2003 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The number of students who sat for the 2003 examination was 6232, 139 more than in 2002. Students were required to answer 30 multiple-choice questions in Part I, and five questions worth a total of 20 marks in Part II.

The 2003 examination was more challenging than the 2002 examination. The mean score (out of 30) for Part I was 17.5, down from 19.6 in 2002; and the number of questions answered correctly by less than 50% of students was seven, up from four in 2002 (though down from ten in 2001). For Part II, the mean score (out of 20) was 10.3, slightly lower than 10.7 in 2002; and the number of question parts for which the mean score was less than 50 per cent of the maximum possible was six (out of nine) compared to two (out of seven) in 2002.

The overall mean and median scores (out of 50) were 28.1 and 28 respectively, compared with 29.5 and 30 in 2002. About 7 per cent of students, compared with 6 per cent in 2002, scored less than 25 per cent of the available marks. The lowest score was three out of 50, obtained by 10 students (compared with a lowest score of 2 obtained by four students in 2002). About 4 per cent of students scored more than 90 per cent of the available marks (compared to about 6 per cent in 2002). Twelve students scored full marks, compared with eight in 2002.

The standard of work on Part II ranged from that which earned no marks at all (or 1 or 2 marks overall) to full or almost full marks. The number of students who exhibit limited algebraic skills remains a concern. Fractions are often poorly written and handled; brackets and the *dx* symbol are often missing. Many students were unable to complete solutions properly in Part II primarily due to limited algebraic skills.

Most students used a pen or wrote ‘boldly’ in pencil to answer Part II. However, there are still some whose faint and/or excessively small writing was hard to decipher. Furthermore, poor handwriting, tending to illegibility, leads to difficulties for assessors. While assessors take care to read what is presented and to give due credit, there are times when it is simply impossible to read. Students must pay more attention to clear presentation of solutions during the year.

Many students were able to use their graphics calculators effectively, but it was clear from answers to Question 4 of Part II that many did not recognise when it is useful, or allowable, to use the technology. In part a of Question 4, the instruction ‘Find the **exact** value of the area …’ means that a numerical value obtained from a numerical integration function will gain no marks. Most students clearly recognised this – formal integration was attempted in almost all cases. On the other hand, in part b, students were required to express a volume as a definite integral and ‘hence find the volume correct to three significant figures’. Students were free to use their graphics calculators to evaluate this integral, but few did so. Students need to understand *when* graphics calculators could, and should, be used.

As was indicated in the 2002 Report, students continue to perform less well on complex number questions than on those from other areas of study. In Part I, of the five complex number questions, only Question 5 was answered correctly by more than 60 per cent of students (the remaining four questions had correct answer rates from 48% to 58%). In Part II, the last question was designed to test understanding of the polar form of complex numbers in a variety of ways, and students performed very poorly. It is evident that more work needs to be done on this topic, and in revision done as preparation for the examination.

SPECIFIC INFORMATION

Part 1 – Multiple choice

This table indicates the approximate percentage of students choosing each distractor. The correct answer is the shaded alternative.

Question	A	B	C	D	E
			%		
1	2	19	15	62	2
2	19	63	7	8	3
3	1	8	23	4	64
4	67	6	6	11	10
5	9	14	4	9	64
6	52	6	29	5	8
7	26	5	9	48	12
8	50	10	23	10	7

9	58	17	7	11	7
10	17	7	66	4	6
11	7	9	7	70	7
12	1	2	7	13	77
13	13	57	5	14	11
14	1	85	4	5	5
15	9	59	10	16	6
16	2	12	35	12	39
17	5	7	60	19	9
18	13	5	4	4	74
19	87	5	5	1	2
20	8	7	17	57	11
21	4	12	11	53	20
22	5	12	27	17	39
23	7	7	71	9	6
24	5	10	72	7	6
25	4	72	14	7	3
26	10	11	45	18	16
27	38	6	18	28	10
28	10	37	19	24	10
29	5	16	10	30	39
30	6	19	8	9	58

Seven questions (7, 16, 22, 26, 27, 28 and 29) were answered correctly by less than 50 per cent of students; in five of these, the correct response was given by less than 40 per cent, which is especially disappointing. Question 30, thought to be a difficult questions, was better done than expected. Students continue to perform particularly well in questions where calculus *techniques* are involved (Questions 11, 12, 23 and 24).

Question 27 was poorly done; one of three questions in which a distractor (A; chosen by 38% of students) was more popular than the correct answer (D). Option A is deceptively attractive, but it was not anticipated that it would turn out to be so popular. Students need to be better versed in Newton's Third Law of action-reaction.

In Question 29, where option E was chosen by 39 per cent, these students clearly recognised that there is no value of t for which $v = 3$; however, they failed to realise that at $t = \frac{4\pi}{3}$, v has a minimum of –3, representing a minimum **velocity**, but corresponding to a maximum **speed** of 3 m/s.

In Question 16, a related rates problem there was a poor understanding of the practical situation. The water in the cone has volume $\frac{\pi r^2 h}{3}$ which, as it is evident that $r = \frac{h}{5}$, becomes $\frac{\pi h^3}{75}$; the correct answer follows from the chain rule. Those who chose option E substituted $r = 10$ to incorrectly give the volume as $\frac{100\pi h}{3}$. At this level it is expected that most students would not fall into this error, but rather recognise that r is a function of h that must be found.

Question 28 proved to be challenging, as it involves acceleration in its various forms. The popular distractors were D and C (C represents constant acceleration is evident from the fact that the second derivative of x with respect to t is 2; D is less obvious, involving recognition that acceleration is the derivative with respect to x of $\frac{v^2}{2}$, which gives $a = \frac{1}{2}$).

In Question 22 the difficulty is that there are two times when the particles are in a N-S line. At time $t = 2$, L is *south* of M; at time $t = 4$, it is *north* of M. This question was written to follow up a common error on the 2002 Examination 2 (Question 1d) and should have been better done.

In Question 26 the incorrect responses varied over the distractors suggesting that resolution of forces is still an area of conceptual difficulty for many students. This may well have been a question where guessing was a typical strategy.

In the complex number Question 7 many students were unable to distinguish between a *factor*, for example $(z - 2)$, and the corresponding *root*, 2. A similar difficulty was noted on the 2001 Examination 2 paper where, in Question 4, many students were unable to appreciate the difference between roots and factors, and indeed between (polynomial) expressions and (polynomial) equations. Familiarity with basic concepts and related terminology is essential.

Three questions were answered correctly by more than 50 per cent of students:

- Question 3 (E, 64%), option C, suggests students were taking just a positive square root
- Question 6 (A, 52%), option C indicates either poor algebraic work with signs, or perhaps a misunderstanding of the meaning of complex conjugate pairs
- Question 8 (A, 50%), option C, indicates some difficulty with understanding of the complex conjugate. As the circle is simply a standard circle translated left by 3 units, so that x is replaced by $(x + 3)$, the effect on both z and its conjugate $\frac{v}{z}$ must be the same.

Part 2 – Short answer

Question 1

a

Marks	0	1	Average
%	17	83	**0.83**

Answer: $4.5 - 2\sin(2t)$ (m/s^2)

A straightforward start. There were some slips and, less often, more serious errors, including antidifferentiation instead of differentiation, and the occasional attempt to use constant acceleration formulas.

b

Marks	0	1	2	Average
%	49	8	43	**0.93**

Answer: 12.5 (N)

There were concise, correct answers that made direct use of properties of circular functions, although some students used relatively long-winded calculus methods. Many students assumed (wrongly) that a minimum must occur at $t = 0$. Some students correctly expressed F in terms of t but then stopped. Others, who had the correct answer for part a, left this part blank, suggesting a difficulty working with Newton's law of motion when the acceleration is not constant.

Question 2

Marks	0	1	2	3	4	Average
%	11	17	17	14	41	**2.59**

Answer: Two branches; asymptotes $x = 0$ (and/or $y = \frac{-3}{x}$), $y = \frac{x}{2}$; intercepts $\left(\pm\sqrt{6}, 0\right)$; no turning points.

Many students were able to produce neat, correct answers by combining use of their graphics calculators with a judicious amount of algebra; others only achieved partial success. The most common errors were: a poor attempt at *showing* asymptotic behaviour (curves 'leaving' the oblique asymptote rather than approaching it); labelling the vertical asymptote as $y = 0$ rather than as $x = 0$; and omitting the oblique straight-line asymptote. A few otherwise completely correct graphs were labelled with non-existent turning points.

Some students had not mastered the correct syntax for their calculator, and produced incorrect shapes (cubic curves being particularly common). If students are to use the technology effectively, it is essential that they learn to make correct use of brackets when entering a function: in this case, $y = (x^2 - 6)/(2x)$.

Question 3

Marks	0	1	2	3	Average
%	30	25	14	31	**1.45**

Answer: $m = -6$, $n = 9$

Many students started correctly, but then lost their way in the algebra later or substituted correctly and did not know what to do next. Equating coefficients to find m and n (or substituting values) appeared foreign to some students. The other most common major error was failure to apply the product rule for differentiation from the beginning.

Question 4

a

Marks	0	1	2	3	Average
%	9	29	30	32	**1.84**

Answer: $2\pi - 4$

Some students did not recognise that the antiderivative involved Tan^{-1}, a logarithm expression being the usual alternative; others used Tan^{-1} incorrectly (getting constant term/s incorrect). Some students did not convert the final answer into an expression involving π, or more typically had the negative of the required area. A few students gave a numerical approximation even though an exact value was required.

b

Marks	**0**	**1**	**2**	**3**	**Average**
%	33	16	23	28	**1.44**

Answer: 4.85

There were many good answers, including ones with sensible use of graphics calculators for the final integration. On the other hand, many students had algebraic difficulties in expressing x^2 in terms of y; had the wrong terminals; or rotated around the x-axis. Most students used formal antidifferentiation at the end, often unsuccessfully.

Of concern were students who changed the fraction $\frac{8}{(1-y)}$ into $\frac{-8}{(y-1)}$ and then integrated this term to give $-8\log_e(y-1)$, which is not correct for the interval of integration, presumably attempting to make the antidifferentiation step simpler (such students nevertheless managed to convert logarithms of negative numbers into a correct final answer; they were not awarded full marks). This was also a problem on the last question of Examination 2. Students must be made more aware of the need to use logarithms in the context of integration with care.

Both parts of Question 4 illustrate the importance of reading a question thoroughly, and comprehending and noting instructions about the form of an answer.

Question 5

a

Marks	**0**	**1**	**2**	**Average**
%	42	28	30	**0.87**

Answer: Point with modulus 2 and angle 2θ (approximately $-\frac{3\pi}{4}$).

Many students were able to place u on the circle with radius 2, but were then unable to locate the angle successfully (unable to realise where 2θ might be, or recognise that 2θ was involved).

b

Marks	**0**	**1**	**Average**
%	79	21	**0.21**

Answer: Point with modulus $\frac{1}{\sqrt{2}}$ (approximately 0.7) and angle $-\theta$ (approximately $\frac{3\pi}{8}$).

Only some students were able to recognise the angle for v, but many were then unable to locate it by modulus. Most students had little idea or made no attempt.

c

Marks	**0**	**1**	**Average**
%	84	16	**0.16**

Answer: Point in the third quadrant with modulus approximately 1.4 and angle of approximately $-\frac{4\pi}{5}$.

Few students able to use or attempt to use the triangle rule or parallelogram rule for addition.

Some students attempted to use an invented value of θ in each part of this question, almost invariably without success. The overall analysis of students' attempts on Question 5 suggests that there is a limited understanding of polar form of complex numbers.

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