2005 Specialist Mathematics
Written Examination 2 (Analysis task)
Suggest answers and solutions

Question 1

a Concentration $= \dfrac{\text{Mass}}{\text{Volume}}$

Mass $= x$

Volume = 20t+10-10t

$= 10t + 10$

Concentration $= \dfrac{x}{10t + 10}$

b Rate of Increase = Inflow – Outflow

Inflow $= \dfrac{20 \times 2}{1 + t^2}$

$= \dfrac{40}{1 + t^2}$

Outflow $= \dfrac{10x}{10 + 10t}$

$= \dfrac{x}{1 + t}$

$$\frac{dx}{dt} = \frac{40}{1 + t^2} - \frac{x}{1 + t}$$

$$\Rightarrow \frac{dx}{dt} + \frac{x}{1 + t} = \frac{40}{1 + t^2}$$

ci

$$x = \frac{40}{1 + t}\text{Tan}^{-1}(x) + \frac{20}{1 + t}\log\left(1 + t^2\right)$$

$$\frac{dx}{dt} = \frac{40}{\left(t^2 + 1\right)(1 + t)} - \frac{40\text{Tan}^{-1}t}{(1 + t)^2} + \frac{20 \times 2t}{\left(t^2 + 1\right)(1 + t)} - \frac{20\log_e\left(1 + t^2\right)}{1 + t^2}$$

$$\frac{dx}{dt} = \frac{40t + 40}{\left(t^2 + 1\right)(1 + t)} - \frac{40\text{Tan}^{-1}t}{(1 + t)^2} - \frac{20\log_e\left(1 + t^2\right)}{1 + t^2}$$

$$= \frac{40(1 + t)}{\left(t^2 + 1\right)(1 + t)} - \frac{40\text{Tan}^{-1}t}{(1 + t)^2} - \frac{20\log_e\left(1 + t^2\right)}{1 + t^2}$$

$$= \frac{40}{1 + t^2} - \frac{40\text{Tan}^{-1}(t)}{(1 + t)^2} - \frac{20\log_e\left(1 + t^2\right)}{(1 + t)^2}$$

cii

$$\frac{dx}{dt} = \frac{40}{1 + t^2} - \frac{40\text{Tan}^{-1}(t)}{(1 + t)^2} - \frac{20\log_e\left(1 + t^2\right)}{(1 + t)^2}$$

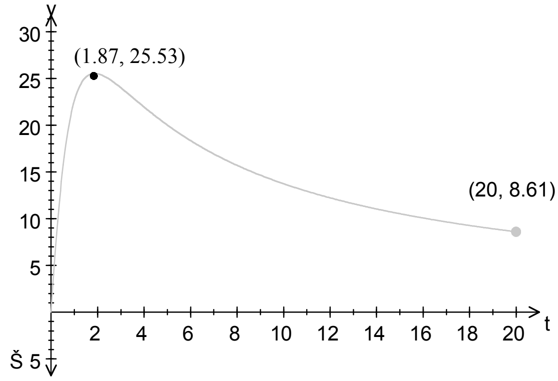
$$= \frac{40}{1 + t^2} - \frac{40\text{Tan}^{-1}(t)}{(1 + t)^2} - \frac{20\log_e\left(1 + t^2\right)}{(1 + t)^2}$$

$$= \frac{40}{1 + t^2} - \frac{1}{1 + t}\left(\frac{40\text{Tan}^{-1}(t)}{(1 + t)^2} - \frac{20\log_e\left(1 + t^2\right)}{(1 + t)^2}\right)$$

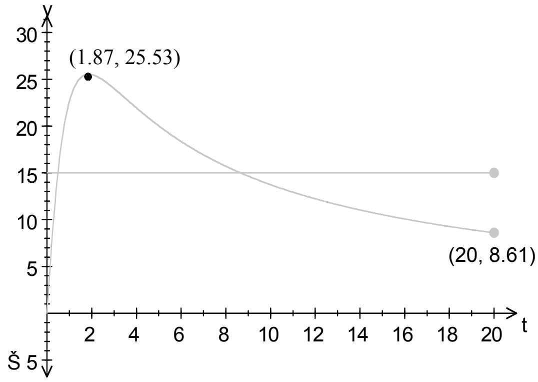
$$= \frac{40}{1 + t^2} - \frac{1}{1 + t}(x)$$

$$\Rightarrow \frac{dx}{dt} + \frac{x}{1 + t} = \frac{40}{1 + t^2}$$

d



ei



Find the point of intersection using a graphics calculator

$t = 0.485$

eii Second Point of Intersection

$t = 8.655$

Chemical remains effective

$8.655 - 0.485 \approx 8.17$ 8.17 (2dp)

Question 2

ai $u = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$|u| = \frac{1}{4} + \frac{3}{4} = 1$

$\arg(u) = \text{Tan}^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \text{Tan}^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

$u = \text{cis}\left(\frac{\pi}{3}\right)$

aii

$u^6 = 1^6 \text{cis}\left(\frac{6\pi}{3}\right)$

$= \text{cis}(2\pi)$

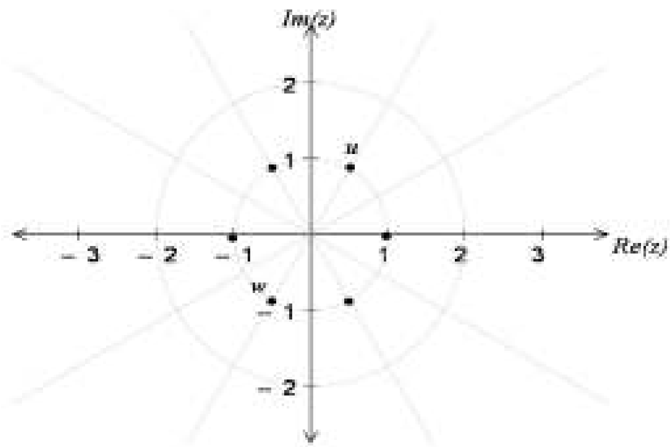
$= 1$

aiii

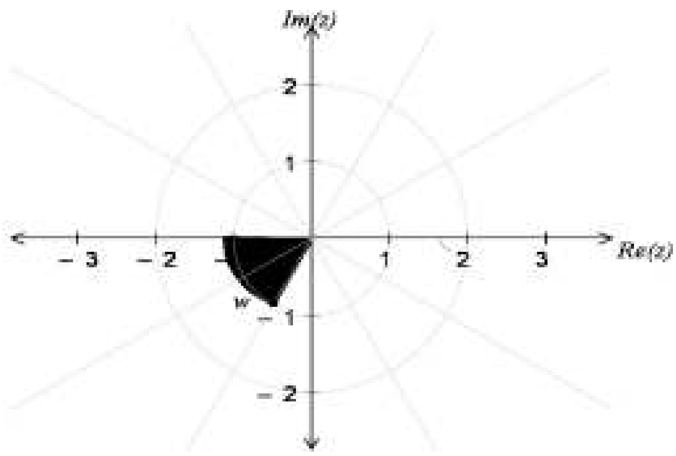
$z^6 - 1 = 0$

$z^6 = \text{cis}(2\pi)$

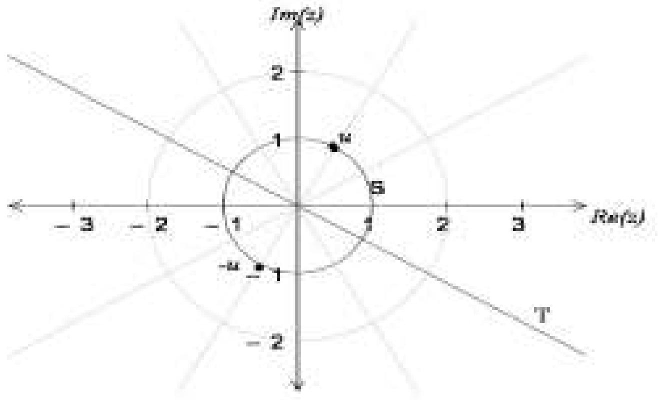
$z = \text{cis}\left(\frac{2n\pi}{6}\right)$



b



ci & cii



ciii

$$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \text{ and } \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

Question 3

a

$A \approx 0.25(2 \times 1.5 + 2 \times 1.25 + 2 \times 0.85 + 0.55)$

$= 1.9875$

b

$$\frac{10x}{(x^2+1)(3x+1)} = \frac{x+A}{x^2+1} + \frac{B}{3x+1}$$

$$= \frac{(x+A)(3x+1) + B(x^2+1)}{(x^2+1)(3x+1)}$$

$$10x \equiv (x+A)(3x+1) + B(x^2+1)$$

Let $x = -\frac{1}{3}$

$$-\frac{10}{3} = \frac{10B}{9}$$

$B = -3$

Let $x = 0$

$0 = A + B$

$\Rightarrow A = 3$

$\therefore A = 3$ and $B = -3$

c

$$\int_0^2 \frac{x+3}{x^2+1} - \frac{3}{3x+1}dx$$

$$= \int_0^2 \frac{x}{x^2+1} + \frac{3}{x^2+1} - \frac{3}{3x+1}dx$$

$$= \left[\tfrac{1}{2}\log_e(x^2+1) + 3\text{Tan}^{-1}(x) - \log_e(3x+1)\right]_0^2$$

$$= \left(\tfrac{1}{2}\log_e(5) + 3\text{Tan}^{-1}(2) - \log_e(7)\right) - (0+0+0)$$

≈ 2.18

d

Using $h(x) = \dfrac{10x}{(x^2+1)(3x+1)}$

At $x = 2$

$$h(2) = \frac{10 \times 2}{(2^2+1)(3 \times 2 + 1)} = \frac{20}{35}$$

$$\frac{10x}{(x^2+1)(3x+1)} = \frac{4}{7}$$

$x \approx .0694$ and $x = 2$

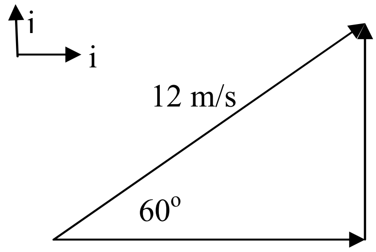
Length of Usable Panel = 2 – 0.0694
= 1.9306

Number of Panel Required $= \frac{100}{1.9306} = 51.79$

Can't purchase part panel, therefore 52 required.

Question 4

a



$\tilde{v}_0 = 12\cos(60^\circ)\tilde{i} + 12\sin(60^\circ)\tilde{j}$

$= 12 \times \frac{1}{2}\tilde{i} + 12 \times \frac{\sqrt{3}}{2}\tilde{j}$

$= 6\tilde{i} + 6\sqrt{3}\,\tilde{j}$

b

$\ddot{\tilde{r}}(t) = -0.1t\tilde{i} - (g - 0.1t)\tilde{j}$

$\dot{\tilde{r}}(t) = -\frac{t^2}{20}\tilde{i} - \left(gt - \frac{t^2}{20}\right)\tilde{j} + \tilde{c}$

$\dot{r}(0) = 6i + 6\sqrt{3}\,j = c$

$\dot{\tilde{r}}(t) = \left(6 - \frac{t^2}{20}\right)\tilde{i} - \left(6\sqrt{3} + gt - \frac{t^2}{20}\right)\tilde{j}$

$\tilde{r}(t) = \left(6t - \frac{t^3}{60}\right)\tilde{i} + \left(6t\sqrt{3} - \frac{gt^2}{2} + \frac{t^3}{50}\right)\tilde{j} + \tilde{c}$

$r(0) = 0 = c$

$$\tilde{r}(t) = \left(6t - \frac{t^3}{60}\right)\tilde{i} + \left(6t\sqrt{3} - \frac{gt^2}{2} + \frac{t^3}{50}\right)\tilde{j}$$

c

To find T, we need to find

$$6t - \frac{t^3}{60} = -\left(6t\sqrt{3} - \frac{gt^2}{2} + \frac{t^3}{50}\right)$$

$$= -6t\sqrt{3} + \frac{gt^2}{2} - \frac{t^3}{50})$$

$$6t + 6t\sqrt{3} - \frac{1}{2}gt^2 = 0$$

$$t\left(6 + 6\sqrt{3} - \frac{1}{2}gt\right) = 0$$

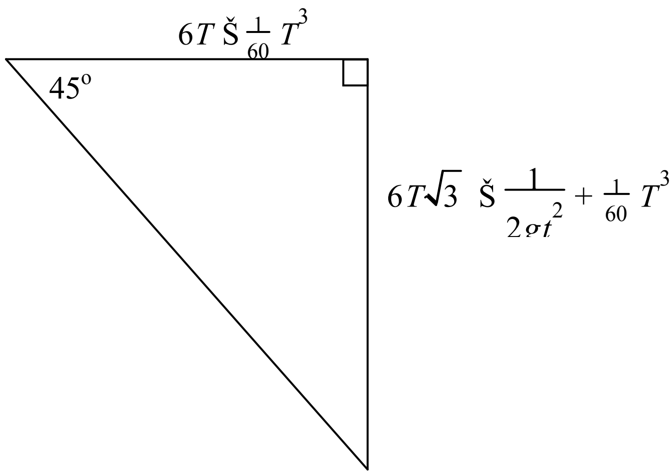
$t = 0$

and

$6 + 6\sqrt{3} = \frac{gt}{2}$

$T = t = \frac{12(1+\sqrt{3})}{g}$

An alternative method



$$\tan(-45^0) = \frac{6T\sqrt{3} - \frac{1}{2}gT^2 + \frac{1}{60}T^3}{6T - \frac{1}{60}T^3}$$

$$-1 = \frac{6T\sqrt{3} - \frac{1}{2}gT^2 + \frac{1}{60}T^3}{6T - \frac{1}{60}T^3}$$

$$-6T + \frac{1}{60}T^3 = 6T\sqrt{3} - \frac{1}{2}gT^2 + \frac{1}{60}T^3$$

$$\begin{aligned} 0 &= 6T\sqrt{3} + 6T - \tfrac{1}{2}gT^2 \\ &= 12T\sqrt{3} + 12T - gT^2 \\ &= T(12\sqrt{3} + 12 - gT) \end{aligned}$$

$$T = 0 \text{ or } T = \frac{12(\sqrt{3}+1)}{g}$$

d

$$\dot{\underset{\sim}{r}}(t) = \left(6 - \frac{t^2}{20}\right)\underset{\sim}{i} - \left(6\sqrt{3} + gt - \frac{t^2}{20}\right)\underset{\sim}{j}$$

at $t = \dfrac{12(1+\sqrt{3})}{g}$

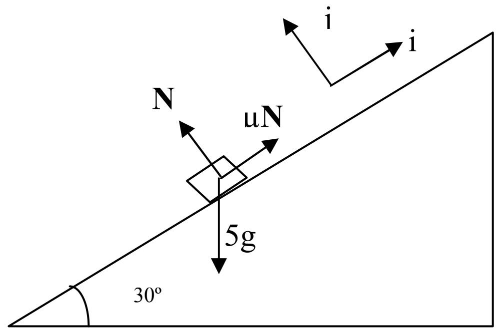
$$\dot{\underset{\sim}{r}}(t) \approx 5.44\underset{\sim}{i} - 42.62\underset{\sim}{j}$$

$$\begin{aligned} |\underset{\sim}{r}(t)| &= \sqrt{5.44^2 + 42.62^2} \\ &= 42.9658 \\ &\approx 43.0 \end{aligned}$$

Question 5

a

$$\begin{aligned} \mu N &= 5g\sin(30^0) \\ &= 2.5g \end{aligned}$$



b

$$\mu\mathbf{N} = 5g\sin(30^\circ) + 0.5 \times 8$$

$$\mu = \frac{5g\sin(30^\circ) + 0.5 \times 8}{\mathbf{N}}$$

$$= \frac{5g\sin(30^\circ) + 0.5 \times 8}{5g\cos(30^\circ)}$$

$$\approx 0.67$$

c

$$T - mg\sin(30^\circ) - \mu mg\cos(30^\circ) = 0.5m$$

$$T = mg\sin(30^\circ) + \mu mg\cos(30^\circ) + 0.5m$$

$$T = m(g\sin(30^\circ) + \mu g\cos(30^\circ) + 0.5)$$

$$m = \frac{T}{g\sin(30^\circ) + \mu g\cos(30^\circ) + 0.5}$$

$$= \frac{160}{g\sin(30^\circ) + \mu g\cos(30^\circ) + 0.5}$$

$$= \frac{160}{4.9 + 5.7 + 0.5}$$

$$\approx 14.4$$

