Victorian Certificate of Education
2004

SPECIALIST MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 1 November 2004

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I
MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 18 pages, with a detachable sheet of miscellaneous formulas in the centrefold and a blank page for rough working.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graph of $y = \dfrac{-x^2+1}{2x}$ has

A. no straight line asymptotes.

B. $y = 2x$ as its only straight line asymptote.

C. $x = 0$ as its only straight line asymptote.

D. $y = 0$ and $y = -\dfrac{1}{2}x$ as its only straight line asymptotes.

E. $x = 0$ and $y = -\dfrac{1}{2}x$ as its only straight line asymptotes.

Question 2

The x-axis is tangent to an ellipse at the point (1, 0) and the y-axis is tangent to the same ellipse at the point (0, –2).

Which one of the following could be the equation of this ellipse?

A. $\dfrac{(x-1)^2}{4} + (y+2)^2 = 1$

B. $\dfrac{(x+1)^2}{4} + (y-2)^2 = 1$

C. $(x-1)^2 + \dfrac{(y+2)^2}{4} = 1$

D. $(x+1)^2 + \dfrac{(y-2)^2}{4} = 1$

E. $(x-2)^2 + \dfrac{(y+1)^2}{4} = 1$

Question 3

Which one of the following is **not** equal to $\tan\left(\frac{\pi}{5}\right)$?

A. $\dfrac{\sin\left(\frac{\pi}{5}\right)}{\cos\left(\frac{\pi}{5}\right)}$

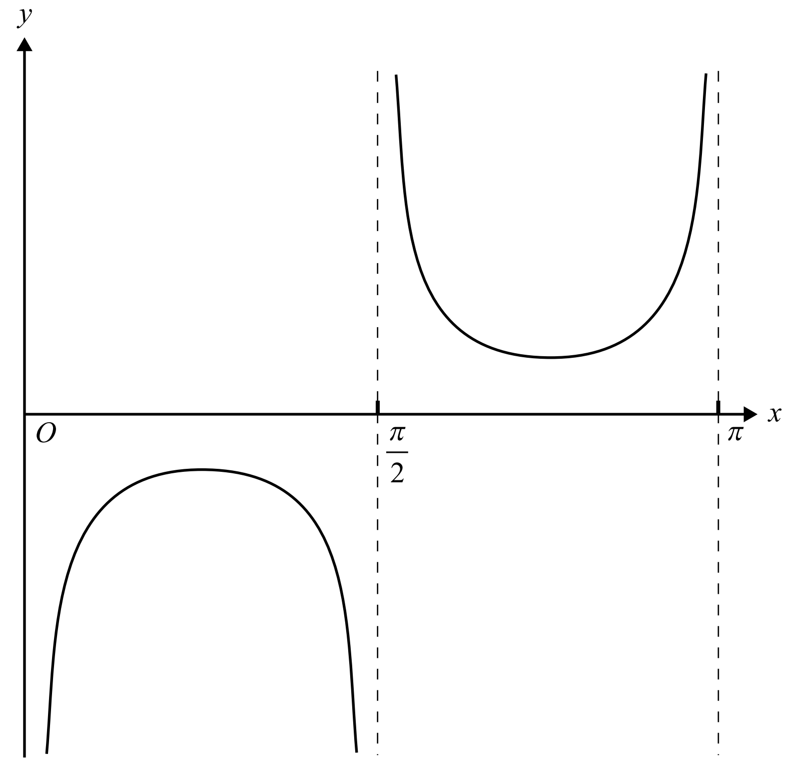
B. $\dfrac{1}{\cot\left(\frac{\pi}{5}\right)}$

C. $\cot\left(\frac{3\pi}{10}\right)$

D. $\dfrac{2\tan\left(\frac{\pi}{10}\right)}{1-\tan^2\left(\frac{\pi}{10}\right)}$

E. $\dfrac{2\tan\left(\frac{2\pi}{5}\right)}{1-\tan^2\left(\frac{2\pi}{5}\right)}$

Question 4



The graph of $y = -\sec(a(x - b))$ is shown above for $0 \le x \le \pi$.
The values of a and b could be

A. $a = 1, b = \frac{\pi}{2}$

B. $a = 1, b = \frac{\pi}{4}$

C. $a = 2, b = \frac{\pi}{2}$

D. $a = 2, b = \frac{\pi}{4}$

E. $a = 2, b = -\frac{\pi}{4}$

Question 5

If $z = x + yi$, where x and y are non-zero real numbers, which one of the following is a real number?

A. $\frac{1}{z}$

B. $\frac{1}{\overline{z}}$

C. $\frac{1}{z - \overline{z}}$

D. $\frac{1}{z} - \frac{1}{\overline{z}}$

E. $\frac{1}{z} + \frac{1}{\overline{z}}$

Question 6

If $\text{Arg}(1 + ai) = -\frac{\pi}{3}$, then the real number a is

A. $-\frac{\pi}{\sqrt{3}}$

B. $-\frac{\sqrt{3}}{2}$

C. $-\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

E. $\sqrt{3}$

Question 7

$P(z)$ is a polynomial in z of degree 4 with real coefficients.

Which one of the following statements **must** be **false**?

A. $P(z) = 0$ has no real roots.

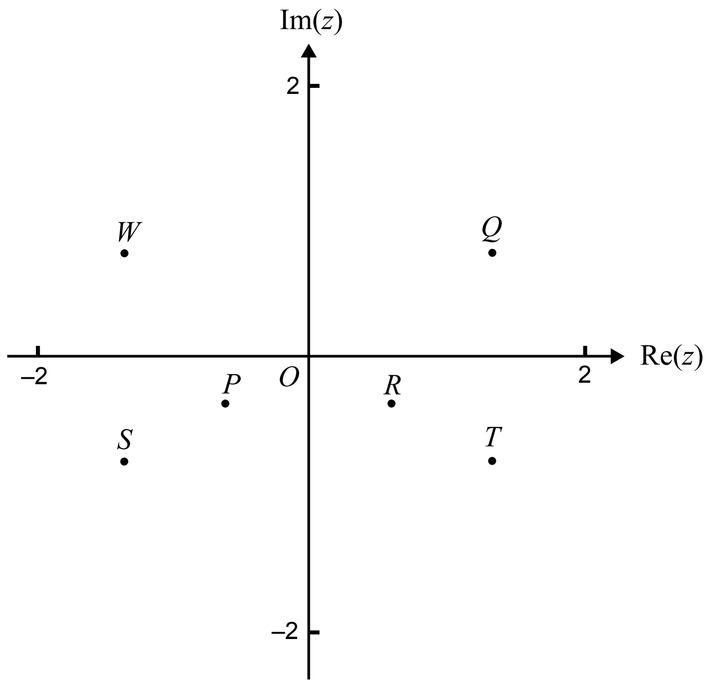
B. $P(z) = 0$ has one real root and three non-real roots.

C. $P(z) = 0$ has one (repeated) real root and two non-real roots.

D. $P(z) = 0$ has two real roots and two non-real roots.

E. $P(z) = 0$ has four real roots.

Question 8



The point W on the Argand diagram above represents a complex number w where $|w| = 1.5$.

The complex number w^{-1} is best represented by the point

A. P

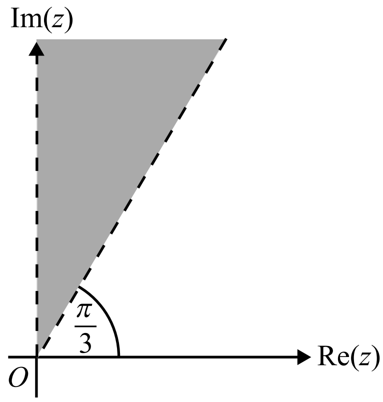
B. Q

C. R

D. S

E. T

Question 9



The shaded region (with boundaries excluded) of the complex plane shown above is best described by

A. $\left\{z\text{: Arg}(z) > \frac{\pi}{3}\right\}$

B. $\left\{z\text{: Arg}(z) > \frac{\pi}{3}\right\} \cup \left\{z\text{: Arg}(z) < \frac{\pi}{2}\right\}$

C. $\left\{z\text{: Arg}(z) > \frac{\pi}{3}\right\} \cap \left\{z\text{: Arg}(z) < \frac{\pi}{2}\right\}$

D. $\left\{z\text{: Arg}(z) > \frac{\pi}{2}\right\} \cup \left\{z\text{: Arg}(z) < \frac{\pi}{3}\right\}$

E. $\left\{z\text{: Arg}(z) > \frac{\pi}{2}\right\} \cap \left\{z\text{: Arg}(z) < \frac{\pi}{3}\right\}$

Question 10

Which one of the following is an antiderivative of $\frac{1}{x^2+16}$?

A. $\log_e\left(x^2+16\right)$

B. $\frac{1}{2x}\log_e\left(x^2+16\right)$

C. $\text{Tan}^{-1}\left(\frac{x}{4}\right)$

D. $\frac{1}{4}\text{Tan}^{-1}\left(\frac{x}{4}\right)$

E. $4\text{Tan}^{-1}\left(\frac{x}{4}\right)$

Question 11

$\int_0^a \left(\sin^2\left(\frac{3x}{2}\right) - \cos^2\left(\frac{3x}{2}\right) \right) dx$ is equal to

A. $-\frac{4}{3}\sin\left(\frac{3a}{4}\right)$

B. $-\frac{1}{3}\sin(3a)$

C. $\frac{1}{3}\sin(3a)$

D. $\frac{1}{3}(1-\sin(3a))$

E. $-\frac{1}{3}(\cos(3a)-1)$

Question 12

With a suitable substitution, $\int_0^{\frac{\pi}{3}} \cos^2(x)\sin^3(x)\,dx$ can be expressed as

A. $\int_{\frac{1}{2}}^{1} u^2\left(1-u^2\right)du$

B. $\int_{1}^{\frac{1}{2}} u^2\left(1-u^2\right)du$

C. $\int_{0}^{\frac{\pi}{3}} u^2\left(1-u^2\right)du$

D. $-\int_{0}^{\frac{\pi}{3}} u^2\left(1-u^2\right)du$

E. $-\int_{0}^{\frac{\sqrt{3}}{2}} u^2\left(1-u^2\right)du$

Question 13

An antiderivative of $\frac{2}{(3-x)^2} - \frac{1}{3-x}$, for $x < 3$, is

A. $\log_e(x-3) - \frac{2}{x-3}$

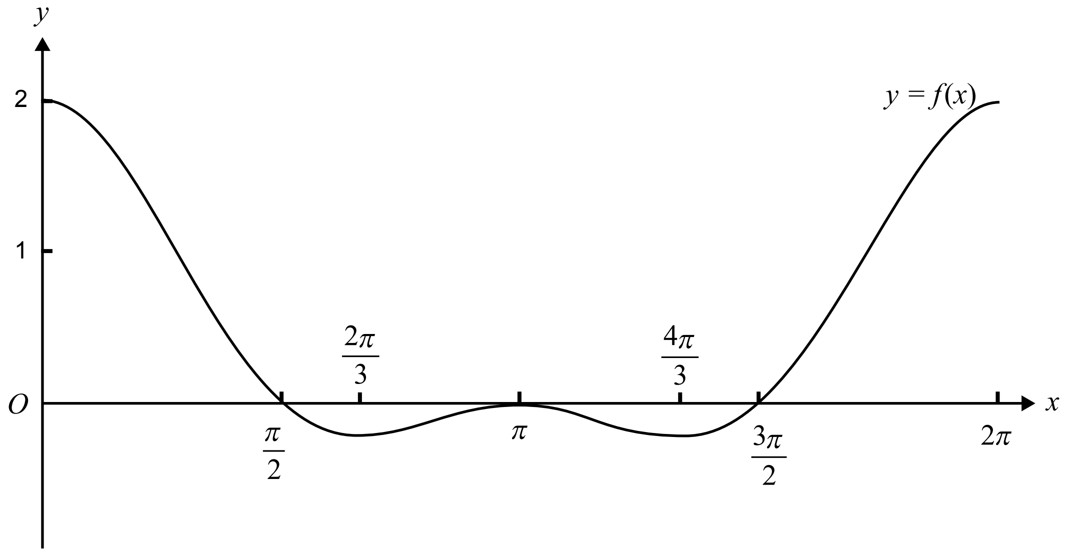
B. $\log_e(x-3) + \frac{2}{x-3}$

C. $\log_e(3-x) - \frac{2}{3-x}$

D. $\log_e(3-x) + \frac{2}{3-x}$

E. $-\log_e(3-x) + \frac{2}{3-x}$

Question 14



The graph of $y = f(x)$ is shown above.
Let $F(x)$ be an antiderivative of $f(x)$.
The stationary points of the graph of $y = F(x)$ could be

A. local maximums at $x = 0$, π and 2π, and local minimums at $x = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$

B. stationary points of inflexion at $x = 0$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$ and 2π, a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$

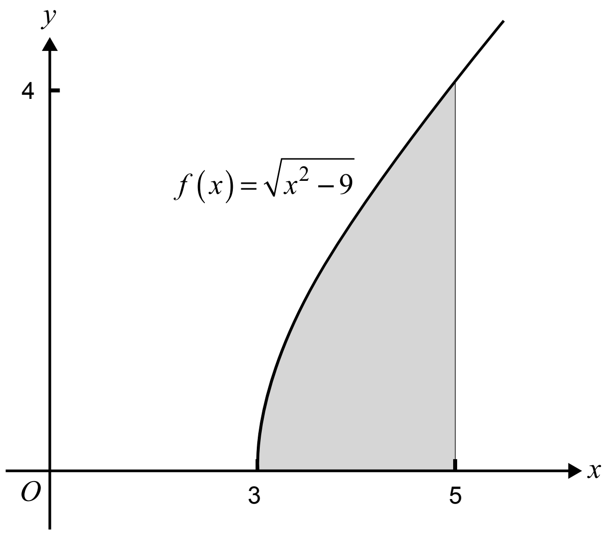
C. stationary points of inflexion at $x = 0$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$ and 2π, a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$

D. a stationary point of inflexion at $x = \pi$, a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$

E. a stationary point of inflexion at $x = \pi$, a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$

The following information relates to Questions 15 and 16.

The graph of $f:[3,\infty)\to R$, where $f(x)=\sqrt{x^2-9}$, is shown below.
The shaded region is bounded by this graph, the x-axis, and the line with equation $x = 5$.



Question 15

The midpoint rule with **two** equal intervals is used to estimate the area of the shaded region.
The value obtained, calculated correct to two decimal places, is

A. 4.65
B. 5.06
C. 5.16
D. 5.29
E. 5.80

Question 16

The shaded region is rotated about the y-axis to form a solid of revolution.
The volume of this solid, in cubic units, is given by

A. $\pi\int_0^4 \left(y^2-9\right)dy$

B. $\pi\int_0^4 \left(34-y^2\right)dy$

C. $\pi\int_0^4 \left(y^2+9\right)dy$

D. $\pi\int_0^4 \left(16-y^2\right)dy$

E. $\pi\int_0^4 \left(5-\sqrt{y^2+9}\right)^2 dy$

Question 17

The vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are given by $\underset{\sim}{p} = 2\underset{\sim}{i} + x\underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{q} = -4\underset{\sim}{i} + y\underset{\sim}{j} - 6\underset{\sim}{k}$, where x and y are real numbers.
The magnitude of vector $\underset{\sim}{p}$ is 4 units, and $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are parallel.
The values of x and y could be

A. $x = \sqrt{3},\ y = -2\sqrt{3}$

B. $x = 3,\ y = -6$

C. $x = \sqrt{3},\ y = 2\sqrt{3}$

D. $x = \sqrt{29},\ y = -\sqrt{29}$

E. $x = -\sqrt{3},\ y = -2\sqrt{3}$

Question 18

The vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are given by $\underset{\sim}{u} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$.

$\underset{\sim}{u} \cdot (\underset{\sim}{u} - 2\underset{\sim}{v})$ is equal to

A. 20

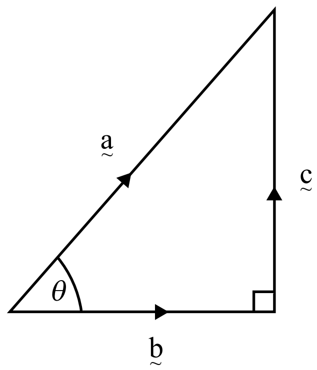
B. 45

C. 52

D. 72

E. 78

Question 19



The right-angled triangle shown above has sides represented by the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.
Which one of the following statements is **false**?

A. $|\underset{\sim}{b}|^2 + |\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2$

B. $\underset{\sim}{b} \cdot (\underset{\sim}{a} - \underset{\sim}{c}) = |\underset{\sim}{b}|^2$

C. $\underset{\sim}{b} \cdot (\underset{\sim}{a} - \underset{\sim}{b}) = |\underset{\sim}{b}||\underset{\sim}{c}|$

D. $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos(\theta)$

E. $\underset{\sim}{a} \cdot \underset{\sim}{c} = |\underset{\sim}{a}||\underset{\sim}{c}|\sin(\theta)$

Question 20

The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = 2\sin(t)\underset{\sim}{i} + \cos(t)\underset{\sim}{j}$, $0 \le t \le \pi$.
The Cartesian equation of the path of the particle is

A. $y = \cos\left(\text{Sin}^{-1}\left(\frac{x}{2}\right)\right), \quad 0 \le x \le 2$

B. $y = \sqrt{1 - \frac{x^2}{4}}, \quad 0 \le x \le 2$

C. $\frac{x^2}{2} + y^2 = 1, \quad -2 \le x \le 2$

D. $\frac{x^2}{4} + y^2 = 1, \quad -2 \le x \le 2$

E. $\frac{x^2}{4} + y^2 = 1, \quad 0 \le x \le 2$

Question 21

The velocity of a particle at time t, $t \geq 0$, is given by $\dot{\underset{\sim}{r}}(t) = 3\sin(2t)\underset{\sim}{i} + 4\underset{\sim}{j}$.

The initial position of the particle is given by $\underset{\sim}{r}(0) = \frac{3}{2}\underset{\sim}{i}$.

The position vector $\underset{\sim}{r}(t)$ of the particle at time t is equal to

A. $-\frac{3}{2}\cos(2t)\underset{\sim}{i} + 4t\underset{\sim}{j}$

B. $\left(3 - \frac{3}{2}\cos(2t)\right)\underset{\sim}{i} + 4t\underset{\sim}{j}$

C. $\left(\frac{3}{2} - \frac{3}{2}\cos(2t)\right)\underset{\sim}{i} + 4t\underset{\sim}{j}$

D. $\frac{3}{2}\cos(2t)\underset{\sim}{i} + 4t\underset{\sim}{j}$

E. $\left(3 - \frac{3}{2}\cos(2t)\right)\underset{\sim}{i}$

Question 22

A body of mass 5 kg slides from rest down a smooth plane inclined at an angle of 30° to the horizontal. The acceleration, in m/s^2, of the body down the plane has magnitude

A. $\frac{\sqrt{3}g}{2}$

B. $\frac{g}{2}$

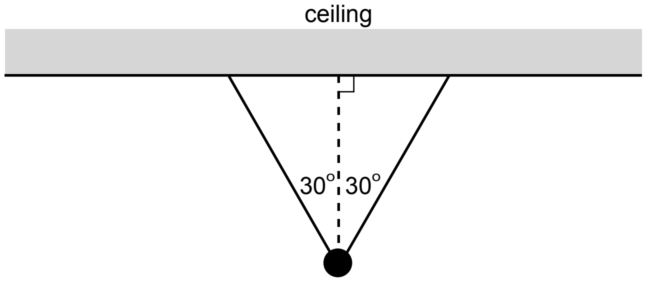
C. 0

D. $\frac{5\sqrt{3}g}{2}$

E. $\frac{5g}{2}$

Question 23

A 10 kg mass is suspended in equilibrium from a horizontal ceiling by two identical light strings. Each string makes an angle of 30° with the vertical as shown below.



The magnitude, in newtons, of the tension in each string is equal to

A. $5g$

B. $10g$

C. $20g$

D. $\dfrac{10g\sqrt{3}}{3}$

E. $\dfrac{20g\sqrt{3}}{3}$

Question 24

A body of mass 5 kg is acted upon by three concurrent coplanar forces $\underset{\sim}{R}$, $\underset{\sim}{S}$ and $\underset{\sim}{T}$, where $\underset{\sim}{R} = 2\underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{S} = \underset{\sim}{i} + 10\underset{\sim}{j}$ and $\underset{\sim}{T} = 3\underset{\sim}{i} - 3\underset{\sim}{j}$. The forces are measured in newtons.

The magnitude of the acceleration of the body, in m/s^2, is

A. 2

B. 4

C. 6

D. 8

E. 10

Question 25

A balloon is rising vertically at a constant speed of 21 metres per second. A stone is dropped from the balloon when it is h metres above the ground. The stone strikes the ground 10 seconds later.

Assuming that air resistance is negligible, the value of h is

A. 210

B. 280

C. 490

D. 700

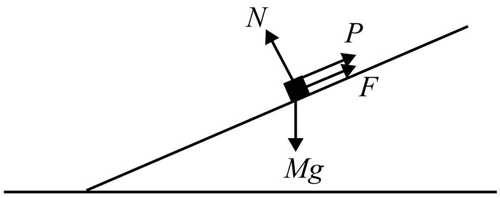
E. 770

Question 26

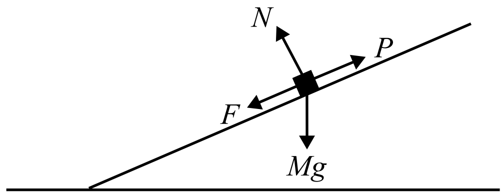
A body of mass M kg is on a rough plane inclined at an angle to the horizontal. The body, which is on the point of sliding down the plane, is held in equilibrium by a force of magnitude P applied parallel to the plane. There is a normal reaction of magnitude N and a frictional force of magnitude F. All forces are measured in newtons.

Which one of the following diagrams shows the forces acting on the body?

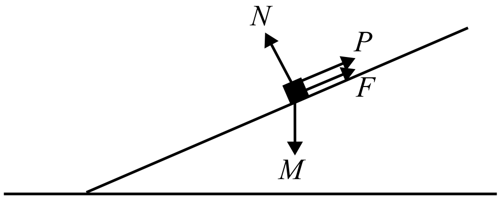
A.



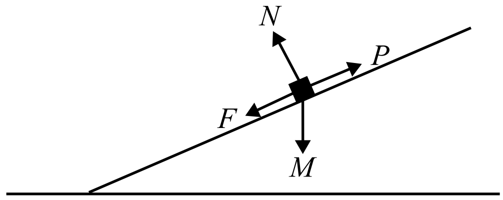
B.



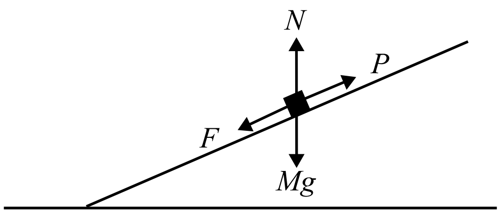
C.



D.



E.



Question 27

A particle is moving in a straight line in such a way that its displacement, x metres, from a fixed origin at time t seconds is given by $x = 2.5t + 9\cos\left(\frac{t}{2}\right),\ t \geq 0$.

If the velocity of the particle at time t seconds is v metres per second, then the minimum value of v is

A. –6.5

B. –2

C. 0

D. 2.5

E. 7

Question 28

Which one of the following differential equations is satisfied by $y = \sin(2x)$?

A. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 4\cos(2x)$

B. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 4y = 4\cos(2x)$

C. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 4\cos(2x)$

D. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 4y = 4\cos(2x)$

E. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = 4\cos(2x)$

Question 29

A jug of water at a temperature of 20°C is placed in a refrigerator. The temperature inside the refrigerator is maintained at 4°C.

When the jug has been in the refrigerator for t minutes, the temperature of the water in the jug is y°C. The rate at which the water's temperature decreases is proportional to the excess of its temperature over the temperature inside the refrigerator.

If k is a positive constant, a differential equation involving y and t is

A. $\frac{dy}{dt} = -k(y-20); \quad t = 0, \ y = 4$

B. $\frac{dy}{dt} = -k(y+4); \quad t = 0, \ y = 20$

C. $\frac{dy}{dt} = -k(y-4); \quad t = 0, \ y = 16$

D. $\frac{dy}{dt} = -k(y+4); \quad t = 0, \ y = 24$

E. $\frac{dy}{dt} = -k(y-4); \quad t = 0, \ y = 20$

Question 30

A particle moves in a straight line. When its displacement from a fixed origin is x m, its velocity is v m/s and its acceleration is a m/s^2.

Given that $a = 16x$, and that $v = -5$ when $x = 0$, the relation between v and x is

A. $v = -4x - 5$

B. $v = 8x^2 - 5$

C. $v = -\sqrt{25 + 16x^2}$

D. $v = \sqrt{25 + 16x^2}$

E. $v = -\sqrt{25 + 32x^2}$

Working space

END OF PART I MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2004

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 1 November 2004

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II
QUESTION AND ANSWER BOOK

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Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
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- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 6 pages.

Instructions
- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part II

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

a. Show that, for $0 < x < \frac{1}{2}$, $\frac{d}{dx}\left(\text{Sin}^{-1}\left(\sqrt{2x}\right)\right) = \frac{1}{\sqrt{2x(1-2x)}}$.

2 marks

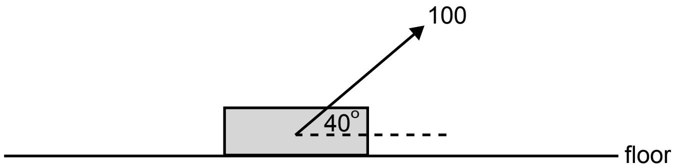
b. **Hence** find the exact value of $\int_{\frac{1}{8}}^{\frac{1}{4}} \frac{1}{\sqrt{2x(1-2x)}}\,dx$.

2 marks

Question 2

A 20 kg crate is pulled across a rough horizontal floor by a force, of magnitude 100 newtons, applied upwards at an angle of 40° to the horizontal.

a. Complete the following diagram so that it shows **all** the forces acting on the crate.



1 mark

b. The coefficient of friction between the crate and the floor is 0.34.

Find the acceleration of the crate, in m/s^2, correct to two decimal places.

4 marks

Question 3

If $f'(x) = 15x\sqrt{2-x}$ and $f(2) = 0$, then $f(x)$ can be written in the form $(ax+b)(2-x)^{\frac{3}{2}}$.

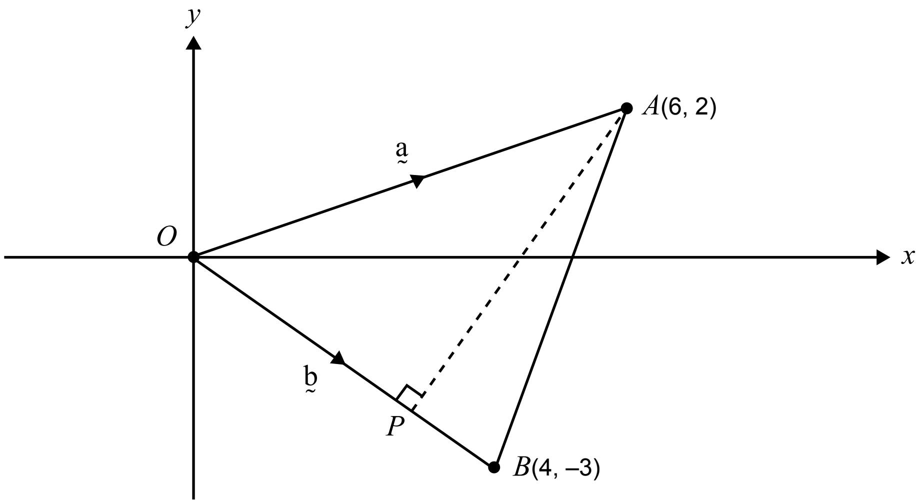
Find the values of a and b.

4 marks

Question 4

Points $O(0, 0)$, $A(6, 2)$ and $B(4, -3)$ form the vertices of a triangle as shown in the diagram below.

The position vectors $\overrightarrow{OA} = \underset{\sim}{a} = 6\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\overrightarrow{OB} = \underset{\sim}{b} = 4\underset{\sim}{i} - 3\underset{\sim}{j}$ are indicated. AP is an altitude of triangle OAB.



a. Find the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$.

2 marks

b. **Hence** find the length of the altitude AP.

2 marks

Question 5

Let $w = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$.

a. Express w in **exact** polar form.

1 mark

b. **Hence** find the least positive integer k for which $w^k = 1$.

2 marks

END OF PART II QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ | $\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ | $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$ | $\int \frac{1}{x} dx = \log_e(x) + c$, for $x > 0$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ | $\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ | $\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ | $\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\text{Sin}^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ | $\int \frac{1}{\sqrt{a^2-x^2}} dx = \text{Sin}^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}\left(\text{Cos}^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$ | $\int \frac{-1}{\sqrt{a^2-x^2}} dx = \text{Cos}^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}\left(\text{Tan}^{-1}(x)\right) = \frac{1}{1+x^2}$ | $\int \frac{a}{a^2+x^2} dx = \text{Tan}^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)\,dx \approx (b-a) f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)\,dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET