Year 2005

VCE

Specialist Mathematics
Trial Examination 1

Suggested Solutions

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ANSWERS TO MULTIPLE CHOICE QUESTIONS

1	A	B	C	D	**E**	**16**	A	B	C	**D**	E
2	A	**B**	C	D	E	**17**	A	B	**C**	D	E
3	A	B	**C**	D	E	**18**	A	**B**	C	D	E
4	A	B	C	**D**	E	**19**	A	B	C	**D**	E
5	A	B	C	D	**E**	**20**	A	**B**	C	D	E
6	**A**	B	C	D	E	**21**	A	B	C	**D**	E
7	A	B	C	**D**	E	**22**	A	B	C	D	**E**
8	**A**	B	C	D	E	**23**	**A**	B	C	D	E
9	A	B	**C**	D	E	**24**	A	B	C	D	**E**
10	A	B	C	**D**	E	**25**	A	**B**	C	D	E
11	**A**	B	C	D	E	**26**	**A**	B	C	D	E
12	A	B	C	D	**E**	**27**	A	B	C	**D**	E
13	A	B	**C**	D	E	**28**	**A**	B	C	D	E
14	A	B	**C**	D	E	**29**	A	**B**	C	D	E
15	**A**	B	C	D	E	**30**	A	B	C	D	**E**

Question 1

$$y = \frac{x^3 + a^3}{2x} = \frac{x^3}{2x} + \frac{a^3}{2x} = \frac{x^2}{2} + \frac{a^3}{2x} \quad \text{since} \quad a > 0$$

$x \to \infty \quad y \to \frac{x^2}{2}$ from above

$x \to -\infty \quad y \to \frac{x^2}{2}$ from below

so $x = 0$ and $y = \frac{x^2}{2}$ are its asymptotes

Answer E.

Question 2

The graph has the equation of the form $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

where the centre is (h, k) now $h = -3$ $k = 2$ and $a = 2$ the distance from the vertices to the centre. The asymptotes are $y = 2 \pm \frac{2}{b}(x+3)$, for this to pass through the origin $x = 0$ and $y = 0$ we require $b = 3$, the correct equation is

$$\frac{(y-2)^2}{4} - \frac{(x+3)^2}{9} = 1$$

Answer B.

Question 3

$$f(x) = \operatorname{cosec}(x) - \cot(x) = \frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)} = \frac{1-\cos(x)}{\sin(x)}$$

$$f(x) = \frac{1 - \left(1 - 2\sin^2\left(\frac{x}{2}\right)\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} = \frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$$

$$f(x) = h(x) = \tan\left(\frac{x}{2}\right)$$

Note that $f(0)$ and $f\left(\frac{\pi}{2}\right)$ are both not defined

f has range $(0, 1)$ and is identical to the function

$h: \left(0, \frac{\pi}{2}\right) \to R$ where $h(x) = \tan\left(\frac{x}{2}\right)$

Answer C.

Question 4

The maximal implied domain of $f(x) = \text{Sin}^{-1}\left(\frac{b}{ax}\right)$

we require

$\left|\frac{b}{ax}\right| \le 1$ or $-1 \le \frac{b}{ax} \le 1$ since $a > 0$ and $b > 0$

$\frac{b}{ax} \le 1$ and $\frac{b}{ax} \ge -1$

$\frac{ax}{b} \ge 1$ and $\frac{ax}{b} \le -1$

$x \ge \frac{b}{a}$ and $x \le -\frac{b}{a}$ that is

$\left(-\infty, -\frac{b}{a}\right] \cup \left[\frac{b}{a}, \infty\right)$

Answer D.

Question 5

$z = x + yi$ and $x, y \in R$ the conjugate $\overline{z} = x - yi$ checking each alternative

A. $|z| = \sqrt{x^2 + y^2}$ $|\overline{z}| = \sqrt{x^2 + (-y)^2} = \sqrt{x^2 + y^2}$ so **A.** is true

B. $\frac{1}{\overline{z}} = \frac{1}{x - yi} = \frac{1}{x - yi} \cdot \frac{x + yi}{x + yi} = \frac{x + yi}{x^2 - y^2 i^2} = \frac{x + yi}{x^2 + y^2} = \frac{z}{|z|^2}$ so **B.** is true

C. $\frac{1}{z} = \frac{1}{x + yi} = \frac{1}{x + yi} \cdot \frac{x - yi}{x - yi} = \frac{x - yi}{x^2 - y^2 i^2} = \frac{x - yi}{x^2 + y^2} = \frac{\overline{z}}{|z|^2}$

so $\frac{1}{z} + \frac{1}{\overline{z}} = \frac{1}{x + yi} + \frac{1}{x - yi} = \frac{z + \overline{z}}{|z|^2}$ **C.** is true

D. $\frac{1}{2}(z + \overline{z}) = \frac{1}{2}((x + yi) + (x - yi)) = x = \text{Re}(z)$ **D.** is true

E. $\frac{i}{2}(z - \overline{z}) = \frac{i}{2}((x + yi) - (x - yi)) = \frac{i}{2}.2iy = -y \ne \text{Im}(z)$ **E.** is false

Answer E.

Question 6

Let $w = r\,\text{cis}\left(-\frac{3\pi}{4}\right)$ $\overline{w} = r\,\text{cis}\left(\frac{3\pi}{4}\right)$ is the point R reflection in the real axis

$i\,\overline{w} = 1\text{cis}\left(\frac{\pi}{2}\right) r\,\text{cis}\left(\frac{3\pi}{4}\right) = r\,\text{cis}\left(\frac{5\pi}{4}\right) = r\,\text{cis}\left(-\frac{3\pi}{4}\right) = w$ or after another rotation of 90° anticlockwise, we are back at the point W

Answer A.

Question 7

If $P(z) = z^2 + bz + c$ and $P(\alpha + i\beta) = 0$ where b, c, α and β are all real non-zero numbers, then checking each alternative

A. by the conjugate root theorem $P(\alpha - i\beta) = 0$ so **A.** is true

B. Since $P(z)$ has complex roots the discriminant

$\Delta = b^2 - 4c < 0$ so $b^2 < 4c$ **B.** is true

C. let $u = \alpha + i\beta$ and $\bar{u} = \alpha - i\beta$ be the roots $u + \bar{u} = 2\alpha$ and

$u.\bar{u} = \alpha^2 - \beta^2 i^2 = \alpha^2 + \beta^2$ the quadratic is $(z-u)(z-\bar{u}) = 0$

expanding gives $z^2 - 2\alpha z + (\alpha^2 + \beta^2) = 0$ *so* $b = -2\alpha$ and $c = \alpha^2 + \beta^2$

C. is true,

E. $P(z)$ has one pair of complex conjugates as its roots, is true

D. is false, the correct statement is $b + 2\alpha = 0$

Answer D.

Question 8

A. is false as it does not include the origin, all of **B. C. D.** and **E** are true they all give $y = x$ which is the line *S* as shown.

Answer A.

Question 9

$z^3 + a^3 = (z+a)(z^2 - az + a^2) = 0$ since $a > 0$ there is one real answer $z = -a$ this is the point *G*, however there are three real roots and furthermore the roots must be equally spaced around the circle by 120°, so the roots are *G* , *C* and *K*

Answer C.

Question 10

$$\int \sec^2(3x)\tan^2(3x)\,dx$$

let $u = \tan(3x)$ $\dfrac{du}{dx} = 3\sec^2(3x)$

$$\sec^2(3x)\,dx = \frac{1}{3}du$$

$$\int \sec^2(3x)\tan^2(3x)\,dx = \frac{1}{3}\int u^2\,du$$

$$\frac{1}{9}u^3 + C = \frac{1}{3}\tan^3(3x) + C$$

However since an antiderivative is asked for , ignore the *C* answer $\dfrac{1}{9}\tan^3(3x)$

Answer D.

Question 11

$y = a\sin(px)$ the period is $T = \dfrac{2\pi}{p}$ so one-half cycle is the first x-intercept $x = \dfrac{\pi}{p}$

$$A = \int_a^b y\,dx$$

$$A = \int_0^{\frac{\pi}{p}} a\sin(px)\,dx$$

$$A = -\frac{a}{p}\Big[\cos(px)\Big]_0^{\frac{\pi}{p}}$$

$$A = -\frac{a}{p}\Big[\cos(\pi) - \cos(0)\Big]$$

$$A = \frac{2a}{p}$$

Answer A.

Question 12

$$V = \pi\int_a^b y^2\,dx$$

$$V = \pi\int_0^{\frac{\pi}{p}} a^2\sin^2(px)\,dx$$

$$V = \frac{\pi a^2}{2}\int_0^{\frac{\pi}{p}} \big(1 - \cos(2px)\big)\,dx$$

$$V = \frac{\pi a^2}{2}\left[x - \frac{1}{2p}\sin(2px)\right]_0^{\frac{\pi}{p}}$$

$$V = \frac{\pi a^2}{2}\left[\left(\frac{\pi}{p} - \frac{1}{2p}\sin(2\pi)\right) - \big(0 - \sin(0)\big)\right]$$

$$A = \frac{\pi^2 a^2}{2p}$$

Answer E.

Question 13

$y = \text{Sin}^{-1}\left(\frac{x}{2}\right)$ at the point where $x = 1$ $y = \text{Sin}^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ the point $P\left(1, \frac{\pi}{6}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{4 - x^2}}$ $m_T = \left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{\sqrt{3}}$ $m_N = -\sqrt{3}$ the equation of the normal is

$y - \frac{\pi}{6} = -\sqrt{3}(x - 1)$

$y = -\sqrt{3}x + \sqrt{3} + \frac{\pi}{6}$

Answer C.

Question 14

$$\int_0^2 \frac{x(2x^2 + 3)}{\sqrt{2x^2 + 1}}\,dx$$

let $u = 2x^2 + 1$ $\frac{du}{dx} = 4x$ so $x.dx = \frac{1}{4}du$

$2x^2 + 3 = u + 2$

change the teminals when $x = 2$ $u = 9$ and $x = 0$ $u = 1$

$\frac{1}{4}\int_1^9 \frac{u + 2}{\sqrt{u}}\,du$ is correct

Answer C.

Question 15

$y = \log_e(2x - 3)$

x	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
y	$\log_e(2)$	$\log_e(4)$	$\log_e(6)$

$A_M = 1(\log_e(2) + \log_e(4) + \log_e(6))$

$A_M = \log_e(2 \times 4 \times 6) = \log_e(48) = \log_e(a)$

$a = 48$

Answer A.

Question 16

Using Euler's method with $\frac{dy}{dx} = f(x) = \log_e(2x-3)$ $\quad x_0 = 2 \quad y_0 = 1 \quad h = 0.25 = \frac{1}{4}$

$$y_1 = y_0 + h\,f(x_0) = 1 + \frac{1}{4}\log_e(1) = 1$$

$$y_2 = y_1 + h\,f(x_1) = 1 + \frac{1}{4}\log_e\left(\frac{3}{2}\right)$$

$$y_3 = y_2 + h\,f(x_2) = 1 + \frac{1}{4}\log_e\left(\frac{3}{2}\right) + \frac{1}{4}\log_e(2) = 1 + \frac{1}{4}\log_e(3)$$

$$y_4 = y_3 + h\,f(x_3) = 1 + \frac{1}{4}\log_e(3) + \frac{1}{4}\log_e\left(\frac{5}{2}\right) = 1 + \frac{1}{4}\log_e\left(\frac{15}{2}\right) \approx 1.5037$$

Or use TI-83 programs

Answer D.

Question 17

$f'(x) = 6\cos^2\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)$ and $f(0) = 0$

$$f(x) = \int 6\cos^2\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)dx$$

let $u = \cos\left(\frac{x}{4}\right) \quad \frac{du}{dx} = -\frac{1}{4}\sin\left(\frac{x}{4}\right) \qquad \sin\left(\frac{x}{4}\right).dx = -4\,du$

$$f(x) = -24\int u^2\,du = -8u^3 + C$$

$f(x) = -8\cos^3\left(\frac{x}{4}\right) + C$ but $f(0) = 0$ so $C = 8$

$$f(x) = 8 - 8\cos^3\left(\frac{x}{4}\right)$$

Answer C.

Question 18

$$A = \int_a^b (y_2 - y_1)\,dx$$

let $y_2 = 4\cos^2(2x)$ and $y_1 = 4\sin^2(2x)$

by symmetry, the **total** shaded area is equal to

$$6\int_0^{\frac{\pi}{8}} \left(4\cos^2(2x) - 4\sin^2(2x)\right)dx$$

$$= 24\int_0^{\frac{\pi}{8}} \left(\cos(4x)\right)dx$$

Answer B.

Question 19

let $\underset{\sim}{a} = 4\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k}$ $\quad \left|\underset{\sim}{a}\right| = \sqrt{16 + 9 + 144} = 13$

so $\hat{\underset{\sim}{a}} = \dfrac{1}{13}\left(4\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k}\right)$

we require

$-26\hat{\underset{\sim}{a}} = -2\left(4\underset{\sim}{i} - 3\underset{\sim}{j} + 12\underset{\sim}{k}\right) = 2\left(-4\underset{\sim}{i} + 3\underset{\sim}{j} - 12\underset{\sim}{k}\right)$

Answer D.

Question 20

let $\underset{\sim}{q} = -\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ now $\left|\underset{\sim}{q}\right| = \sqrt{1 + 4 + 4} = 3$

so $\hat{\underset{\sim}{q}} = \dfrac{1}{3}\left(-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}\right)$

the scalar resolute of $\underset{\sim}{p}$ in the direction of $\underset{\sim}{q}$ is $\underset{\sim}{p}.\hat{\underset{\sim}{q}} = -3$

the vector resolute of $\underset{\sim}{p}$ perpendicular $\underset{\sim}{q}$ is $\underset{\sim}{p} - \left(\underset{\sim}{p}.\hat{\underset{\sim}{q}}\right)\hat{\underset{\sim}{q}} = 2\underset{\sim}{i} - \underset{\sim}{k}$

$\underset{\sim}{p} + 3\hat{\underset{\sim}{q}} = 2\underset{\sim}{i} - \underset{\sim}{k}$

$\underset{\sim}{p} + \left(-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}\right) = 2\underset{\sim}{i} - \underset{\sim}{k}$

$\underset{\sim}{p} = \left(2\underset{\sim}{i} - \underset{\sim}{k}\right) - \left(-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}\right)$

$\underset{\sim}{p} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

Answer B.

Question 21

$\underset{\sim}{r}(t) = 2\,\mathrm{Cos}^{-1}(3t)\underset{\sim}{i} + 6\sin(2t)\underset{\sim}{j} + 3\underset{\sim}{k}$

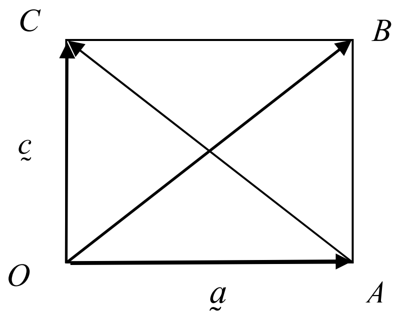
$\dot{\underset{\sim}{r}}(t) = \dfrac{-6}{\sqrt{1 - 9t^2}}\underset{\sim}{i} + 12\cos(2t)\underset{\sim}{j}$

The initial direction of the motion of the particle is $\dot{\underset{\sim}{r}}(0)$

$\dot{\underset{\sim}{r}}(0) = -6\underset{\sim}{i} + 12\underset{\sim}{j}$

Answer D.

Question 22



A. is true $\underset{\sim}{a}.\underset{\sim}{c}=0$ the vectors are perpendicular (it is a square)

B. is true $\left|\underset{\sim}{a}\right|=\left|\underset{\sim}{c}\right|$ the lengths are equal (it is a square)

C. is true $\left|\underset{\sim}{a}\right|^2+\left|\underset{\sim}{c}\right|^2=\left|\underset{\sim}{a}+\underset{\sim}{c}\right|^2$ Pythagorus's Theorem on the diagonal $\overrightarrow{OB}=\underset{\sim}{a}+\underset{\sim}{c}$

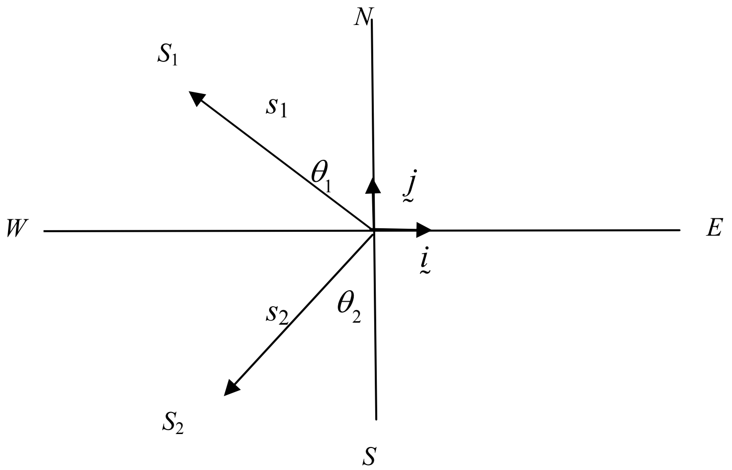
D. is true $\dfrac{\underset{\sim}{a}.\left(\underset{\sim}{a}+\underset{\sim}{c}\right)}{\left|\underset{\sim}{a}\right|\left|\underset{\sim}{a}+\underset{\sim}{c}\right|}=\dfrac{\sqrt{2}}{2}$ the angle between $\overrightarrow{OA}=\underset{\sim}{a}$ and $\overrightarrow{OB}=\underset{\sim}{a}+\underset{\sim}{c}$ is 45^0

$$\cos\left(45^0\right)=\frac{\overrightarrow{OA}.\overrightarrow{OB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|}=\frac{\sqrt{2}}{2}=\frac{\underset{\sim}{a}.\left(\underset{\sim}{a}+\underset{\sim}{c}\right)}{\left|\underset{\sim}{a}\right|\left|\underset{\sim}{a}+\underset{\sim}{c}\right|}$$

E. is false, for vectors $\underset{\sim}{a}\neq\underset{\sim}{c}$

Answer E.

Question 23



$$\overrightarrow{OS_1}=-s_1\sin\left(\theta_1\right)\underset{\sim}{i}+s_1\cos\left(\theta_1\right)\underset{\sim}{j} \quad \text{and} \quad \overrightarrow{OS_2}=-s_2\sin\left(\theta_2\right)\underset{\sim}{i}-s_2\cos\left(\theta_2\right)\underset{\sim}{j}$$

$$\overrightarrow{S_1S_2}=\overrightarrow{OS_2}-\overrightarrow{OS_1}=\left(s_1\sin\left(\theta_1\right)-s_2\sin\left(\theta_2\right)\right)\underset{\sim}{i}-\left(s_1\cos\left(\theta_1\right)+s_2\cos\left(\theta_2\right)\right)\underset{\sim}{j}$$

Answer A.

Question 24

A. is true, the particle moves on a circle.

$\underset{\sim}{r}(t) = a\cos(\omega t)\underset{\sim}{i} + a\sin(\omega t)\underset{\sim}{j}$

the parametric equations are given by

$x = a\cos(\omega t)$ (1)

$y = a\sin(\omega t)$ (2) squaring and adding gives

$x^2 + y^2 = a^2$ as the cartesian equation of the curve centre at the origin radius a

B. is true, the speed of the particle is constant.

the speed is given by $\left|\dot{\underset{\sim}{r}}(t)\right|$

velocity vector is $\dot{\underset{\sim}{r}}(t) = -\omega a\sin(\omega t)\underset{\sim}{i} + \omega a\cos(\omega t)\underset{\sim}{j}$

$\left|\dot{\underset{\sim}{r}}(t)\right| = \sqrt{(\omega a\sin(\omega t))^2 + (\omega a\cos(\omega t))^2} = \sqrt{\omega^2 a^2\left(\sin^2(\omega t) + \cos^2(\omega t)\right)}$

$\left|\dot{\underset{\sim}{r}}(t)\right| = \omega a$ this is independant of t and is constant

C. is true,

the acceleration vector is in the opposite direction to the position vector.

$\ddot{r}(t) = -\omega^2 a\cos(\omega t)\underset{\sim}{i} - \omega^2 a\sin(\omega t)\underset{\sim}{j} = -\omega^2\left(a\cos(\omega t)\underset{\sim}{i} + a\sin(\omega t)\underset{\sim}{j}\right) = -\omega^2\underset{\sim}{r}(t)$

D. is true, the velocity vector is perpendicular to the position vector.

velocity vector is $\dot{\underset{\sim}{r}}(t) = -\omega a\sin(\omega t)\underset{\sim}{i} + \omega a\cos(\omega t)\underset{\sim}{j}$

position vector is $r(t) = a\cos(\omega t)\underset{\sim}{i} + a\sin(\omega t)\underset{\sim}{j}$

Now $\underset{\sim}{r}(t)\,\dot{\underset{\sim}{r}}(t) = \left(-\omega a\sin(\omega t)\underset{\sim}{i} + \omega a\cos(\omega t)\underset{\sim}{j}\right)\left(a\cos(\omega t)\underset{\sim}{i} + a\sin(\omega t)\underset{\sim}{j}\right)$

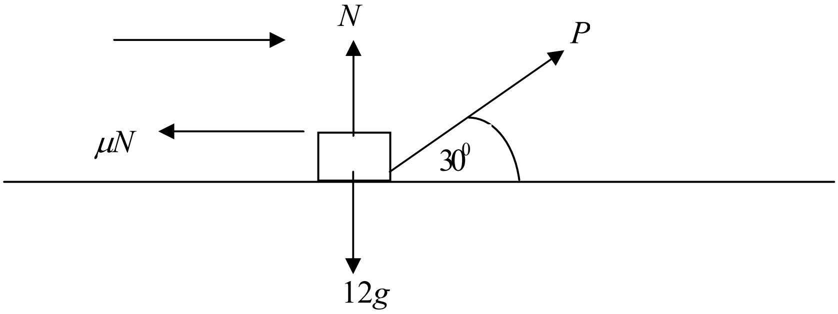
$= -\omega a^2\sin(\omega t)\cos(\omega t) + \omega a^2\sin(\omega t)\cos(\omega t) = 0$

Since $\underset{\sim}{r}(t).\dot{\underset{\sim}{r}}(t) = 0$

E. Is false, the acceleration vector is not constant.

Answer E.

Question 25



resolving parallel to the plane (1) $P\cos\left(30^0\right) - 0.25N = 0$

resolving perpendicular to the plane (2) $P\sin 30^0 + N - 12g = 0$

from (2) $N = 12g - P\sin\left(30^0\right)$ substituting into (1) gives

$P\cos\left(30^0\right) - 0.25\left(12g - P\sin\left(30^0\right)\right) = 0$

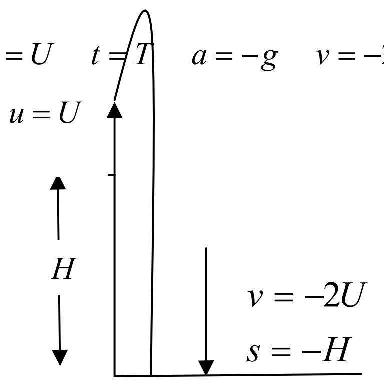
$P\left(\cos\left(30^0\right) + 0.25\sin\left(30^0\right)\right) = 0.25 \text{ x } 12g$

$P = \dfrac{3g}{\cos\left(30^0\right) + 0.25\sin\left(30^0\right)} = 29.67$ newton

Answer B.

Question 26

$u = U \quad t = T \quad a = -g \quad v = -2U \quad$ and $\quad s = -H$



using $v = u + at$

$-2U = U - gT$

$3U = gT$

$T = \dfrac{3U}{g}$

using $s = ut + \dfrac{1}{2}at^2$

$-H = UT - \dfrac{1}{2}gT^2$

$H + UT - \dfrac{1}{2}gT^2 = 0$

$T = \dfrac{3U}{g}$ and $H + UT - \dfrac{1}{2}gT^2 = 0$

Answer A.

Question 27

$x = 3\log_e\left(1+2e^{-3t}\right)-6t$

$v(t) = \frac{dx}{dt} = \frac{-18e^{-3t}}{1+2e^{-3t}} - 6$

$v(0) = \frac{-18}{3} - 6 = -12$

Answer D.

Question 28

$\dot{\underset{\sim}{r}}(t) = 2\sin(2t)\underset{\sim}{i} + 2\cos(2t)\underset{\sim}{j}$ integrating

$\underset{\sim}{r}(t) = -\cos(2t)\underset{\sim}{i} + \sin(2t)\underset{\sim}{j} + \underset{\sim}{c}$ to find $\underset{\sim}{c}$

$\underset{\sim}{r}(0) = -\underset{\sim}{i} + \underset{\sim}{c} = \underset{\sim}{i} + 2\underset{\sim}{j}$ so $\underset{\sim}{c} = 2\underset{\sim}{i} + 2\underset{\sim}{j}$

$\underset{\sim}{r}(t) = \left(2-\cos(2t)\right)\underset{\sim}{i} + \left(2+\sin(2t)\right)\underset{\sim}{j}$

$x = 2-\cos(2t)$ $y = 2+\sin(2t)$ are the parametric equations

$\cos(2t) = 2-x$ and $\sin(2t) = y-2$ eliminating t

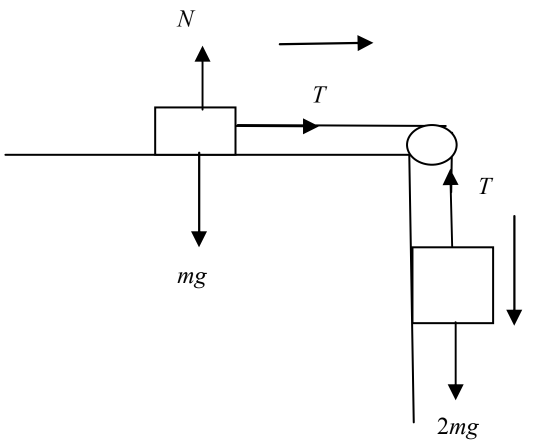
$\cos^2(2t)+\sin^2(2t) = 1$

$(x-2)^2 + (y-2)^2 = 1$

The particle moves on a circle with centre at $(2,2)$ and radius 1

Answer A.

Question 29



resolving using Newton's 2nd law

(1) $2mg - T = 2ma$

(2) $T = ma$ substituting into (1)

$2mg - ma = 2ma$

$3ma = 2mg$

$a = \frac{2g}{3}$

Answer B.

Question 30

All of **A. B. C.** and **D.** are incorrect, they all use constant acceleration formulae, when the acceleration is not constant $a = 2x$

$$a = \frac{d\left(\frac{1}{2}v^2\right)}{dx} = 2x$$

$\frac{1}{2}v^2 = \int 2x\, dx = x^2 + C_1$ now when $t = 0$ $v = 2$ $x = \sqrt{2}$

$2 = 2 + C_1$ $C_1 = 0$

$v^2 = 2x^2$

$v = \pm\sqrt{2}\,x$ since $v > 0$ when $x > 0$ take $+$

$$v = \frac{dx}{dt} = \sqrt{2}\,x$$

$$\int \frac{dx}{x} = \int \sqrt{2}\,.dt$$

$\log_e(x) = \sqrt{2}\,t + C_2$ now when $t = 0$ $x = \sqrt{2}$

$C_2 = \log_e\left(\sqrt{2}\right)$

$\log_e(x) = \sqrt{2}\,t + \log_e\left(\sqrt{2}\right)$

$$\log_e\left(\frac{x}{\sqrt{2}}\right) = \sqrt{2}\,t$$

$$\frac{x}{\sqrt{2}} = e^{\sqrt{2}\,t}$$

$x = \sqrt{2}\,e^{\sqrt{2}t}$

Answer E.

Question 1

$y = Ax\sin(3x)$ differentiating using the product rule gives

$\frac{dy}{dx} = A\sin(3x) + 3Ax\cos(3x)$ differentiating again using the product rule on the last term

$\frac{d^2y}{dx^2} = 3A\cos(3x) + 3A\cos(3x) - 9Ax\sin(3x)$

$\frac{d^2y}{dx^2} = 6A\cos(3x) - 9Ax\sin(3x)$ substituting

$\frac{d^2y}{dx^2} + 9y = 6A\cos(3x) - 9Ax\sin(3x) + 9Ax\sin(3x) = 6A\cos(3x) = 12\cos(3x)$

so that $6A = 12$

$A = 2$

2 marks

Question 2

a. Let $y = \text{Cos}^{-1}(x^2) = \text{Cos}^{-1}(u)$ where $u = x^2$

$\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$ and $\frac{du}{dx} = 2x$ using the Chain Rule

$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{-2x}{\sqrt{1-x^4}}$

2 marks

b. Since $\frac{d}{dx}\left(\text{Cos}^{-1}(x^2)\right) = \frac{-2x}{\sqrt{1-x^4}}$ it follows that

$$\int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{1-x^4}} = -\frac{1}{2}\left[\text{Cos}^{-1}(x^2)\right]_0^{\frac{1}{\sqrt{2}}}$$

$$\int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{1-x^4}} = -\frac{1}{2}\left[\text{Cos}^{-1}\left(\frac{1}{2}\right) - \text{Cos}^{-1}(0)\right] = -\frac{1}{2}\left[\frac{\pi}{3} - \frac{\pi}{2}\right] = \frac{\pi}{12}$$

2 marks

Question 3

a. $\underset{\sim}{a} = -5\underset{\sim}{i} + 2\underset{\sim}{j} + z\underset{\sim}{k}$ the length of $\underset{\sim}{a}$ is $\left|\underset{\sim}{a}\right| = \sqrt{25+4+z^2} = 6$

$\sqrt{29+z^2} = 6$ squaring both sides gives

$29+z^2 = 36$

$z^2 = 7$

$z = \pm\sqrt{7}$ and both answers are acceptable.

2 marks

b. The vector $\underset{\sim}{a}$ makes an angle of $\chi = \text{Cos}^{-1}\left(-\frac{\sqrt{7}}{6}\right)$ with the z-axis

$\cos(\chi) = \frac{z}{\left|\underset{\sim}{a}\right|} = \frac{z}{\sqrt{29+z^2}} = -\frac{\sqrt{7}}{6}$

$6z = -\sqrt{7}\sqrt{29+z^2}$ squaring both sides gives

$7\left(29+z^2\right) = 36z^2$

$7\text{x}29 + 7z^2 = 36z^2$

$29z^2 = 7\text{x}29$

$z^2 = 7$

$z = \pm\sqrt{7}$ however since the angle χ is obtuse we require $z < 0$

$z = -\sqrt{7}$ is the only answer.

2 marks

Question 4

a. $\frac{u}{v} = \frac{3-ki}{k-2i}$ multiplying by the conjugate of v

$\frac{u}{v} = \frac{3-ki}{k-2i} \text{ x } \frac{k+2i}{k+2i} = \frac{3k+6i-k^2i-2ki^2}{k^2-4i^2} = \frac{3k+2k+\left(6-k^2\right)i}{k^2+4}$

$\frac{u}{v} = \frac{5k}{k^2+4} + \frac{\left(6-k^2\right)i}{k^2+4}$ if $\frac{u}{v}$ is real we require the imaginary part to be zero

$6-k^2 = 0$ so that

$k = \pm\sqrt{6}$

2 marks

b. $S = \{ z : |z-u| = |z-v| \}$ substituting for u and v gives

$|z-(3-ki)| = |z-(k-2i)|$ with $z = x+iy$

$|(x-3)+(y+k)i| = |(x-k)+(y+2)i|$ from the definition

$\sqrt{(x-3)^2+(y+k)^2} = \sqrt{(x-k)^2+(y+2)^2}$ squaring and expanding both sides

$x^2-6x+9+y^2+2ky+k^2 = x^2-2kx+k^2+y^2+4y+4$ simplifying

$-6x+9+2ky = -2kx+4y+4$

$(2k-6)x+(2k-4)y+5=0$ comparing this with T

$T = \{ z : 6\text{Re}(z)+8\text{Im}(z)+5=0 \}$ since

$z = x+yi$ $\text{Re}(z)=x$ and $\text{Im}(z)=y$ it follows that

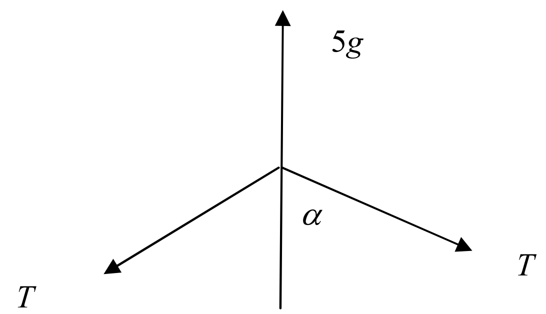
$2k-6=6$ and $2k-4=8$ so that

$k=6$

2 marks

Question 5

a. Let T be the tensions in the strings, by the symmetry both tension are equal. The reaction of the hook is the normal which is equal and opposite to the weight force of the painting. The situation is



since $\tan(\alpha) = \frac{45}{15} = 3$ from Pythagorus $\cos(\alpha) = \frac{1}{\sqrt{10}}$

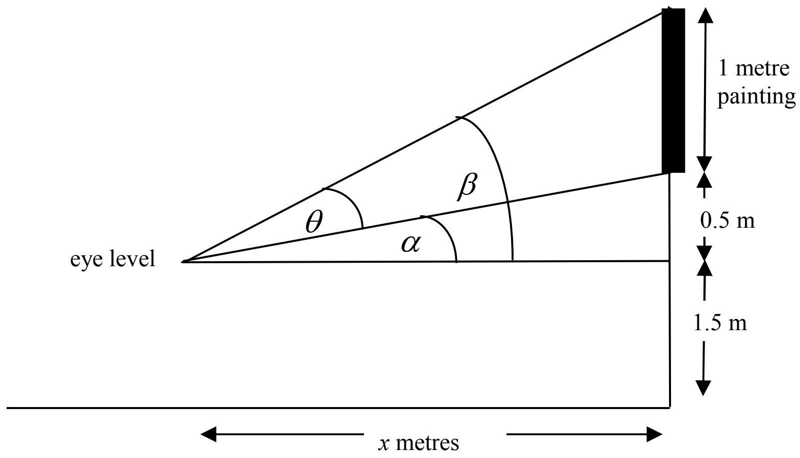
resolving vertically gives

$2T\cos(\alpha) = 5g$

$T = \frac{5g}{2\cos(\alpha)} = \frac{5 \text{ x } 9.8}{2 \text{ x} \frac{1}{\sqrt{10}}} = \frac{5\sqrt{10}\,g}{2} = 77.48$ newtons

2 marks

b.



In the diagram let $\tan(\beta) = \frac{1.5}{x} = \frac{3}{2x}$ and $\tan(\alpha) = \frac{0.5}{x} = \frac{1}{2x}$ and $\theta = \beta - \alpha$

$$\tan(\theta) = \tan(\beta - \alpha) = \frac{\tan(\beta) - \tan(\alpha)}{1 + \tan(\beta)\tan(\alpha)}$$

$$\tan(\theta) = \frac{\frac{3}{2x} - \frac{1}{2x}}{1 + \frac{1}{2x}.\frac{3}{2x}} = \frac{\frac{2}{2x}}{1 + \frac{3}{4x^2}} = \frac{1}{x}.\frac{4x^2}{(4x^2+3)} = \frac{4x}{3+4x^2}$$

1 mark

c. For maximum viewing angle $\frac{d}{dx}(\tan\theta) = \frac{d}{dx}\left(\frac{4x}{3+4x^2}\right) = 0$

differentiating using the quotient rule

$$\frac{d}{dx}(\tan\theta) = \frac{4(3+4x^2) - 8x.4x}{(3+4x^2)^2} = \frac{4\left[3+4x^2-8x^2\right]}{(3+4x^2)^2} = 0$$

$$\frac{d}{dx}(\tan\theta) = \frac{4(3-4x^2)}{(3+4x^2)^2} = 0$$

so $3 = 4x^2 \quad x^2 = \frac{3}{4} \quad x = \frac{\sqrt{3}}{2}$ since $x > 0$

$$\tan\theta = \frac{4.\frac{\sqrt{3}}{2}}{3+3} = \frac{\sqrt{3}}{3} \qquad \text{so} \qquad \theta = \text{Tan}^{-1}\left(\frac{\sqrt{3}}{3}\right) = 30^0$$

3 marks

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