Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse:	$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$
hyperbola:	$\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2 x+\sin^2 x=1$

$1+\tan^2 x=\sec^2 x$ 　　　　　　　　$\cot^2 x+1=\operatorname{cosec}^2 x$

$\sin(x+y)=\sin x\cos y+\cos x\sin y$ 　　　$\sin(x-y)=\sin x\cos y-\cos x\sin y$

$\cos(x+y)=\cos x\cos y-\sin x\sin y$ 　　　$\cos(x-y)=\cos x\cos y+\sin x\sin y$

$\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$ 　　　　$\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}$

$\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$

$\sin 2x=2\sin x\cos x$ 　　　　　　　$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1,1]$	$[-1,1]$	R
range	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ 　　　　　　　$-\pi<\operatorname{Arg} z\le\pi$

$z_1 z_2=r_1 r_2\operatorname{cis}(\theta_1+\theta_2)$ 　　　　$\dfrac{z_1}{z_2}=\dfrac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis} n\theta$ 　　(de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e x + c, \text{ for } x > 0$$

$$\frac{d}{dx}\left(\sin ax\right) = a\cos ax$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}\left(\tan ax\right) = a\sec^2 ax$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax + c$$

$$\frac{d}{dx}\left(\mathrm{Sin}^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \mathrm{Sin}^{-1}\frac{x}{a} + c, a > 0$$

$$\frac{d}{dx}\left(\mathrm{Cos}^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \mathrm{Cos}^{-1}\frac{x}{a} + c, a > 0$$

$$\frac{d}{dx}\left(\mathrm{Tan}^{-1}x\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \mathrm{Tan}^{-1}\frac{x}{a} + c$$

product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x)dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\frac{1}{2}v^2$$

constant (uniform) acceleration: $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r \qquad\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\qquad\qquad \underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\qquad \underset{\sim}{R} = m\underset{\sim}{a}$

friction: $\qquad\qquad\quad F \quad \mu N$

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

END OF FORMULA SHEET

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$