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SPECIALIST MATHS TRIAL EXAMINATION 2 2003 SOLUTIONS

Question 1

a. **i.** $|u| = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$ **(1 mark)**

ii. Method 1

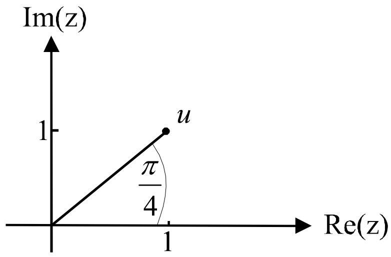
$\text{Arg } u = \tan^{-1}\left(\frac{1}{1}\right)$ (first quadrant)

$= \frac{\pi}{4}$

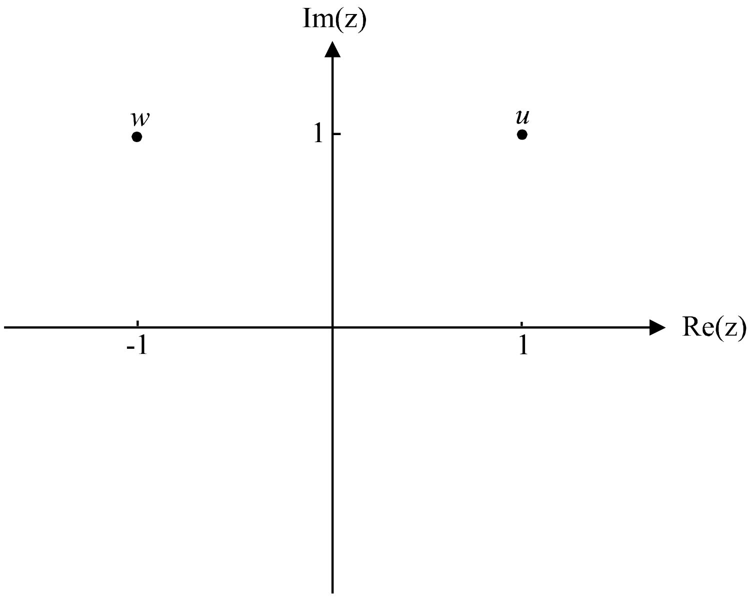
Method 2

From the diagram,

$\text{Arg} u = \frac{\pi}{4}$ **(1 mark)**



iii. Multiplying a complex number by i has the effect of rotating it in an anticlockwise direction by $\frac{\pi}{2}^c$. So, start at u and after 5 anticlockwise turns of $\frac{\pi}{2}$, you land at w as shown.



(1 mark)

Alternatively, $w = ui^5 = ui = (1+i)i = -1+i$

iv. Hence means use what you have just found therefore we must take a graphical rather than an algebraic approach.

We know that the roots of the equation $z^4 + 4 = 0$ will be evenly spaced about the origin on the Argand diagram. **(1 mark)**

Given that *u* and *w* are roots and that there are two others, from the graph, they must be $-1-i$ and $1-i$.

(1 mark)

b. i. If *mi* is a solution to the equation

$$z^2 - (1+5i)z + 3i - 6 = 0$$

then $m^2 i^2 - (1+5i)mi + 3i - 6 = 0$

$$-m^2 - mi + 5m + 3i - 6 = 0$$

so $-m^2 + 5m - 6 + i(-m+3) = 0 + 0i$

$$-m^2 + 5m - 6 = 0 \quad \text{AND} \quad -m + 3 = 0 \qquad \textbf{(1 mark)}$$

$$(-m+2)(m-3) = 0 \qquad\qquad m = 3$$

$$m = 2 \text{ or } m = 3$$

So $m = 3$ since the value of *m* must satisfy the real and imaginary parts.

(1 mark) for rejecting $m = 2$

ii. Since $3i$ is a solution to the equation then $z - 3i$ is a factor.

$$\begin{array}{r|l} & z - 1 - 2i \\ z - 3i & z^2 - z - 5iz + 3i - 6 \\ & \underline{z^2 \quad - 3iz} \\ & -z - 2iz + 3i \\ & \underline{-z \quad + 3i} \\ & -2iz \quad -6 \\ & \underline{-2iz \quad -6} \end{array}$$

So,

$z^2 - (1+5i)z + 3i - 6 = 0$

becomes

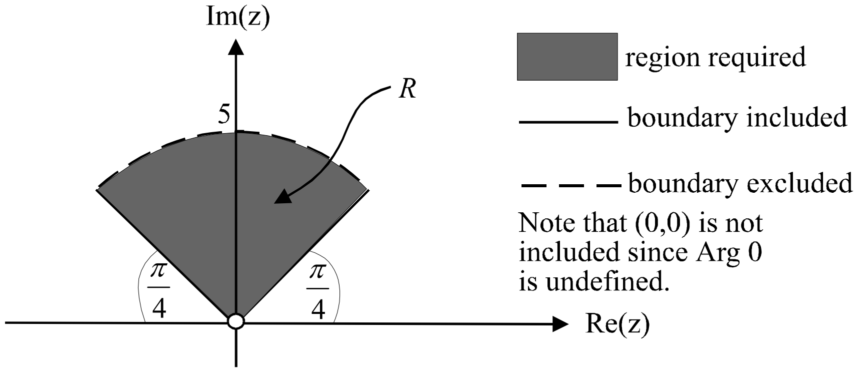
$(z - 3i)(z - 1 - 2i) = 0$

The solutions are $z = 3i$ and $z = 1 + 2i$

(1 mark)

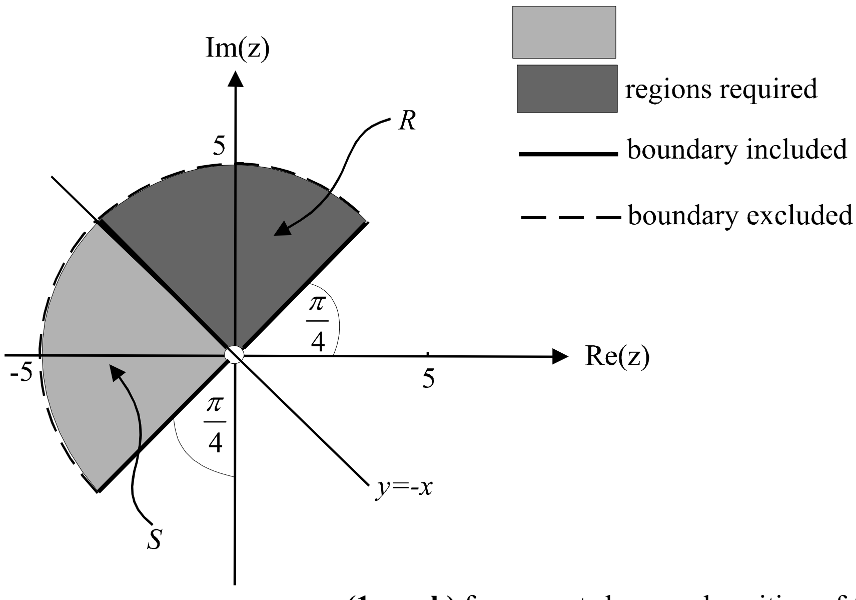
c. **i.** The region defined by $\{z : |z| < 5\}$ is a circle with boundary excluded and with centre $0 + 0i$ and radius 5 units.

The region defined by $\left\{z : \frac{\pi}{4} \le \text{Arg}\, z \le \frac{3\pi}{4}\right\}$ is a wedge with vertex at $0 + 0i$ which is not included, and included boundaries with equations $y = x,\ x \in (0, \infty)$ and $y = -x,\ x \in (-\infty, 0)$. The region R is shown below.



1 mark) for correct curved boundary
(1 mark) for correct straight line boundaries

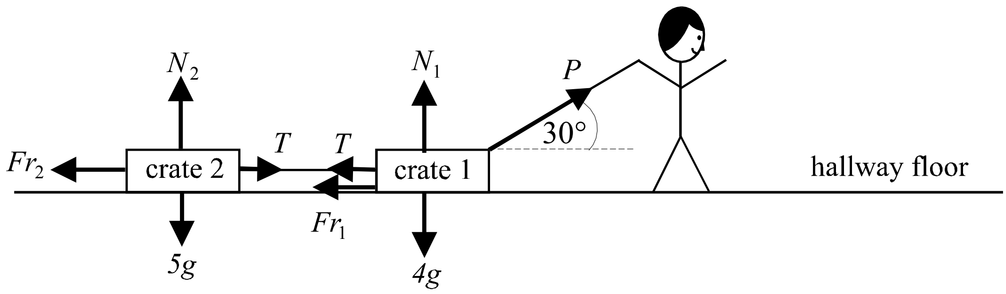
ii.
$$|z - 3 - i| = |z + 1 + 3i|$$
So, $$|x + iy - 3 - i| = |x + iy + 1 + 3i|$$
$$\sqrt{(x-3)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y+3)^2}$$
$$x^2 - 6x + 9 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 + 6y + 9$$
$$-8x - 8y = 0$$
$$y = -x$$
The region R is reflected in the line $y = -x$. **(1 mark)**



(1 mark) for correct shape and position of S
Total 12 marks

Question 2

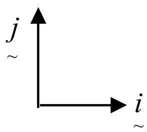
a.



(1 mark) for correct forces around crate 1
(1 mark) for correct forces around crate 2

b. $\underset{\sim}{R} = m\underset{\sim}{a}$

Using



as our convention for direction we have, resolving around crate 1,

$$\left(P\cos 30^\circ - T - Fr_1\right)\underset{\sim}{i} + \left(N_1 + P\sin 30^\circ - 4g\right)\underset{\sim}{j} = 4a\,\underset{\sim}{i}$$ **(1 mark)**

Since $P = 43.55$, $\mu = 0.5$ and $a = 0.5$,

$$\frac{43.55\sqrt{3}}{2} - T - 0.5N_1 = 4a \text{ and } N_1 + 43.55 \times \frac{1}{2} - 4g = 0$$ **(1 mark)**

$$N_1 = 4g - 21.775$$
$$= 17.425$$

So, $T = 21.775\sqrt{3} - 0.5 \times 17.425 - 4 \times 0.5$
$= 27.0N$ (correct to 1 decimal place) **(1 mark)**

c. Because Zac is accelerating at a constant rate of $0.5\,\text{ms}^{-2}$, we can use the formula

$$s = ut + \frac{1}{2}at^2$$ **(1 mark)**

where $s = 20$, $u = 0$, and $a = 0.5$

So $20 = 0 + \frac{1}{2} \times 0.5 \times t^2$

$t = \pm\sqrt{80}$ but $t \geq 0$

So, $t = 8.9$ seconds (to 1 decimal place)

(1 mark)

d. The crates are on the point of sliding when $Fr_1 = \mu N_1$.
Prior to that point, $Fr_1 < \mu N_1$. **(1 mark)**

Resolving around crate 1 and remembering that $a = 0$ now, we have

$$\left(P\cos 30^\circ - T - Fr_1\right)\underset{\sim}{i} + \left(N_1 + P\sin 30^\circ - 4g\right)\underset{\sim}{j} = 4\times 0\,\underset{\sim}{i}$$

So,

$$\frac{P\sqrt{3}}{2} - T - 0.5N_1 = 0 \text{ and } N_1 + \frac{P}{2} - 4g = 0$$

$$N_1 = 4g - \frac{P}{2}$$

So $\dfrac{P\sqrt{3}}{2} - T - 0.5\left(4g - \dfrac{P}{2}\right) = 0$

$$\frac{P\sqrt{3}}{2} - T - 2g + \frac{P}{4} = 0$$

$$P\left(\frac{\sqrt{3}}{2} + \frac{1}{4}\right) = T + 2g$$

$$\frac{2\sqrt{3}+1}{4}P = T + 2g$$

$$P = \frac{4}{2\sqrt{3}+1}\left(T + 2g\right) \quad -*$$ **(1 mark)**

Resolving around crate 2, we have

$$\left(T - Fr_2\right)\underset{\sim}{i} + \left(N_2 - 5g\right)\underset{\sim}{j} = 5\times 0\times\underset{\sim}{i}$$ **(1 mark)**

So $T = \mu N_2$ and $N_2 = 5g$

$$= 0.5\times 5g$$
$$= 2.5g$$

In *, gives

$$P = \frac{4}{2\sqrt{3}+1}\times 4.5g$$
$$= \frac{18g}{2\sqrt{3}+1} \text{ newtons}$$

(1 mark)

Alternatively, a single equation can be used to solve this by considering the two crates as a single 9 kg entity.

$$\underset{\sim}{R} = m\underset{\sim}{a}$$

$$(P\cos 30^\circ - \mu N)\underset{\sim}{i} + (N + P\sin 30^\circ - 9g)\underset{\sim}{j} = 0\underset{\sim}{i}$$

Equating components, we obtain $N = 9g - \dfrac{P}{2}$

$$\frac{\sqrt{3}}{2}P - \frac{1}{2}(9g - \frac{P}{2}) = 0$$

$$\sqrt{3}P - 9g + \frac{P}{2} = 0$$

$$P(2\sqrt{3}+1) = 18g \quad \text{so, } P = \frac{18g}{2\sqrt{3}+1} \text{ newtons}$$

Total 11 marks

Question 3

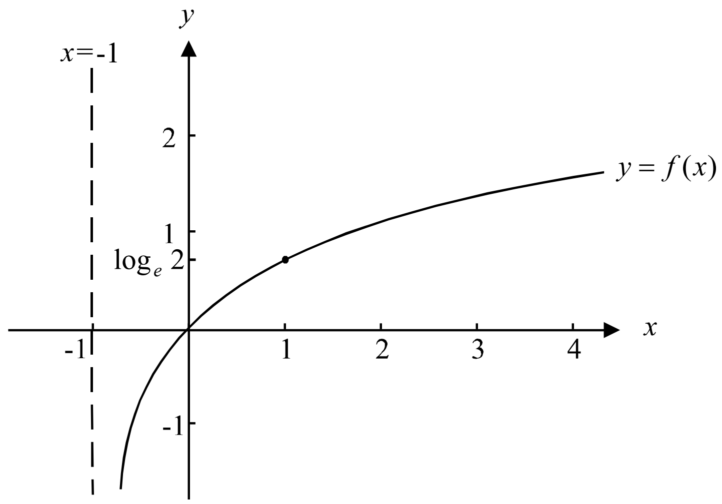
a. For the function *f* to be defined, we require that

$$x+1>0$$
$$x>-1$$

So the maximal domain of *f* is $(-1,\infty)$.

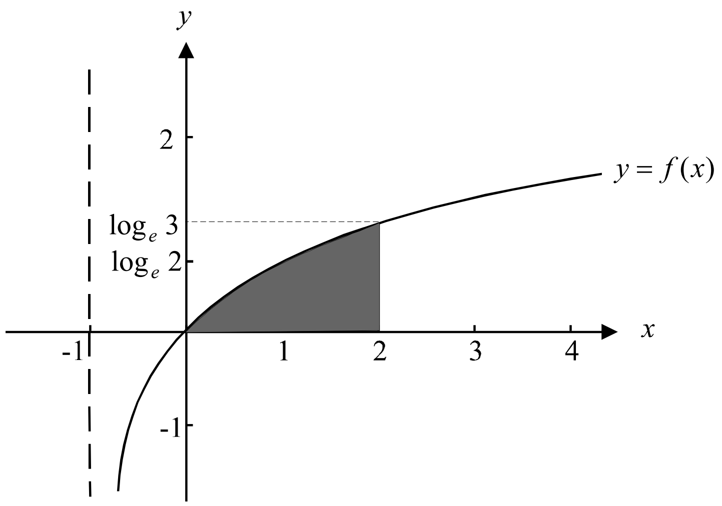
(1 mark)

b.



(1 mark)

c. The integral $\int_0^2 \log_e(x+1)\,dx$ gives the area shaded in the diagram below.



(1 mark)

The rectangle with corner points $(0,0), (2,0), (2,\log_e(3))$ and $(0,\log_e(3))$ has an area given by $2\times\log_e(3)$. Clearly, the area of the shaded region is less than the area of the rectangle and hence $\int_0^2 \log_e(x+1)\,dx < 2\log_e(3)$.

(1 mark)

d. Following the same reasoning as in part **c**. we have

$\int_0^4 \log_e(x+1)\,dx < 4\log_e(5)$, since the rectangle "surrounding" the area described by

$\int_0^4 \log_e(x+1)\,dx$ will have a width of 4 and a height of $\log_e(5)$.

(1 mark)

e. **i.**

Let $y = x\log_e(x+1)$

$$\frac{dy}{dx} = x \times \frac{1}{x+1} + \log_e(x+1)$$

$$= \frac{x}{x+1} + \log_e(x+1)$$

(1 mark)

ii.

Since $\frac{dy}{dx} = \frac{x}{x+1} + \log_e(x+1)$

then $\int_0^2 \frac{dy}{dx}\,dx = \int_0^2 \frac{x}{x+1}\,dx + \int_0^2 \log_e(x+1)\,dx$ _____*

Method 1

Now, $\int_0^2 \frac{x}{x+1}\,dx = \int_1^3 \frac{u-1}{u}\frac{du}{dx}\,dx$

$= \int_1^3 \left(1-u^{-1}\right)du$

$= [u - \log_e(u)]_1^3$

$= (3-\log_e(3)) - (1-\log_e(1))$

$= 2 - \log_e(3)$

where $u = x+1$

so $x = u-1$

and $\frac{du}{dx} = 1$

Also $x = 2, u = 3$

$x = 0, u = 1$

(1 mark)

Method 2

$\int_0^2 \frac{x}{x+1}\,dx = \int_0^2 (1 - \frac{1}{x+1})dx$ Using long division

$$\begin{array}{r} 1 \\ x+1\overline{)x} \\ x+1 \\ -1 \end{array}$$

$= [x - \log_e(x+1)]_0^2$

$= (2-\log_e(3)) - (0-\log(1))$

$= 2 - \log_e(3)$

So, * becomes

$$[x\log_e(x+1)]_0^2 = 2 - \log_e(3) + \int_0^2 \log_e(x+1)\,dx$$

$$\int_0^2 \log_e(x+1)\,dx = (2\log_e(3) - 0) - (2 - \log_e(3))$$

$$= 3\log_e(3) - 2$$

(1 mark)

f. Note that we are rotating about the y-axis and so the terminals must be on the y-axis.
When $x = 2,\ f(2) = \log_e(3)$ and similarly $f(0) = 0$.

Volume required $= \pi \int_0^{\log_e(3)} x^2\, dy$

(1 mark) for terminals

Now, $y = \log_e(x+1)$

so, $e^y = x + 1$

$x = e^y - 1$

So volume required $= \pi \int_0^{\log_e(3)} \left(e^y - 1\right)^2 dy$ **(1 mark)** for integrand

$$= \pi \int_0^{\log_e(3)} \left(e^{2y} - 2e^y + 1\right) dy$$

$$= \pi \left[\frac{e^{2y}}{2} - 2e^y + y \right]_0^{\log_e(3)}$$

$$= \pi \left\{ \left(\frac{e^{2\log_e(3)}}{2} - 2e^{\log_e(3)} + \log_e(3) \right) - \left(\frac{e^0}{2} - 2e^0 + 0 \right) \right\}$$ **(1 mark)**

$$= \pi \left\{ \frac{e^{\log_e(9)}}{2} - 6 + \log_e(3) - \frac{1}{2} + 2 \right\}$$

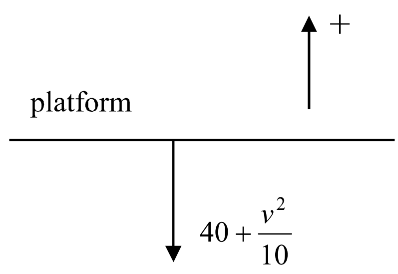
$$= \pi \left(\frac{9}{2} - \frac{9}{2} + \log_e(3) \right)$$

$= \pi \log_e(3)$ cubic units **(1 mark)**

Total 12 marks

Question 4

a. Take the origin to be that part on the platform from which the particle is projected. The downward forces acting on the particle are the gravitational and air resistance forces. Taking the positive direction as upwards, and using the equation of motion, $F = ma$ we obtain



$-\left(40 + \frac{v^2}{10}\right) = 4\ddot{y}$ where $\ddot{y}$ is vertical acceleration.

So, $\ddot{y} = -10 - \frac{v^2}{40}$ as required. **(1 mark)**

b. The maximum height is reached when $v = 0$.

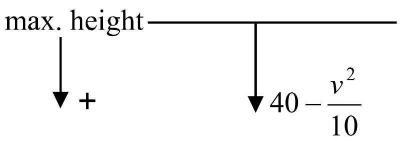
Now, $v^2 = 100\left(5e^{\frac{-y}{20}} - 4\right)$

becomes $0 = 100\left(5e^{\frac{-y}{20}} - 4\right)$

So,
$$e^{\frac{-y}{20}} = \frac{4}{5}$$
$$e^{\frac{y}{20}} = \frac{5}{4}$$
$$\log_e\left(\frac{5}{4}\right) = \frac{y}{20}$$
$$y = 20\log_e\left(\frac{5}{4}\right)$$

So the maximum height reached is $20\log_e\left(\frac{5}{4}\right)$ metres. **(1 mark)**

c. We need to consider the downward part of the particle's journey. Taking its maximum height as the origin, the downward direction as positive, and using the equation of motion $F = ma$,



we have $40 - \frac{v^2}{10} = 4\ddot{y}$ where $\ddot{y}$ is the vertical acceleration. So, $\ddot{y} = 10 - \frac{v^2}{40}$ **(1 mark)**

Now, given that $\ddot{y} = v\frac{dv}{dy}$

We have,
$$v\frac{dv}{dy} = 10 - \frac{v^2}{40}$$
$$\frac{dv}{dy} = \frac{10}{v} - \frac{v}{40}$$
$$= \frac{400 - v^2}{40v}$$
$$\frac{dy}{dv} = \frac{40v}{400 - v^2}$$

$$\int \frac{dy}{dv}\,dv = 40\int \frac{v}{400-v^2}\,dv \qquad \text{let } u = 400 - v^2$$

$$y = 40\int -\frac{1}{2}\frac{du}{dv}\cdot u^{-1}\,dv \qquad \frac{du}{dv} = -2v$$

$$= -20\int u^{-1}\,du$$

$$y = -20\log_e\left(400-v^2\right)+c$$

When $y = 0$, $v = 0$

$$0 = -20\log_e 400 + c$$

$$c = 20\log_e 400$$

$$y = -20\log\ \left(400-v^2\right) + 20\log_e 400$$

$$= 20\log_e\left(\frac{400}{400-v^2}\right)$$

(1 mark)

From part **b.**, the particle returns to the platform when it has travelled $20\log_e\left(\frac{5}{4}\right)$ metres. So, when $y = 20\log_e\left(\frac{5}{4}\right)$

We have $$20\log_e\left(\frac{5}{4}\right) = 20\log_e\left(\frac{400}{400-v^2}\right)$$

So, $$\frac{5}{4} = \frac{400}{400-v^2}$$

$$5\left(400-v^2\right) = 1600$$

$$2000 - 5v^2 = 1600$$

$$v^2 = 80$$

$$v = \pm\sqrt{80}$$

So, the particle is travelling at a speed of $4\sqrt{5}$ metres per second when it returns to the platform. **(1 mark)**

d. **i.** We need an expression for velocity as a function of time. Looking at the motion of the particle from when it reached its maximum height, we have, from part **c.**

$$\ddot{y} = 10 - \frac{v^2}{40}$$

Now, $$\frac{dv}{dt} = \frac{400-v^2}{40}$$

So, $$\frac{dt}{dv} = \frac{40}{400-v^2}$$

Now, $$\frac{40}{400-v^2} = \frac{40}{(20-v)(20+v)}$$

Let $$\frac{40}{(20-v)(20+v)} \equiv \frac{A}{20-v} + \frac{B}{20+v}$$

$$\equiv \frac{A(20+v)+B(20-v)}{(20-v)(20+v)}$$

True iff $40 \equiv A(20+v)+B(20-v)$

Put $v=-20$, $40=40B$, $B=1$

Put $v=20$, $40=40A$, $A=1$

So, $\dfrac{40}{400-v^2}=\dfrac{1}{20-v}+\dfrac{1}{20+v}$

So, $\dfrac{dt}{dv}=\dfrac{1}{20-v}+\dfrac{1}{20+v}$ **(1 mark)**

$$\int \frac{dt}{dv}dv=\int \frac{1}{20-v}dv+\int \frac{1}{20+v}dv$$

$$t=-\log_e(20-v)+\log_e(20+v)+c$$

When $t = 0$, $v = 0$ $\quad 0=-\log_e 20+\log_e 20+c$

$$c=0$$

So $t=\log_e(\dfrac{20+v}{20-v})$

So, $e^t=\dfrac{20+v}{20-v}$ **(1 mark)**

$$20e^t-ve^t=20+v$$

$$-ve^t-v=20-20e^t$$

$$v(e^t+1)=20e^t-20$$

$$v=\frac{20e^t-20}{e^t+1}$$

$v=\dfrac{20(e^t-1)}{e^t+1}$ **(1 mark)**

ii. Now, $v=\dfrac{20(e^t-1)}{e^t+1}$

So, $v=20\left(1-\dfrac{2}{e^t+1}\right)$ since

$$\begin{array}{r|l} & 1 \\ e^t+1 & e^t-1 \\ & \underline{e^t+1} \\ & -2 \end{array}$$

As, $t\to\infty, \dfrac{2}{e^t+1}\to 0$

So, $v\to 20$ from below.

So the terminal velocity of the particle is 20 metres per second.

(1 mark)

e. For that part of the particle's journey where it is falling downward, we have from part **d.** that $t=\ln(\dfrac{20+v}{20-v})$.

From part **c.** the speed of the particle when it returned to the platform was $4\sqrt{5}$ metres per second.

So when $v=4\sqrt{5}$, (since we assume that downwards is positive).

$$t=\log_e(\frac{20+4\sqrt{5}}{20-4\sqrt{5}})$$

$$=0.96 \text{ seconds (correct to 2 decimal places.)}$$

(1 mark)

Total 10 marks

Question 5

a. The naval exercise began when $t = 0$

$$\underset{\sim}{r}_A(0) = 5\sin(0)\underset{\sim}{i} + 5\cos(0)\underset{\sim}{j}$$
$$= 0\underset{\sim}{i} + 5\underset{\sim}{j}$$

The aircraft carrier *A* was positioned 5 km due north of the fleet command ship when the exercise started.

(1 mark)

b. From **a.**

$\underset{\sim}{r}_A(0) = 0\underset{\sim}{i} + 5\underset{\sim}{j}$

Also,

$\underset{\sim}{r}_B(0) = 2\cos(0)\underset{\sim}{i} + 3\sin(0)\underset{\sim}{j}$

$= 2\underset{\sim}{i} + 0\underset{\sim}{j}$

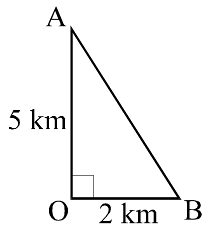
So aircraft carrier *B* is positioned 2 km due East of the fleet command ship.

So at the start of the naval exercise *A* and *B* are as shown in the diagram

Distance required

$= \sqrt{25 + 4}$

$= \sqrt{29}$ km



(1 mark)

c. $\underset{\sim}{r}_A(t) = 5\sin(2t)\underset{\sim}{i} + 5\cos(2t)\underset{\sim}{j}$

So, $\dot{\underset{\sim}{r}}_A(t) = 10\cos(2t)\underset{\sim}{i} - 10\sin(2t)\underset{\sim}{j}$ **(1 mark)**

So, $\dot{\underset{\sim}{r}}_A(0) = 10\cos(0)\underset{\sim}{i} - 10\sin(0)\underset{\sim}{j}$

$= 10\underset{\sim}{i} + 0\underset{\sim}{j}$

So the aircraft carrier *A* is moving due East when the naval exercise begins.

(1 mark)

d.

$$\underset{\sim}{r}_A\left(\frac{\pi}{6}\right) = 5\sin\left(\frac{\pi}{3}\right)\underset{\sim}{i} + 5\cos\left(\frac{\pi}{3}\right)\underset{\sim}{j}$$
$$= \frac{5\sqrt{3}}{2}\underset{\sim}{i} + \frac{5}{2}\underset{\sim}{j}$$
$$\underset{\sim}{r}_B\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{3}\right)\underset{\sim}{i} + 3\sin\left(\frac{\pi}{3}\right)\underset{\sim}{j}$$
$$= \underset{\sim}{i} + \frac{3\sqrt{3}}{2}\underset{\sim}{j}$$ **(1 mark)**

$$\cos\theta = \frac{\underset{\sim A}{r}\left(\frac{\pi}{6}\right)\cdot\underset{\sim B}{r}\left(\frac{\pi}{6}\right)}{\left|\underset{\sim A}{r}\left(\frac{\pi}{6}\right)\right|\left|\underset{\sim B}{r}\left(\frac{\pi}{6}\right)\right|}$$ **(1 mark)**

$$= \left(\frac{5\sqrt{3}}{2}\times 1+\frac{5}{2}\times\frac{3\sqrt{3}}{2}\right)\div\left(\sqrt{\frac{75}{4}+\frac{25}{4}}\sqrt{1+\frac{27}{4}}\right)$$

$$= \left(\frac{5\sqrt{3}}{2}+\frac{15\sqrt{3}}{4}\right)\div\left(\sqrt{\frac{100}{4}}\sqrt{\frac{31}{4}}\right)$$

$$= \frac{25\sqrt{3}}{4}\div\frac{5\sqrt{31}}{2}$$

$$= \frac{25\sqrt{3}}{4}\times\frac{2}{5\sqrt{31}}$$

$$\cos\theta = \frac{5\sqrt{3}}{2\sqrt{31}}$$

$$\theta = 38^\circ 57'$$

So the required angle is 38°57'. (to the nearest minute) **(1 mark)**

e. **i.** $\underset{\sim A}{r}(t) = 5\sin(2t)\underset{\sim}{i} + 5\cos(2t)\underset{\sim}{j}$

$x = 5\sin(2t) \qquad y = 5\cos(2t)$

$\frac{x}{5} = \sin(2t) \qquad \frac{y}{5} = \cos(2t)$

$\frac{x^2}{25} = \sin^2(2t) \qquad \frac{y^2}{25} = \cos^2(2t)$

So, $\frac{x^2}{25}+\frac{y^2}{25} = \sin^2(2t)+\cos^2(2t)$

$\frac{x^2}{25}+\frac{y^2}{25} = 1$

$x^2 + y^2 = 25$

(1 mark)

ii. The path is circular with centre at O and a radius of 5 km. Also, we know from part **c.** that it is clockwise motion.

(1 mark)

f. Method 1 – "Hence…"

$\underset{\sim}{r}_B(t) = 2\cos(2t)\underset{\sim}{i} + 3\sin(2t)\underset{\sim}{j}$

$x = 2\cos(2t) \qquad y = 3\sin(2t)$

$x^2 = 4\cos^2(2t) \qquad y^2 = 9\sin^2(2t)$

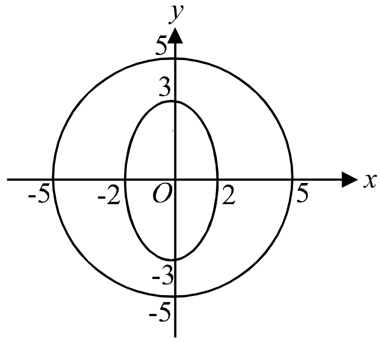
So, $\dfrac{x^2}{4} + \dfrac{y^2}{9} = \cos^2(2t) + \sin^2(2t)$

$\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ **(1 mark)**

The path of aircraft carrier B is that of an ellipse with a semi-minor axis length of 2 km and a semi-major axis length of 3 km and "centre" at O. From part **e.** we know that the path of aircraft carrier A is circular.

From the diagram, we see that the two aircraft carriers cannot collide.

(1 mark)



Method 2 – "Otherwise…"

The two aircraft carriers will collide iff

$5\sin(2t) = 2\cos(2t)$ AND $5\cos(2t) = 3\sin(2t)$

$\dfrac{\sin(2t)}{\cos(2t)} = \dfrac{2}{5}$ AND $\dfrac{5}{3} = \dfrac{\sin(2t)}{\cos(2t)}$

$\tan(2t) = \dfrac{2}{5}$ AND $\tan(2t) = \dfrac{5}{3}$

(1 mark)

Since this is not possible, that is the $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components are not the same for both the carriers at the same point in time, then the two aircraft carriers cannot collide.

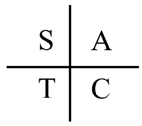
(1 mark)

g. **i.** The helicopter can only land on aircraft carrier A if they are at the same position at the same time.
So, the helicopter can land on aircraft carrier A if

$$5\sin(2t) = 5\sqrt{3}\cos(2t) \text{ AND } 5\cos(2t) = \frac{5\sqrt{3}}{3}\sin(2t) \text{ AND } \cos(t) + \frac{1}{2} = 0$$

(1 mark)

Now, $5\sin(2t) = 5\sqrt{3}\cos(2t)$



$$\frac{\sin(2t)}{\cos(2t)} = \sqrt{3}$$

$$\tan(2t) = \sqrt{3}$$

$$2t = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, ...$$

$$t = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, ...$$

Also, $5\cos(2t) = \frac{5\sqrt{3}}{3}\sin(2t)$

$$\frac{\sin(2t)}{\cos(2t)} = \sqrt{3}$$

$$\tan(2t) = \sqrt{3}$$

$$t = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, ...$$

Also $\cos(t) = -\frac{1}{2}$

$t = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$ **(1 mark)** for examining each component

There are values of t (ie $t = \frac{2\pi}{3}, \frac{8\pi}{3}, ...$) that satisfies all three position components. Therefore the helicopter and aircraft carrier A can be at the same position at the same time and hence the helicopter (barring technical difficulties) can land.

ii. Hence the least value of t when this happens is at $t = \frac{2\pi}{3}$ hours .

(So, $T = \frac{2\pi}{3}$.)

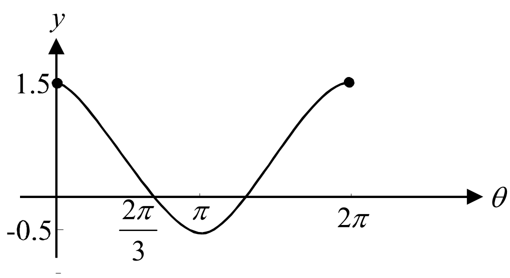
(1 mark)

h. Consider the vertical component of the position function of the helicopter.

Graph the function $y = \cos(t) + \frac{1}{2}$.

At $t = \frac{2\pi}{3}$, ie at $t = T$, $y = 0$.

For $T < t < \frac{4\pi}{3}$, $y < 0$.



This would mean that the helicopter would be under the water.

(1 mark)
Total 15 marks