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SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

(FACTS, SKILLS AND APPLICATIONS TASK)

2003

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of Part I and Part II.
Part I consists of 30 multiple-choice questions and should be answered on the detachable answer sheet on page 22 of this exam. This section of the paper is worth 30 marks.
Part II consists of 6 short-answer questions, all of which should be answered in the spaces provided. Part II begins on page 16 of this exam. This section of the paper is worth 20 marks.
There is a total of 50 marks available.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Students may bring up to two A4 pages of pre-written notes into the exam.

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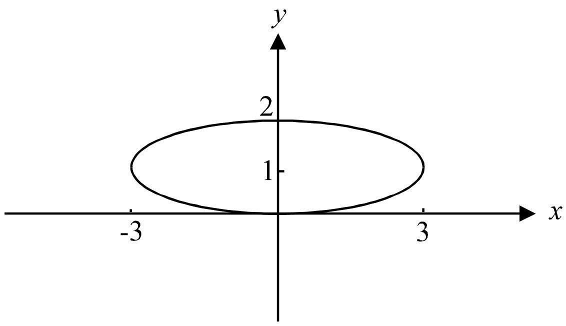
PART I

Question 1



The equation of the ellipse shown in the diagram above is

A. $\frac{x^2}{3}+\frac{y^2}{2}=1$

B. $\frac{x^2}{2}+\frac{(y+1)^2}{3}=1$

C. $\frac{(x-1)^2}{\sqrt{3}}+\frac{y^2}{\sqrt{3}}=1$

D. $\frac{x^2}{9}+\frac{(y-1)^2}{1}=1$

E. $\frac{x^2}{9}+\frac{(y-1)^2}{4}=1$

Question 2

Which one of the following equations would have a graph that has an asymptote of $y=\frac{5}{3}x$?

A. $\frac{x^2}{3}-\frac{y^2}{5}=1$

B. $\frac{x^2}{9}-\frac{y^2}{25}=1$

C. $\frac{x^2}{9}+\frac{y^2}{25}=1$

D. $\frac{x^2}{25}-\frac{y^2}{9}=1$

E. $\frac{(x-2)^2}{25}+\frac{y^2}{9}=1$

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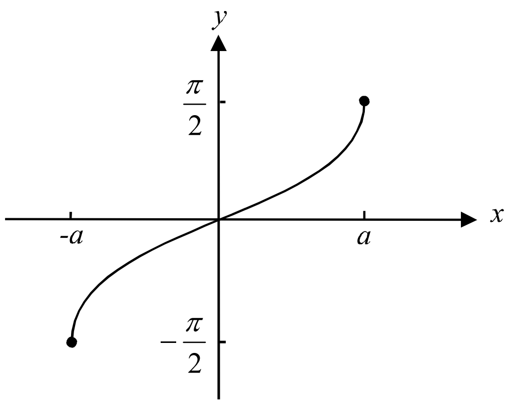
Question 3



The equation of the graph shown above could be

A. $y = \mathrm{Tan}(ax)$

B. $y = \mathrm{Cos}^{-1}\left(\frac{x}{a}\right)$

C. $y = \mathrm{Sin}^{-1}\left(\frac{x}{a}\right)$

D. $y = \mathrm{Cos}^{-1}(ax)$

E. $y = \mathrm{Sin}^{-1}(ax)$

Question 4

If $y = \mathrm{cosec}(2x-1)$, then $\frac{dy}{dx}$ is equal to

A. $2\tan^2(2x-1)$

B. $-\mathrm{cosec}^2(2x-1)$

C. $\frac{-\cot(2x-1)}{\sin(2x-1)}$

D. $-2\cot(2x-1)\mathrm{cosec}(2x-1)$

E. $2\tan(2x-1)\sec(2x-1)$

Question 5

If $z = 4-3i$ then $z + zi - \bar{z} + |z|$ is equal to

A. $2-2i$

B. $8-2i$

C. $8+4i$

D. $16+10i$

E. $3+\sqrt{5}-2i$

Question 6

If $u = 1 + \sqrt{3}i$ and $v = 2\text{cis}\left(\frac{\pi}{6}\right)$ then $\frac{u}{v}$ is equal to

A. $\text{cis}\left(-\frac{\pi}{6}\right)$

B. $\text{cis}\left(\frac{\pi}{6}\right)$

C. $2\text{cis}\left(\frac{\pi}{6}\right)$

D. $\text{cis}\left(\frac{\pi}{2}\right)$

E. $2\text{cis}\left(\frac{\pi}{2}\right)$

Question 7

Given that $z - 3i$ is a factor of $P(z) = z^3 + 2z^2 + 9z + 18$, which one of the following statements is **not** true?

A. $P(3i) = 0$
B. $P(-3i) = 0$
C. $P(z) = 0$ has 3 roots
D. $P(z) = 0$ has 2 complex roots
E. $P(z) = 0$ has 2 real roots

Question 8

The solutions to the equation $z^2 = 2 + 2i$ where z is a complex number are

A. $-2\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$ and $2\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$

B. $-\sqrt[4]{8}\text{cis}\left(\frac{\pi}{4}\right)$ and $\sqrt[4]{8}\text{cis}\left(\frac{\pi}{4}\right)$

C. $\sqrt[4]{8}\text{cis}\left(\frac{\pi}{2}\right)$ and $\sqrt[4]{8}\text{cis}\left(\frac{3\pi}{2}\right)$

D. $2\text{cis}\left(\frac{\pi}{8}\right)$ and $\sqrt[2]{2}\text{cis}\left(\frac{3\pi}{8}\right)$

E. $\sqrt[4]{8}\text{cis}\left(\frac{\pi}{8}\right)$ and $\sqrt[4]{8}\text{cis}\left(\frac{9\pi}{8}\right)$

Question 9

An antiderivative of $\dfrac{3x}{\sqrt{x^2-2}}$ is

A. $-3\sqrt{x^2-2}$

B. $3\sqrt{x^2-2}$

C. $\dfrac{-3}{4}\sqrt{x^2-2}$

D. $6x\left(x^2-2\right)$

E. $\dfrac{-3x}{\left(x^2-2\right)^{\frac{3}{2}}}+\dfrac{1}{3\left(x^2-2\right)}$

Question 10

$\int x\sqrt{2x-1}\,dx$ equals

A. $\dfrac{3x-1}{\sqrt{2x-1}}+c$

B. $\dfrac{4x^{\frac{5}{2}}}{5}-\dfrac{2x^{\frac{3}{2}}}{3}+c$

C. $\dfrac{(2x-1)^{\frac{5}{2}}}{10}+\dfrac{(2x-1)^{\frac{3}{2}}}{6}+c$

D. $\dfrac{(2x-1)^{\frac{5}{2}}}{5}+\dfrac{(2x-1)^{\frac{3}{2}}}{3}+c$

E. $\dfrac{4(2x-1)^{\frac{5}{2}}}{5}+\dfrac{2(2x-1)^{\frac{3}{2}}}{3}+c$

Question 11

If $\dfrac{dy}{dx}=\cos^3(3x)$ then y is equal to

A. $\dfrac{\cos^4(3x)}{12}+c$

B. $\dfrac{\cos^4(3x)}{4}+c$

C. $x-\dfrac{\sin^3(3x)}{9}+c$

D. $\dfrac{\sin(3x)}{3}-\dfrac{\sin^3(3x)}{9}+c$

E. $\dfrac{\sin(3x)}{3}-\dfrac{\sin^3(3x)}{3}+c$

Question 12

$\int_{1}^{\sqrt{3}} \frac{1}{x^2+3}\,dx$ is equal to

A. $\frac{\pi}{36}$
B. $\frac{\sqrt{3}\pi}{36}$
C. $\frac{\pi}{12}$
D. $\frac{\sqrt{3}\pi}{6}$
E. $\sqrt{3}$

Question 13

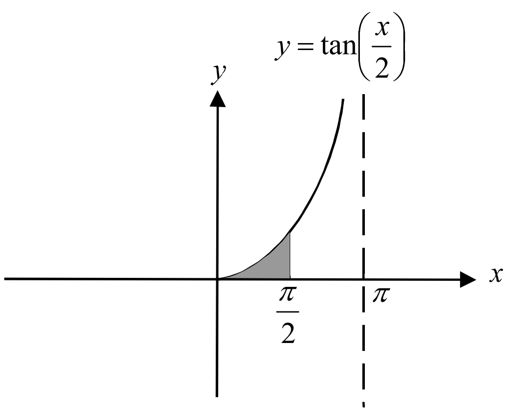
The value of

$$\int_{0.5}^{1} \frac{\log_e(2x)}{\sin(2x)}\,dx,$$

correct to 4 decimal places is

A. 0.0572
B. 0.1134
C. 0.1997
D. 0.2342
E. 0.2871

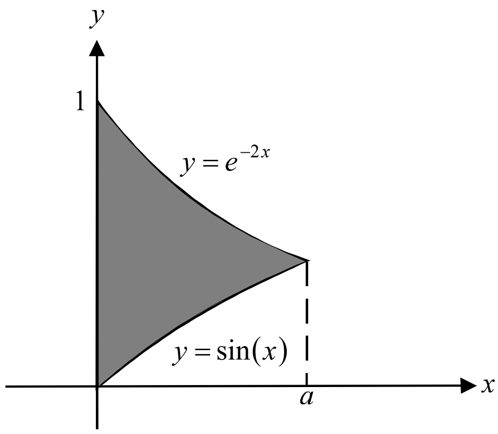
Question 14



The graph of $y=\tan\left(\frac{x}{2}\right)$ is shown in the diagram above. The mid-point rule with two equal intervals is used to approximate the shaded area shown in the diagram above. The approximate area found, in square units and correct to 2 decimal places is

A. 0.68
B. 0.87
C. 1.36
D. 1.78
E. 2.22

Question 15



The graphs of $y = \sin(x)$, $0 \le x \le a$ and $y = e^{-2x}$, $0 \le x \le a$ are shown above.
The area of the shaded region is

A. $-\dfrac{1}{e^{2a}} - \cos(a) - \dfrac{1}{2}$

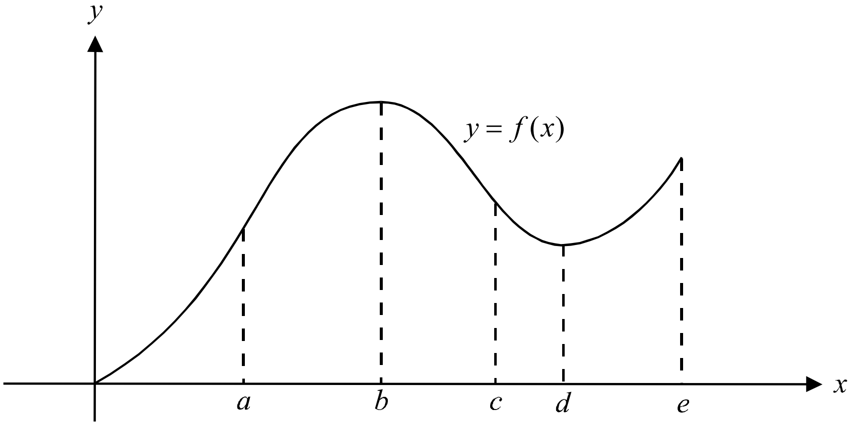
B. $-\dfrac{1}{2e^{2a}} + \cos(a) - \dfrac{1}{2}$

C. $-\dfrac{1}{e^{2a}} + \cos(a) - 3$

D. $-\dfrac{2}{e^{2a}} - \cos(a) + 3$

E. $\dfrac{1}{2e^{2a}} - \cos(a) + \dfrac{1}{2}$

Question 16



The graph of the function *f* is shown above for $0 \le x \le e$.
For this function, $f'(b) = 0$, $f'(d) = 0$ and there is a point of inflection at $x = a, x = c$ and $x = e$.
It is true to say that $f''(x) > 0$ for

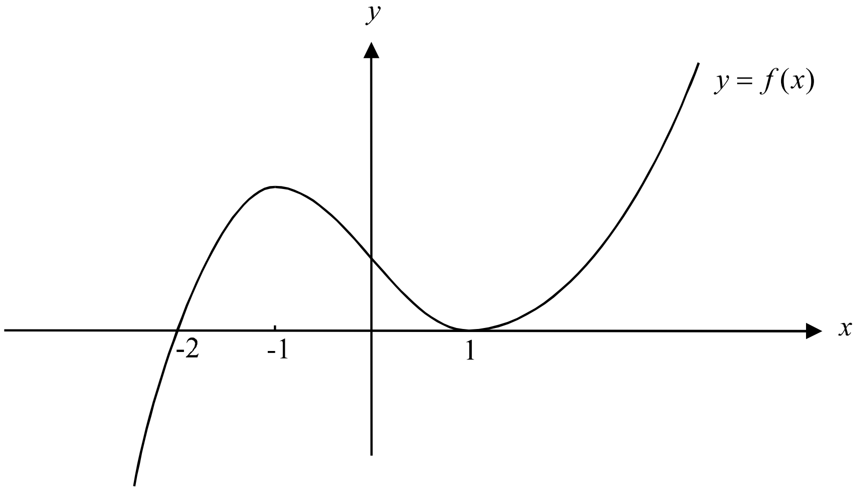
A. $0 < x < a$
B. $a < x < c$
C. $c < x < e$
D. $a < x < b$ and $d < x < e$
E. $0 < x < a$ and $c < x < e$

Question 17

An approximation to the solution of the differential equation $(x-1)\frac{dy}{dx} = e^{-3x}$ with $y = 1$ at $x = 0$ is found using Euler's method with a step size of 0.2. When $x = 0.4$, the value obtained for *y* is closest to

A. –0.5037
B. 0.4963
C. 0.6628
D. 0.7093
E. 1.7093

Question 18

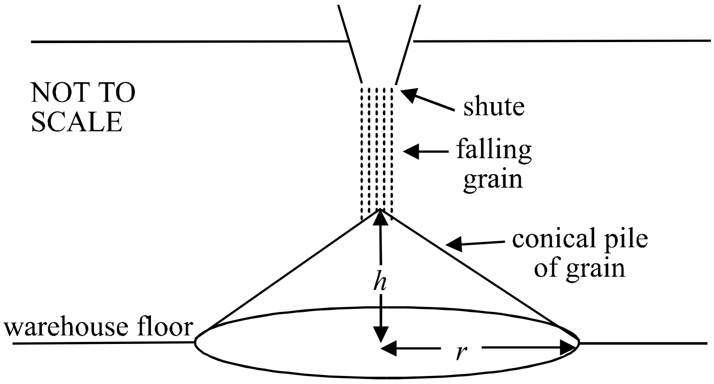


The graph of $y = f(x)$ is shown above. If $F(x)$ was an antiderivative of $f(x)$, then which one of the following set of features would the graph of $y = F(x)$ have?

A. a local maximum at $x = -1$ and a local minimum at $x = 1$.
B. a local maximum at $x = -2$ and a local maximum at $x = 1$.
C. a stationary point of inflection at $x = 0$, a local minimum at $x = 1$ and a local maximum at $x = -1$.
D. a stationary point of inflection at $x = -2$ and a local maximum at $x = 1$.
E. a stationary point of inflection at $x = 1$ and a local minimum at $x = -2$.

Question 19

Grain is being dropped onto a warehouse floor through a chute in the roof of the warehouse.



The pile of grain is in the shape of a cone with radius, r, of the circular base twice the height, h, of the cone.
The volume, V, of grain delivered to the pile per second is $2\text{m}^3\text{s}^{-1}$.
The rate at which the height of the pile is increasing when the height of the pile is 2 metres is given by

A. $\dfrac{1}{8\pi}\text{ms}^{-1}$

B. $\dfrac{1}{4\pi}\text{ms}^{-1}$

C. $\dfrac{1}{2\pi}\text{ms}^{-1}$

D. $4\pi\,\text{ms}^{-1}$

E. $8\pi\,\text{ms}^{-1}$

Question 20

A particle moves in a straight line with velocity, v, and a displacement x from a fixed origin O. Its acceleration is given by $\dfrac{1}{x+1}$. Given that the particle started from rest at O, then the displacement of the particle when it is moving at $\sqrt{2}\text{ms}^{-1}$ is given by

A. $e^{\sqrt{2}}$

B. $e-1$

C. $e^{\sqrt{2}}-1$

D. e^2-1

E. $\log_e\left(1+\sqrt{2}\right)$

Question 21

If $\overrightarrow{OP} = -3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\overrightarrow{PQ} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$, then $\overrightarrow{OQ}$ equals

A. $-\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$

B. $\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$

C. $\underset{\sim}{i} - 4\underset{\sim}{j} + 7\underset{\sim}{k}$

D. $-5\underset{\sim}{i} - 4\underset{\sim}{j} - 7\underset{\sim}{k}$

E. $5\underset{\sim}{i} - 4\underset{\sim}{j} + 7\underset{\sim}{k}$

Question 22

A vector, which has a magnitude of 5 and is parallel to $2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ is

A. $10\underset{\sim}{i} - 15\underset{\sim}{j} - \underset{\sim}{k}$

B. $\frac{1}{\sqrt{14}}\left(2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}\right)$

C. $\frac{5}{\sqrt{14}}\left(2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}\right)$

D. $\frac{5\sqrt{6}}{6}\left(2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}\right)$

E. $\frac{\sqrt{5}\sqrt{14}}{5}\left(2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}\right)$

Question 23

The vector resolute of $2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ perpendicular to $-\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$ is

A. $\frac{3}{2}\underset{\sim}{i} - \frac{1}{2}\underset{\sim}{k}$

B. $-\frac{1}{2}\underset{\sim}{i} + \underset{\sim}{j} - \frac{3}{2}\underset{\sim}{k}$

C. $\frac{3}{2}\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j} - \frac{1}{2}\underset{\sim}{k}$

D. $\frac{5}{2}\underset{\sim}{i} - 2\underset{\sim}{j} + \frac{5}{2}\underset{\sim}{k}$

E. $\frac{7}{\sqrt{14}}\left(-\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}\right)$

Question 24

The position vector of a particle at time t is given by

$$\underset{\sim}{r}(t) = \text{Sin}^{-1}\left(\frac{t}{3}\right)\underset{\sim}{i} + \text{Cos}^{-1}\left(\frac{t}{3}\right)\underset{\sim}{j} \qquad 0 \le t \le 3 .$$

The speed of the particle at time $t = 1$ is

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{\sqrt{2}}{2}$

D. $\frac{\sqrt{3}}{2}$

E. 1

Question 25

The acceleration vector of a particle at time t is given by $\ddot{\underset{\sim}{r}}(t) = e^{-2t}\,\underset{\sim}{i} + \cos(2t)\,\underset{\sim}{j}, \quad t \ge 0$.

Also, $\dot{\underset{\sim}{r}}(t) = -\frac{1}{2}\underset{\sim}{i}$ and $\underset{\sim}{r}(t) = \frac{1}{4}\underset{\sim}{i}$ when $t = 0$. The position vector of the particle is given by

A. $\underset{\sim}{r} = 4e^{-2t}\,\underset{\sim}{i} - 4\cos(2t)\,\underset{\sim}{j}$

B. $\underset{\sim}{r} = \frac{e^{-2t}}{-2}\underset{\sim}{i} - \frac{\sin(2t)}{2}\underset{\sim}{j}$

C. $\underset{\sim}{r} = \frac{e^{-2t}}{4}\underset{\sim}{i} - \cos(2t)\,\underset{\sim}{j}$

D. $\underset{\sim}{r} = \frac{e^{-2t}}{4}\underset{\sim}{i} + \frac{1}{4}(1 - \cos(2t))\,\underset{\sim}{j}$

E. $\underset{\sim}{r} = \frac{e^{-2t}}{4}\underset{\sim}{i} + \frac{1}{4}(1 + \cos(2t))\,\underset{\sim}{j}$

Question 26

A particle is acted on by a force of magnitude $\sqrt{2}$ newtons acting in a south-westerly direction and by a second force of 7 newtons acting due south. The magnitude of the resulting force in newtons is

A. $\sqrt{37}$

B. $2\sqrt{11}$

C. $\sqrt{51}$

D. $\sqrt{58}$

E. $\sqrt{65}$

Question 27

A particle of mass 3 kg is moving in a straight line. The velocity of the particle decreases from $20\,\text{ms}^{-1}$ to 15ms^{-1} over a period of 4 seconds. The change of momentum of the particle in kg ms^{-1} in the direction of the straight line is

A. -15
B. $\frac{-15}{4}$
C. $\frac{-5}{3}$
D. 5
E. 20

Question 28

A particle of mass 2 kg is being dragged across a rough horizontal surface by a force of magnitude 20 newtons acting at an angle of 30° to the horizontal. The acceleration of the particle is 3ms^{-2}. The coefficient of friction between the particle and the rough surface is equal to

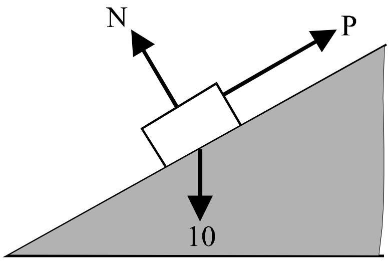
A. $\frac{10}{g}$
B. $\frac{5}{g-7}$
C. $\frac{7}{g-5\sqrt{3}}$
D. $\frac{5\sqrt{3}-3}{g}$
E. $\frac{5\sqrt{3}-3}{g-5}$

Question 29

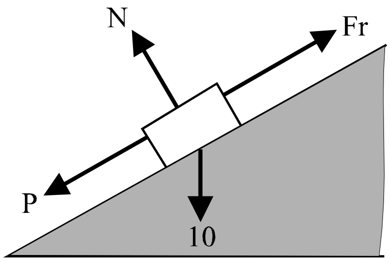
A particle of mass 10 kg is at rest on a rough inclined plane. A pulling force of *P* newtons is acting on the body parallel to the inclined plane. A normal force of *N* newtons together with a frictional force of *Fr* also act on the particle.

Which one of the following diagrams could show correctly the forces acting on the particle?

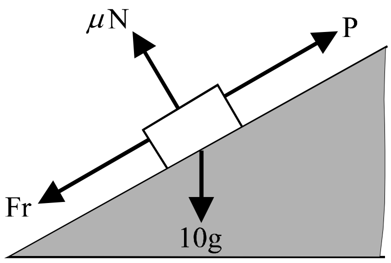
A.



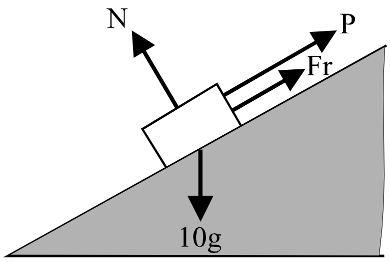
B.



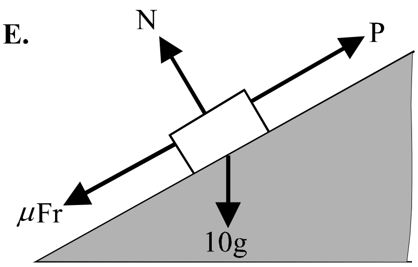
C.



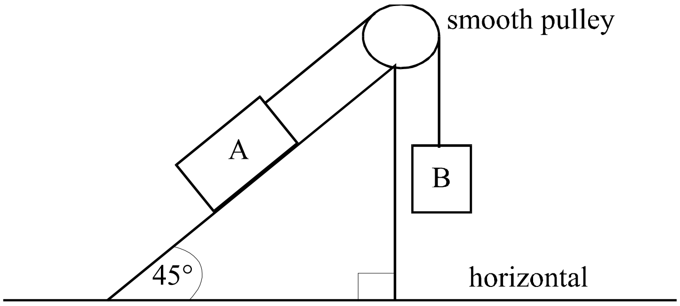
D.



E.



Question 30



Two particles *A* and *B* with masses of m_A and m_B respectively are connected by a light inextensible string, which passes over a smooth pulley indicated in the diagram above. Particle *A* rests on a rough plane inclined at an angle of 45° to the horizontal. The system is on the point of slipping down the inclined plane. If μ is the coefficient of friction between the plane and particle *A*, *T* is the tension in the string and *N* is the normal force, then *T*, in newtons, is given by

A. m_B

B. μm_A

C. $\dfrac{m_A g}{\sqrt{2}}$

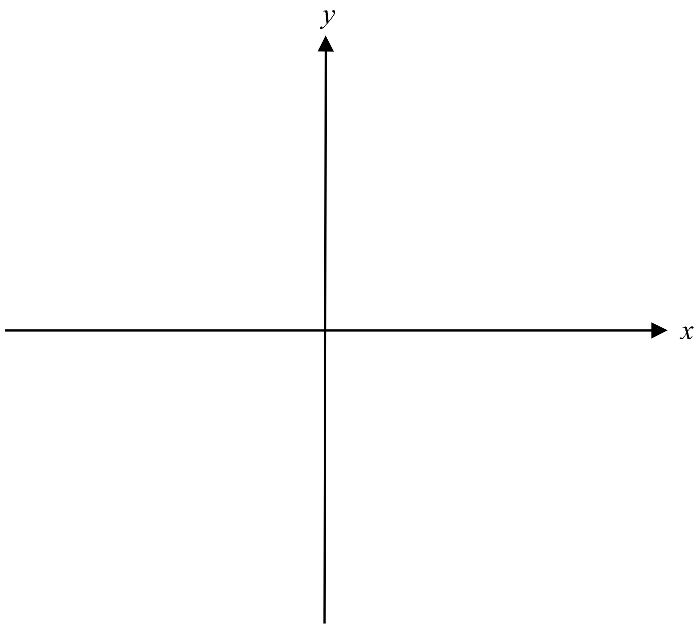
D. $m_A g\left(\mu + \dfrac{1}{\sqrt{2}}\right)$

E. $\dfrac{m_A g}{\sqrt{2}}(1-\mu)$

PART II

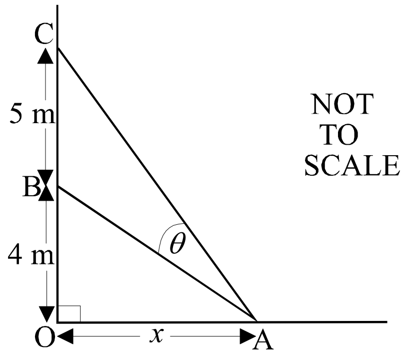
Question 1

Sketch the graph of $y = 3x^2 + \frac{2}{x^2}$ on the set of axes below, labelling clearly all the features of the graph. Express answers correct to 1 decimal place where appropriate.



2 marks

Question 2



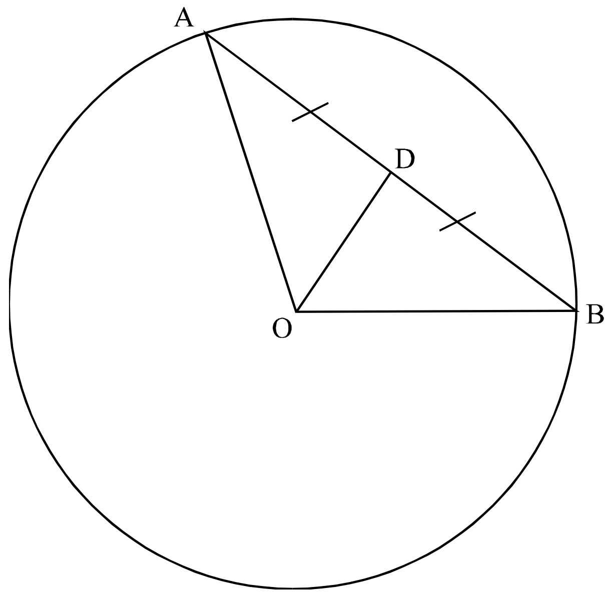
A building is being constructed from pre-cast concrete slabs. A vertical wall panel is being supported by two metal struts which are anchored at the variable point A on the horizontal floor. The distance OB is 4 metres, the distance BC is 5 metres, the distance AO is x metres and the angle BAC is θ radians.

Use an appropriate trigonometric identity to show that

$$\tan\theta = \frac{5x}{x^2 + 36}$$

2 marks

Question 3



The circle shown in the diagram above has its centre at O and the chord AB is bisected by OD.

Let $\overrightarrow{OA} = \underset{\sim}{u}$ and $\overrightarrow{OB} = \underset{\sim}{v}$.

Use a vector method to prove that $\angle BDO$ is a right angle.

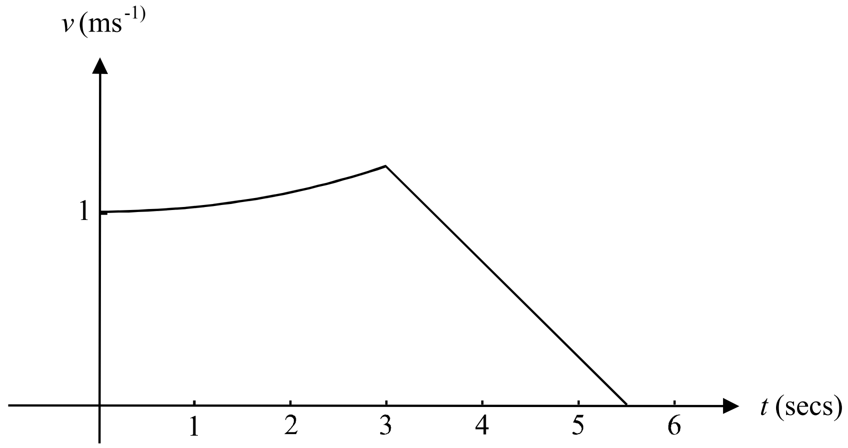
3 marks

Question 4

The velocity-time graph of a particle travelling in a straight line is given by

$$v(t)=\begin{cases}\dfrac{5}{\sqrt{25-t^2}} & 0\le t<3\\ -0.5t+2.75 & 3\le t\le 5.5\end{cases}$$

The graph of this function is shown below.



i. Find the acceleration of the particle at $t = 2$. Express your answer as an exact value.

1 mark

ii. Find the distance travelled by the particle on its journey. Express your answer as an exact value.

3 marks

Question 5

Let $p(z) = z^4 - 6z^3 + 16z^2 - 22z + 15$.

i. Find the remainder when $p(z)$ is divided by $z - 3i$.

1 mark

ii. Given that $2 - i$ is a solution to the equation $p(z) = 0$, find all the solutions to this equation.

3 marks

Question 6

A tank initially contains 1000 litres of a solution made up of water and salt with a concentration of 10 grams per litre. A solution of water and salt with a concentration of 15 grams per litre is flowing into the tank at a rate of 20 litres per minute. The mixture in the tank is constantly stirred and is therefore kept uniform. The solution flows out of the tank at the rate of 20 litres per minute. Let Q be the quantity of salt present in the tank after t minutes.

i. Show that the differential equation for Q in terms of t is given by

$$\frac{dQ}{dt} = 300 - \frac{Q}{50}.$$

1 mark

ii. Hence find an expression for Q in terms of t.

4 marks

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ▬ C D E

The answer selected is B. Only one answer should be selected.

1. A B C D E	11. A B C D E	21. A B C D E
2. A B C D E	12. A B C D E	22. A B C D E
3. A B C D E	13. A B C D E	23. A B C D E
4. A B C D E	14. A B C D E	24. A B C D E
5. A B C D E	15. A B C D E	25. A B C D E
6. A B C D E	16. A B C D E	26. A B C D E
7. A B C D E	17. A B C D E	27. A B C D E
8. A B C D E	18. A B C D E	28. A B C D E
9. A B C D E	19. A B C D E	29. A B C D E
10. A B C D E	20. A B C D E	30. A B C D E

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