} - (7 - 5t)\underset{\sim}{j} - 4\underset{\sim}{k}.$$

The velocity vector of the particle at time t is given by

A. $6t\underset{\sim}{i} + 5\underset{\sim}{j}$

B. $6t\underset{\sim}{i} - 5\underset{\sim}{j}$

C. $6t\underset{\sim}{i} - 5\underset{\sim}{j} - 4\underset{\sim}{k}$

D. $(6t - 2)\underset{\sim}{i} - 2\underset{\sim}{j} - 4\underset{\sim}{k}$

E. $(t^3 - 2t)\underset{\sim}{i} - (7t - \frac{5}{2}t^2)\underset{\sim}{j} - 4t\underset{\sim}{k}$

Question 26

A particle initially at the origin starts from rest at $t = 0$. The particle moves in a straight line in such a way that its acceleration at time t is given by $e^{-0.1t}\,\underset{\sim}{i} + (6t)\,\underset{\sim}{j}$.

The velocity of the particle at time t is given by

A. $-(0.1e^{-0.1t})\,\underset{\sim}{i} + 6\,\underset{\sim}{j}$

B. $-(10e^{-0.1t})\,\underset{\sim}{i} + (3t^2)\,\underset{\sim}{j}$

C. $10(1 - e^{-0.1t})\,\underset{\sim}{i} + (3t^2)\,\underset{\sim}{j}$

D. $0.1(1 - e^{-0.1t})\,\underset{\sim}{i} + (3t^2)\,\underset{\sim}{j}$

E. $10(10 - t - 10e^{-0.1t})\,\underset{\sim}{i} + (t^3)\,\underset{\sim}{j}$

Question 27

A body of mass 5 kg is acted on by a force of variable magnitude. The acceleration, a m/s^2, of the body at time t s, $t \geq 0$, is given by $a = 20 - 10\cos(2t)$.

The maximum value of the magnitude of the force, measured in newtons, is

A. 20

B. 30

C. 50

D. 100

E. 150

Question 28

A body of mass 10 kg, at rest on a rough horizontal surface, is acted on by a force of magnitude 14 N acting at an angle of 30° to the horizontal.

If the body is on the point of moving, the coefficient of friction between the body and the surface is equal to

A. $\frac{\sqrt{3}}{13}$

B. $\frac{\sqrt{3}}{14}$

C. $\frac{\sqrt{3}}{15}$

D. $\frac{7}{\sqrt{3}}$

E. $\frac{1}{14 - \sqrt{3}}$

Question 29

A particle is held in equilibrium by three concurrent coplanar forces $\underset{\sim}{P}$, $\underset{\sim}{Q}$ and $\underset{\sim}{R}$.

$\underset{\sim}{P}$ has magnitude 5 newtons and acts in the east direction. $\underset{\sim}{Q}$ has magnitude 5 newtons and acts in the north direction.

The magnitude and direction of $\underset{\sim}{R}$ are, respectively

A. 5 newtons, southeast

B. $5\sqrt{2}$ newtons, northeast

C. $5\sqrt{2}$ newtons, southwest

D. 10 newtons, southwest

E. 10 newtons, northeast

Question 30

A body of mass 200 kg is lowered vertically by a crane. The crane cable exerts a constant force of 1000 N vertically upwards on the body.

The magnitude of the acceleration of the body, in m/s^2, is

A. 0.49

B. 4.8

C. 5.0

D. 9.8

E. 14.8

END OF PART 1 MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2005

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 31 October 2005

Reading time: 11.45 am to 12.00 noon (15 minutes)
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PART II
QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	20

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten), one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 9 pages.

Instructions
- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions for Part II

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Consider the hyperbola with equation

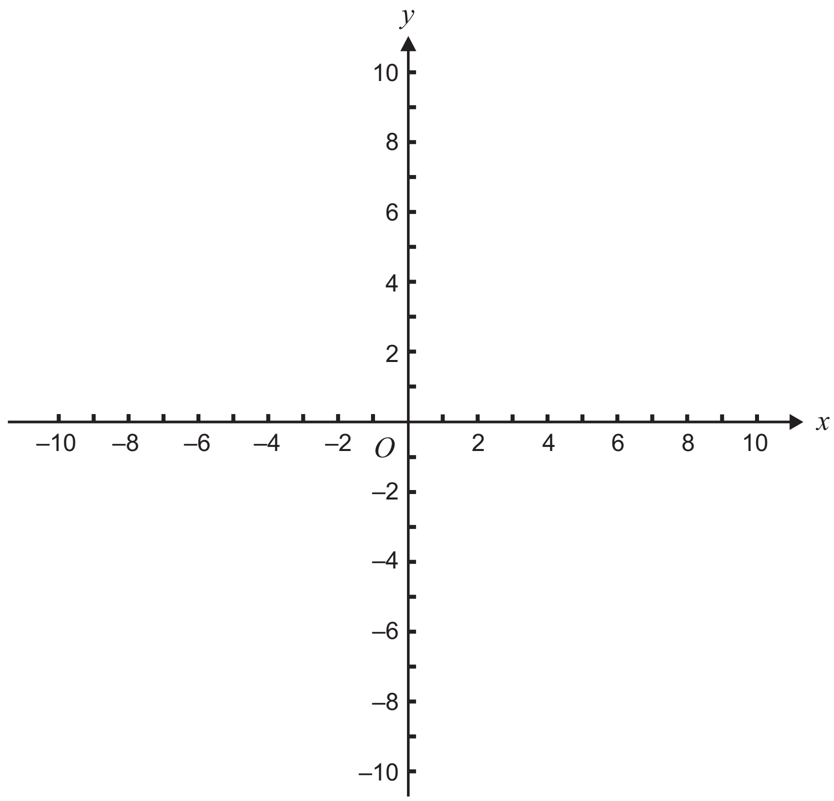
$$\frac{(x-c)^2}{9}-\frac{(y-3)^2}{4}=1\text{, where } c \text{ is a real constant.}$$

The equation of one of the asymptotes of this hyperbola is $y=\frac{2}{3}x+5$.

a. Show that $c=-3$.

1 mark

b. Sketch the hyperbola on the following set of axes, clearly showing the asymptotes.



3 marks

Question 2

$y = e^{2x}\cos(x)$ is a solution of the differential equation

$$\frac{d^2 y}{dx^2} + k\frac{dy}{dx} + y = -2e^{2x}\sin(x)\ ,$$

where $k \in R$.

Find the value of k.

4 marks

Question 3

The vector resolute of $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v}$ is $3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$.

The vector resolute of $\underset{\sim}{u}$ perpendicular to $\underset{\sim}{v}$ is $2\underset{\sim}{i} + x\underset{\sim}{j} + 2\underset{\sim}{k}$.

a. Show that $x = 4$.

1 mark

b. **Hence** find $\underset{\sim}{u}$.

2 marks

Question 4

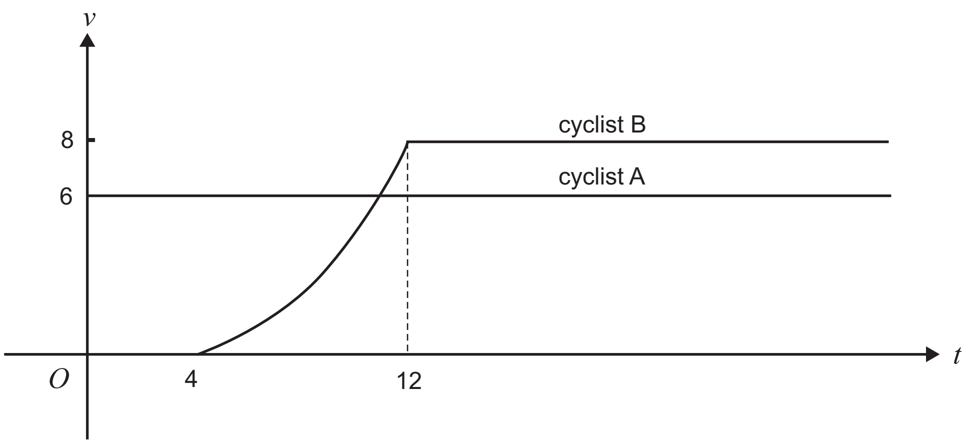
At time $t = 0$, cyclist A, travelling at a speed of 6 m/s along a straight bicycle path, passes cyclist B who is stationary.

Four seconds later, at $t = 4$, cyclist B accelerates in the direction of cyclist A for 8 seconds in such a way that her speed, v m/s, is given by $v = (t-4)\tan\left(\frac{\pi}{48}t\right)$.

a. Show that cyclist B accelerates to a speed of 8 m/s.

1 mark

Cyclist B then maintains her speed of 8 m/s. The velocity-time graph that represents this situation is shown below.



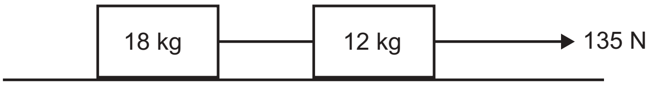
b. Find the time at which cyclist B passes cyclist A, correct to the nearest tenth of a second.

3 marks

Question 5

Two boxes made of the same material have masses 12 kg and 18 kg. They are joined by a light rope as shown in the diagram.

The boxes are pulled along a rough horizontal plane with an acceleration of 0.5 m/s^2 by a horizontal force of magnitude 135 N.



a. Show that the coefficient of friction between the boxes and the plane is 0.41, correct to two decimal places.

3 marks

b. Find the magnitude of the tension in the light rope that joins the two boxes.

2 marks

END OF PART II QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $-\pi < \text{Arg}\, z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \log_e(x) + c$, for $x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\mathrm{Sin}^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \mathrm{Sin}^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\mathrm{Cos}^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\, dx = \mathrm{Cos}^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\mathrm{Tan}^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\, dx = \mathrm{Tan}^{-1}\left(\frac{x}{a}\right) + c$

product rule:	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule:	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule:	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
mid-point rule:	$\int_a^b f(x)\, dx \approx (b-a) f\left(\frac{a+b}{2}\right)$
trapezoidal rule:	$\int_a^b f(x)\, dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$
Euler's method:	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration:	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
constant (uniform) acceleration:	$v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET