2004 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The number of students who sat for the 2004 examination was 6159, 73 less than in 2003 but 66 more than in 2002. As in previous years, students were required to answer 30 multiple-choice questions in Part I, and five questions worth a total of 20 marks in Part II.

Students' overall performance on the 2004 examination was better than that in 2003, and very similar to the performance on the 2002 examination. The mean score for Part I was 19.5 out of 30, up from 17.5 in 2003 but slightly lower than the 19.6 mean in 2002. Only four questions were answered correctly by less than 50% of students, which was the same as in 2002 and down from seven questions in 2003. For Part II, the mean score was 10.8 out of 20, up from 10.3 in 2003 and 10.7 in 2002. Three out of nine question parts had a mean score of less than 50% of the maximum possible, compared to six out of nine in 2003 and two out of seven in 2002.

The overall mean and median scores were 31.0 and 32.5 out of 50, compared to 28.1 and 28 in 2003 and 29.5 and 30 in 2002. This is the first time that either the mean or the median has exceeded 60% of the maximum score. About six per cent of students scored less than 25% of the available marks, compared to seven per cent in 2003 and six per cent in 2002. Eight students received only two marks out of 50, which was the lowest score obtained, compared to a lowest score of three which was obtained by 10 students in 2003. Very high scores were significantly more common than in the past. About 10% of the students scored more than 90% of the available marks (compared to four per cent in 2003 and six per cent in 2002), and 40 students scored full marks, compared to 12 in 2003 and eight in 2002.

As usual, the quality of students' responses in Part II varied widely, with many students giving exemplary answers to most or all of the questions, while others seemed out of their depth and were only able to give a few correct answers. The better students set out their work in a clear and logical manner and included all the necessary steps. Students who skipped steps often made errors.

Once again it was clear that many students were severely hampered by poor algebraic skills. This was most evident in Question 3. Also, sloppy notation continued to be all too common. For example, tildes were often missing from vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ and the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ in Question 4; and *dx* was often omitted in antidifferentiations and integration.

There is a continuing need to ensure that students understand and respond to instructions, particularly when asked to provide **exact** answers, use **calculus**, or **show working** in questions where more than one mark is available. For example, students who gave the correct answer to Questions 1b, 4a, 4b or 5b but did not provide any working received no marks for that question part. The meaning of 'hence' in a question statement should also be emphasised.

Finally, students should be reminded that in questions where they have to **show** a given result (for example, Question 1a), the onus is on them to include enough working to make it clear that they know how to derive the result, whether or not it is given.

SPECIFIC INFORMATION

Part 1 – Multiple-choice

The table below indicates the number of students who chose each option. The correct answer is indicated by shading.

	A	B	C	D	E	No Answer
Question	**%**	**%**	**%**	**%**	**%**	**%**
1	10	1	7	8	73	0
2	9	3	81	5	2	0
3	1	2	11	6	80	0
4	5	4	9	73	9	0
5	2	4	8	9	77	0
6	2	9	80	4	5	0
7	17	48	9	15	11	0
8	53	6	10	14	16	0
9	2	19	73	3	4	0
10	3	5	5	84	4	0
11	5	64	14	10	7	1
12	59	19	6	5	10	1
13	4	5	24	53	14	0
14	5	10	7	69	8	1
15	4	8	77	9	1	0
16	9	4	62	16	9	0
17	75	5	11	2	6	1
18	2	3	9	82	4	0
19	2	16	50	3	28	0
20	11	5	5	28	51	1
21	11	70	12	5	2	0
22	7	65	2	10	16	0
23	9	7	4	67	12	1
24	64	8	10	7	11	1
25	14	46	21	16	3	1
26	70	23	3	2	2	0
27	4	59	10	22	4	1
28	2	11	76	5	5	1
29	4	12	9	4	69	1
30	5	22	42	24	6	1

The mean score was 19.5 and the standard deviation was 6.4. Four questions (Questions 7, 16, 25 and 30) were answered correctly by less than 50% of students, but only Question 16, which was on the volume of a solid of revolution, was answered correctly by less than 40% of students. In general, students performed consistently across the various Areas of Study. Male and female mean scores were similar on most questions, but there was a marked difference on Question 25 (motion under gravity), which was answered correctly by 51% of male students, but by only 38% of female students.

Question 7 was the first question that was answered incorrectly by a majority of students, with only 48% of students answering correctly. As can be seen in the table above, each of the four options was chosen by a significant proportion of the 52% of students who gave an incorrect response. It is likely that many of these students guessed the answer rather than giving responses based on any particular misconception.

Question 16 was the worst-answered question, with only 16% of students responding correctly. This was surprising, given that it could be regarded as a routine application of calculus. The majority of students, 63%, chose option C which was the ‘interior’ (though it is actually open-ended) of the solid of revolution formed by rotating the shaded region about the y-axis. In no other question was an incorrect option anywhere near as popular as the correct answer.

Question 25, which was on motion under gravity, was answered correctly by 46% of students. Option C, which took the initial velocity of the stone to be zero, was chosen by 22% of students and was the most popular incorrect option.

Options D (which used the incorrect sign for the initial velocity) and A (zero acceleration) were also chosen by significant proportions of students (16% and 14% respectively).

Question 30 was the second worst-answered question and the final one that was not answered correctly by a majority of students, with 58% of students answering incorrectly. However, considering this was a difficult question, students performed reasonably well. As well as requiring knowledge of the various forms for expressing acceleration, the negative direction of motion gives this question an extra dimension. Option D (which gave the incorrect sign) was chosen by 24% of students and option B (which took acceleration as $\frac{dv}{dx}$) was chosen by 22% of students.

Five other questions deserve comment because of the unusually high popularity of one of the incorrect options, even though they were answered correctly by a majority of students. In Question 13, 24% of students chose option C. This expression gave the incorrect sign for the second term, which was obtained by ignoring the fact that $\frac{du}{dx} = -1$ when $u = 3 - x$ (although this was accounted for in the first term of the expression).

In Question 19, 28% of students chose option E, which indicated that they did not recognise that $\cos(\frac{\pi}{2} - \theta) = \sin\theta$.

In Question 20, 29% of students chose option D, which gave the correct equation but an incorrect domain, since $0 \le t \le \pi$ implies that $0 \le x \le 2$.

In Question 26, 23% of students chose option B, overlooking the fact that the body is on the point of sliding **down** the plane.

Finally, in Question 27, 22% of students chose option D, which was obtained by putting $t = 0$ in the expression for velocity to 'find' its minimum value – a common type of mistake noted over the years that clearly requires attention from teachers and students.

Part II – Short answer

Question 1

1a

Marks	0	1	2	Average
%	37	11	52	1.2

Use the chain rule, with $u = \sqrt{2x}$.

Many students just wrote $\frac{d}{dx}\left(\text{Sin}^{-1}(\sqrt{2x}\right) = \left(\frac{1}{\sqrt{2x}}\right)\left(\frac{1}{\sqrt{1-2x}}\right)$. This meant that it was unclear whether they had correctly applied the chain rule to differentiate $\text{Sin}^{-1}\left(\sqrt{2x}\right)$, or whether they had 'fudged' the answer by 'factorising' $\frac{1}{\sqrt{2x(1-2x)}}$. A common mistake was to work with $\left(\sqrt{2}\right)x$ instead of with $\sqrt{(2x)}$.

1b

Marks	0	1	2	Average
%	22	19	59	1.5

$\frac{\pi}{12}$

Most students recognised that $\text{Sin}^{-1}\left(\sqrt{2x}\right)$ is an antiderivative of the integrand. However, many students could get no further than $\text{Sin}^{-1}\left(\sqrt{\frac{1}{2}}\right)-\text{Sin}^{-1}\left(\sqrt{\frac{1}{4}}\right)$ or $\text{Sin}^{-1}\left(\frac{1}{\sqrt{2}}\right)-\text{Sin}^{-1}\left(\frac{1}{2}\right)$, while others once again misinterpreted $\sqrt{2x}$ as $\left(\sqrt{2}\right)x$.

Question 2

2a

Marks	0	1	Average
%	17	83	**0.9**

Weight force ($20g$ N) vertically down, normal reaction force (N N) vertically up, and friction force (F N) horizontally left.

This question was reasonably well done; however, some students seemed to think that the horizontal and vector resolutes of the applied force (100 N) were also acting on the crate.

2b

Marks	0	1	2	3	4	Average
%	22	6	19	7	46	**2.7**

1.59 m/s^2

The most common error was to omit the vertical component (100 sin 40°) of the applied force when resolving forces, which gave $N = 20g$ and, eventually, $a = 0.50$. It was disturbing to note that many students used equations (presumably from their pre-written notes) that applied to a different situation (for example, a body being pulled up an inclined plane).

Question 3

Marks	0	1	2	3	4	Average
%	39	5	18	17	21	**1.9**

$a = –6$, $b = –8$

The vast majority of students tackled this question by antidifferentiating $15x\sqrt{2-x}$, although a few took the alternative (and possibly easier) approach of differentiating $(ax+b)(2-x)^{\frac{3}{2}}$.

The two most common errors were to use $\frac{du}{dx}=1$ when substituting $u = 3-x$ (or to omit it altogether), and to not make allowance for a constant of integration (even though it turned out to be zero). Some students antidifferentiated 'in parts', obtaining $\left(\frac{15x^2}{2}\right)\left(-\frac{2}{3}(2-x)^{\frac{3}{2}}\right)$, while many students who obtained the correct antiderivative, $6(2-x)^{\frac{5}{2}}-20(2-x)^{\frac{3}{2}}$, had difficulty trying to express it in the form $(ax+b)(2-x)^{\frac{3}{2}}$.

Question 4

4a

Marks	0	1	2	Average
%	20	15	65	**1.6**

$\frac{18}{5}$

This was the second best-answered question (after Question 2a) in Part II. The most common error was to find the **vector** resolute instead of the scalar resolute. Some students evaluated the correct scalar resolute expression, $(6\underset{\sim}{i}+2\underset{\sim}{j})\cdot(\frac{4}{5}\underset{\sim}{i}-\frac{3}{5}\underset{\sim}{j})$, as the **vector** $(\frac{24}{5}\underset{\sim}{i}-\frac{6}{5}\underset{\sim}{j})$.

4b

Marks	0	1	2	Average
%	51	17	32	**0.9**

$AP^2 = OA^2 - OP^2 = 40 - \left(\frac{18}{5}\right)^2$, since *OP* is the scalar resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$. So *AP* = 5.2.

Less than half of the students recognised that the answer to part a was the length *OP*. Many of these students did not then take the easy route and apply Pythagoras' theorem in Δ*OAP*, instead using it to find $\overrightarrow{OP}$, then $\overrightarrow{AP}$ and *AP*. Other students proceeded similarly after finding $\overrightarrow{OP}$ as the vector resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$ from scratch. Some students left their answer as the vector $\overrightarrow{AP} = -\frac{78}{25}\underset{\sim}{i} - \frac{104}{25}\underset{\sim}{j}$.

Question 5

5a

Marks	0	1	Average
%	43	57	**0.6**

$w = \text{cis}\left(-\frac{5\pi}{6}\right)$, or $w = \text{cis}\left(\frac{7\pi}{6}\right)$, etc.

The most common error was to have the argument in the wrong quadrant; for example, $w = \text{cis}\left(-\frac{\pi}{6}\right)$ and $w = \text{cis}\left(\frac{5\pi}{6}\right)$. When finding the polar form of a complex number, students should be encouraged to do a rough plot of the number to locate the correct quadrant for the argument.

5b

Marks	0	1	2	Average
%	68	18	14	**0.5**

$k = 12$

Most students made little real progress in this question. Many of those who did know what they were doing either ignored, or didn't understand, the requirement that *k* be an **integer**, and gave $\frac{12}{5}$ or $\frac{12}{7}$ as their answer.