VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2002

SPECIALIST MATHEMATICS
Written examination 1
(Facts, skills and applications)

Monday 4 November 2002

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I
MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 18 pages, with a detachable sheet of miscellaneous formulas in the centrefold and two blank pages for rough working.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part I

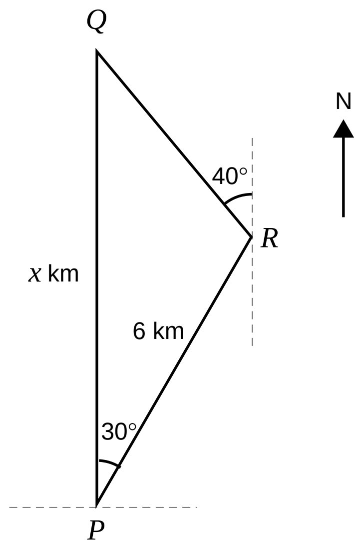
Answer **all** questions in pencil, on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

Question 1

A ship leaves a port, *P*, and sails 6 km on a heading of N30˚E to position *R*. It then heads N40˚W until it reaches a port, *Q*, which is directly north of *P*.



The distance *x* km from *P* to *Q* is given by

A. $\dfrac{x}{\sin(40^\circ)} = \dfrac{6}{\sin(110^\circ)}$

B. $\dfrac{x}{\sin(30^\circ)} = \dfrac{6}{\sin(110^\circ)}$

C. $\dfrac{x}{\sin(30^\circ)} = \dfrac{6}{\sin(40^\circ)}$

D. $\dfrac{x}{\sin(40^\circ)} = \dfrac{6}{\sin(30^\circ)}$

E. $\dfrac{x}{\sin(110^\circ)} = \dfrac{6}{\sin(40^\circ)}$

Question 2

The graph of $y = \dfrac{x^2 + 4}{x^2}$ has

A. a single asymptote $x = 0$, and no turning points.

B. two asymptotes $x = 0$ and $y = 0$, and no turning points.

C. two asymptotes $x = 0$ and $y = 1$, and no turning points.

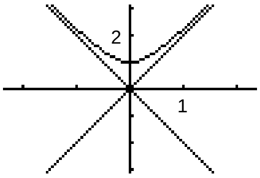
D. a single asymptote $x = 0$, and turning points at $x = \pm\sqrt{2}$.

E. a single asymptote $x = 0$, and intercepts at $x = \pm 2$.

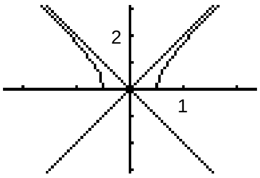
Question 3

Which one of the following could be the graph of the curve with equation $4x^2 - y^2 + 1 = 0$?

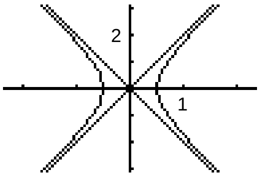
A.



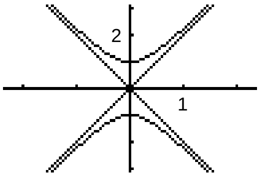
B.



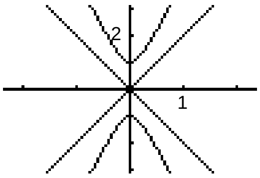
C.



D.



E.



Question 4

Let $f: (0, \frac{\pi}{2}) \rightarrow R$ where $f(x) = \sec^2(x) + \text{cosec}^2(x)$.

Which one of the following statements is true?

A. f is identical to the function $h: (0, \frac{\pi}{2}) \rightarrow R$ where $h(x) = \sec^2(x)\text{cosec}^2(x)$

B. f is identical to the function $g: (0, \frac{\pi}{2}) \rightarrow R$ where $g(x) = \tan^2(x) + \cot^2(x)$

C. f is equal to 1 for all values of x in $(0, \frac{\pi}{2})$

D. f has range $[1, \infty)$

E. f has range $[2, \infty)$

Question 5

The exact value of $\text{Sin}^{-1}(\frac{1}{\sqrt{2}})\text{Cos}^{-1}(\frac{1}{2})$ is

A. 0.822

B. 2700

C. $\frac{7\pi}{24}$

D. $\frac{\pi^2}{24}$

E. $\frac{\pi^2}{12}$

Question 6

If $z = x + yi$, where x and y are non-zero real numbers, which one of the following **must** be a pure imaginary number (that is, a complex number with real part zero)?

A. $z - \bar{z}$

B. $z + \bar{z}$

C. iz

D. $i\bar{z}$

E. $\text{Im}(\bar{z})$

Question 7

If $z = -2i$, then $|z^2|$ and $\text{Arg}(z^2)$ are respectively

A. 4 and 0

B. 4 and π

C. 4 and $-\pi$

D. -4 and π

E. -4 and $-\pi$

Question 8

Given that $(z - 2 + i)$ is a factor of $P(z) = 2z^3 - 7z^2 + 6z + 5$, which one of the following statements must be true?

A. $P(-2 + i) = 0$

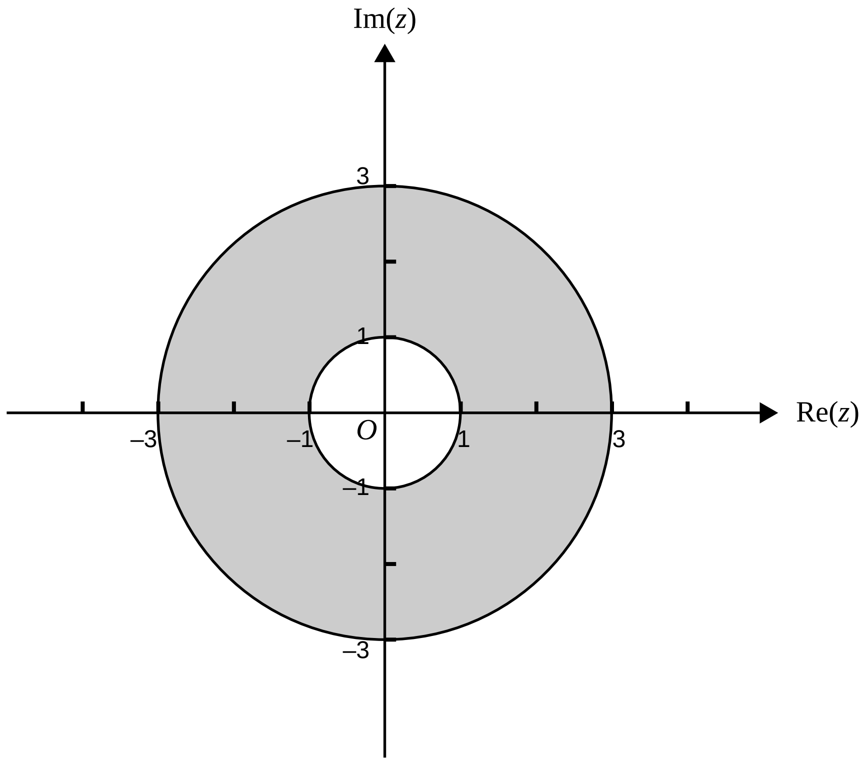
B. $P(-2 - i) = 0$

C. $P(z) = 0$ has no real roots

D. $P(z) = 0$ has one real root and two complex roots

E. $P(z) = 0$ has two real roots and one complex root

Question 9



Given that $z \in C$, the shaded region (with boundaries included) is best described by

A. $\{z: 1 \le z\bar{z} \le 3\}$

B. $\{z: 1 \le z\bar{z} \le 9\}$

C. $\{z: 1 \le |z| \le 9\}$

D. $\{z: 1 \le |z|^2 \le 3\}$

E. $\{z: (|z| \le 3) - (|z| \le 1)\}$

Question 10

$\dfrac{3x+4}{(x-4)^2}$ expressed in partial fractions has the form

A. $\dfrac{A}{(x+4)} + \dfrac{B}{(x-4)}$

B. $\dfrac{A}{(x-4)} + \dfrac{B}{(x-4)}$

C. $\dfrac{A}{(x-4)} + \dfrac{B}{(x-4)^2}$

D. $\dfrac{A}{(x-4)} + \dfrac{Bx+C}{(x-4)^2}, \ B \ne 0$

E. $\dfrac{A}{(x-4)^2} + \dfrac{B}{(x-4)^2}$

Question 11

The derivative of $x \operatorname{Cos}^{-1}(x)$ with respect to x is $\operatorname{Cos}^{-1}(x) - \dfrac{x}{\sqrt{1-x^2}}$.

It follows that an antiderivative of $\operatorname{Cos}^{-1}(x)$ is

A. $x \operatorname{Cos}^{-1}(x) + \displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$

B. $\displaystyle\int x \operatorname{Cos}^{-1}(x)dx + \int \frac{x}{\sqrt{1-x^2}}\,dx$

C. $x \operatorname{Cos}^{-1}(x) - \displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$

D. $\displaystyle\int x \operatorname{Cos}^{-1}(x)dx + \frac{x}{\sqrt{1-x^2}}$

E. $x \operatorname{Cos}^{-1}(x) + \dfrac{x}{\sqrt{1-x^2}}$

Question 12

An antiderivative of $\dfrac{e^{2x}}{2e^{2x}-1}$ (for $e^{2x} > \dfrac{1}{2}$) is

A. $\dfrac{1}{2}(x - e^{2x})$

B. $4\log_e(2e^{2x} - 1)$

C. $2\log_e(2e^{2x} - 1)$

D. $\dfrac{1}{2}\log_e(2e^{2x} - 1)$

E. $\dfrac{1}{4}\log_e(2e^{2x} - 1)$

Question 13

Using an appropriate substitution, $\int_0^4 (2x + 3)\sqrt{2x + 1}\, dx$ is equal to

A. $\frac{1}{2}\int_0^4 (u+2)\sqrt{u}\, du$

B. $\frac{1}{2}\int_1^3 (u+2)\sqrt{u}\, du$

C. $\frac{1}{2}\int_1^9 (u+2)\sqrt{u}\, du$

D. $\int_1^9 (u+2)\sqrt{u}\, du$

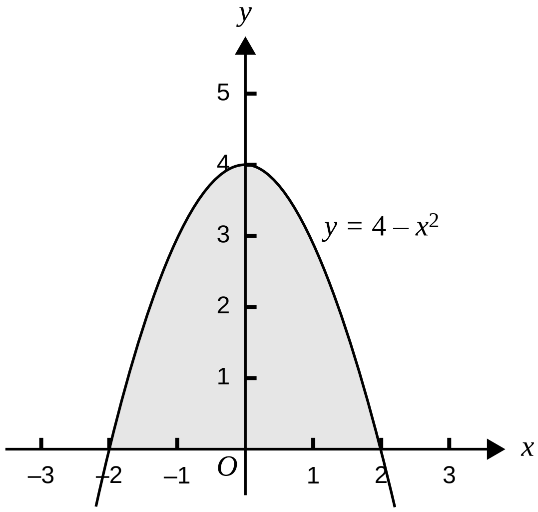
E. $2\int_1^9 (u+2)\sqrt{u}\, du$

Question 14

The value of $\int_0^1 \frac{x^2 - 2x}{2\cos(x)}\, dx$, correct to four decimal places, is

A. –0.4369
B. –0.3334
C. –0.2622
D. –0.0484
E. 0.4369

Question 15



The shaded region is bounded by the x-axis and the curve with equation $y = 4 - x^2$.
This region is rotated about the y-axis to form a solid of revolution.
The volume of the solid, in cubic units, is

A. 6π

B. 8π

C. $\frac{32\pi}{3}$

D. 16π

E. $\frac{64\pi}{3}$

Question 16

If $y = e^{kx}$ satisfies the differential equation $\frac{d^2y}{dx^2} = 5\frac{dy}{dx} - 6y$, the possible values for k are

A. –6 and 1
B. –1 and 6
C. –5 and 6
D. –3 and –2
E. 2 and 3

Question 17

The radius of a sphere is increasing at a rate of 3 cm/min.
When the radius is 8 cm, the rate of increase, in cm^3/min, of the volume of the sphere is

A. $85\frac{1}{3}\pi$

B. 256π

C. $682\frac{2}{3}\pi$

D. 768π

E. 2048π

Question 18

To reach a lookout from a campsite, a bushwalker walks 2 km due south up a slope which is inclined at 30° to the horizontal.

Let $\underset{\sim}{i}$ be a unit vector due east, $\underset{\sim}{j}$ a unit vector due north, and $\underset{\sim}{k}$ a unit vector vertically up.

The position vector of the lookout relative to the campsite is

A. $-\underset{\sim}{j}+\sqrt{3}\,\underset{\sim}{k}$

B. $-\sqrt{3}\,\underset{\sim}{j}+\underset{\sim}{k}$

C. $-2\underset{\sim}{j}+\underset{\sim}{k}$

D. $\sqrt{2}(-\underset{\sim}{j}+\underset{\sim}{k})$

E. $-2\underset{\sim}{j}+\sqrt{3}\,\underset{\sim}{k}$

Question 19

A vector perpendicular to $2\underset{\sim}{i}-2\underset{\sim}{j}-3\underset{\sim}{k}$, and with magnitude 4, is

A. $\frac{4}{3}(\underset{\sim}{i}-2\underset{\sim}{j}-2\underset{\sim}{k})$

B. $\frac{4}{9}(\underset{\sim}{i}-2\underset{\sim}{j}+2\underset{\sim}{k})$

C. $4(\underset{\sim}{i}-2\underset{\sim}{j}+2\underset{\sim}{k})$

D. $\frac{4}{3}(-2\underset{\sim}{i}+\underset{\sim}{j}-2\underset{\sim}{k})$

E. $\frac{4}{9}(-2\underset{\sim}{i}+\underset{\sim}{j}-2\underset{\sim}{k})$

Question 20

The vector resolute of $-\underset{\sim}{i}-\underset{\sim}{j}+3\underset{\sim}{k}$ in the direction of $\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$ is $-\underset{\sim}{i}-2\underset{\sim}{j}+2\underset{\sim}{k}$.

The vector resolute of $-\underset{\sim}{i}-\underset{\sim}{j}+3\underset{\sim}{k}$ perpendicular to $\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$ is

A. $\underset{\sim}{j}+\underset{\sim}{k}$

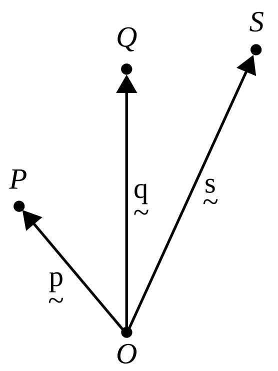
B. $-\underset{\sim}{j}-\underset{\sim}{k}$

C. $-2\underset{\sim}{i}-3\underset{\sim}{j}+5\underset{\sim}{k}$

D. $2\underset{\sim}{i}+3\underset{\sim}{j}-5\underset{\sim}{k}$

E. $\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$

Question 21



Let the three distinct points P, Q and S have non-zero position vectors $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{s}$ respectively.

To prove that P, Q and S lie in a straight line, it is sufficient to show that

A. $\underset{\sim}{s} = k(\underset{\sim}{q} - \underset{\sim}{p}),\ k \in R$

B. $(\underset{\sim}{q} - \underset{\sim}{p}) = k(\underset{\sim}{s} - \underset{\sim}{q}),\ k \in R$

C. $(\underset{\sim}{q} - \underset{\sim}{p}).(\underset{\sim}{s} - \underset{\sim}{q}) = 0$

D. $\underset{\sim}{p} + \underset{\sim}{q} + \underset{\sim}{s} = \underset{\sim}{0}$

E. $\underset{\sim}{p} + \underset{\sim}{q} + \underset{\sim}{s} \neq \underset{\sim}{0}$

Question 22

The velocity of a particle at time t, $t \geq 0$, is given by $\underset{\sim}{v}(t) = \sin(t-1)\underset{\sim}{i} + 5\underset{\sim}{j} - 3e^{(1-t)}\underset{\sim}{k}$.

The acceleration of the particle when $t = 1$ is

A. $\underset{\sim}{i} + 3\underset{\sim}{k}$

B. $-\underset{\sim}{i} + 3\underset{\sim}{k}$

C. $\underset{\sim}{i} - 3\underset{\sim}{k}$

D. $\underset{\sim}{i} + 5\underset{\sim}{j} + 3\underset{\sim}{k}$

E. $-\underset{\sim}{i} + 5\underset{\sim}{j} + 3\underset{\sim}{k}$

Question 23

At time $t \geq 0,$ a particle has displacement

$$\underset{\sim}{r}(t) = (3t - 6)\underset{\sim}{i} - (t^2 - 6t - 16)\underset{\sim}{j}$$

where $\underset{\sim}{i}$ is a horizontal unit vector and $\underset{\sim}{j}$ is a unit vector in the vertically up direction.

The particle reaches its maximum height when *t* is

A. 1
B. 2
C. 3
D. 8
E. 10

Question 24

The acceleration of a particle at time *t*, $t \geq 0$, is given by $\underset{\sim}{a}(t) = 2\sin(t)\underset{\sim}{i}$.

The velocity of the particle when $t = \pi$ is $2\underset{\sim}{i} + 2\underset{\sim}{j}$.

The **initial velocity** of the particle is

A. $-2\underset{\sim}{i}$

B. $2\underset{\sim}{j}$

C. $-2\underset{\sim}{i} + 2\underset{\sim}{j}$

D. $2\underset{\sim}{i} + 2\underset{\sim}{j}$

E. $6\underset{\sim}{i} + 2\underset{\sim}{j}$

Question 25

A particle is acted on by two forces, one of magnitude 5 newtons acting due north, and the other of magnitude *x* newtons acting due east. The magnitude of the resultant force is 8 newtons.

The number *x* is

A. 3
B. $\sqrt{39}$
C. 7
D. $\sqrt{89}$
E. 13

Question 26

A particle of mass 2 kg has a horizontal velocity component of magnitude 12 m/s, and a vertical velocity component of magnitude 5 m/s.

The magnitude, in kg m/s, of the momentum of the particle is

A. 10
B. 14
C. 24
D. 26
E. 34

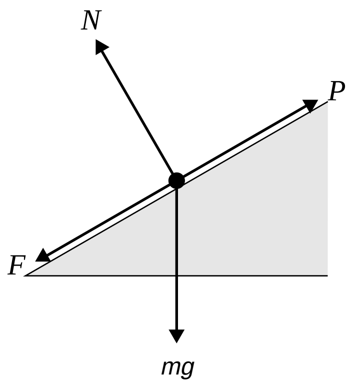
PART I – continued

Question 27

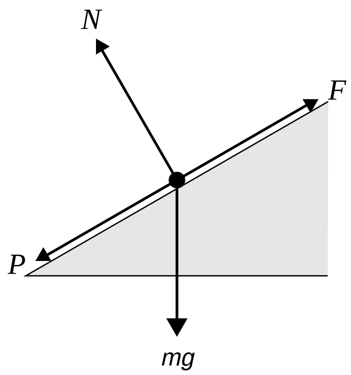
A particle of mass m kg rests on a rough inclined plane. The particle is stationary. There is a normal reaction of magnitude N newtons, and F newtons is the magnitude of the force due to friction. Where present, P newtons is the magnitude of a force applied to the particle parallel to the plane.

Which one of the following diagrams **cannot** be a correct representation of the forces acting on the particle?

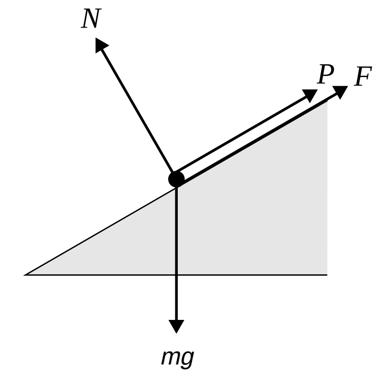
A.



B.



C.



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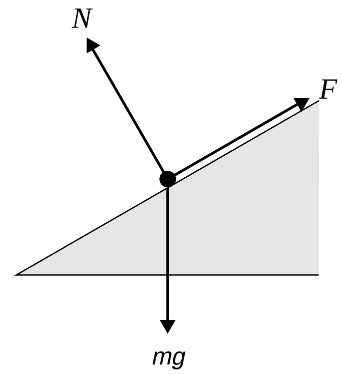
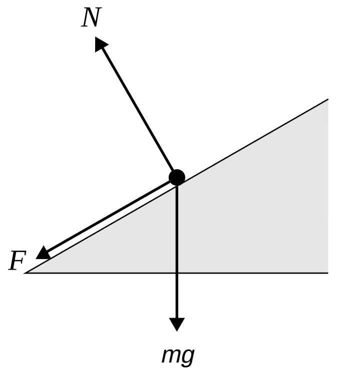
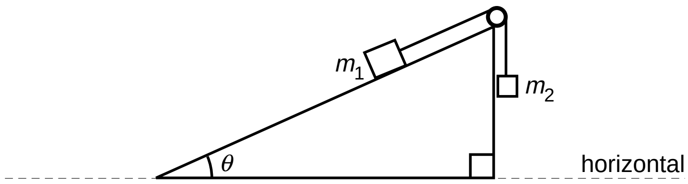


E.



Question 28

Two particles of masses m_1 and m_2 are connected by a light string that passes over a smooth pulley as shown in the diagram. The particle of mass m_1 rests on a smooth, inclined plane.



If the system is in equilibrium, $\frac{m_1}{m_2}$ is equal to

A. $\sin(\theta)$

B. $\cos(\theta)$

C. $\frac{1}{\sin(\theta)}$

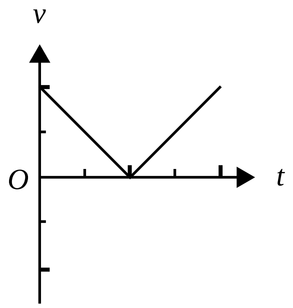
D. $\frac{1}{\cos(\theta)}$

E. 1

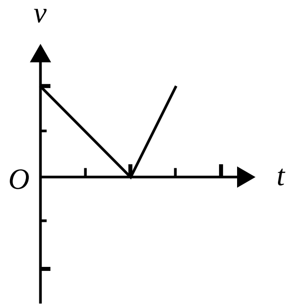
Question 29

A ball is dropped from rest on to a concrete floor and bounces vertically to half its drop height.
Which one of the following velocity-time graphs could represent the motion of the ball?

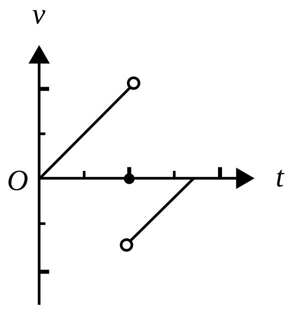
A.



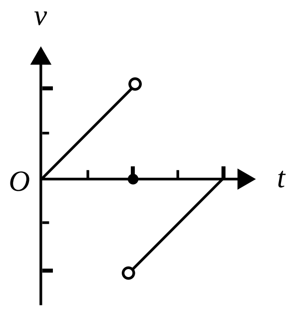
B.



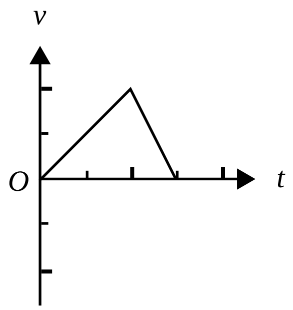
C.



D.



E.



Question 30

A 150 litre cylinder of air contains 20% oxygen. The amount of oxygen in the cylinder is to be increased by pumping in pure oxygen at a constant rate of 10 litres/minute, while removing the uniformly mixed air at the same rate.

If P litres is the volume of oxygen in the cylinder at time t minutes after the pumping begins, a differential equation for P in terms of t is

A. $\frac{dP}{dt} = 8P;\ t = 0, P = 30$

B. $\frac{dP}{dt} = 8P;\ t = 0, P = 150$

C. $\frac{dP}{dt} = 30 + 10t;\ t = 0, P = 30$

D. $\frac{dP}{dt} = 10 - \frac{P}{15};\ t = 0, P = 30$

E. $\frac{dP}{dt} = 10 - \frac{P}{15};\ t = 0, P = 150$

Working space

Working space

END OF PART I MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2002

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 4 November 2002

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II
QUESTION AND ANSWER BOOK

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Materials supplied
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Instructions
- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

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Instructions for Part II

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

Where an **exact** answer is required for a question, appropriate working must be shown.

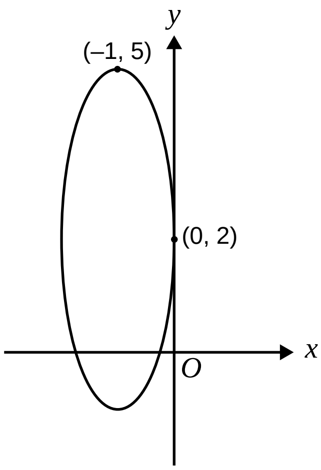
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The diagram shows an ellipse with axes of symmetry parallel to the coordinate axes.



a. Write down the equation of the ellipse.

2 marks

b. Describe a sequence of transformations which, when applied to the unit circle with equation $x^2 + y^2 = 1$, produces this ellipse.

2 marks

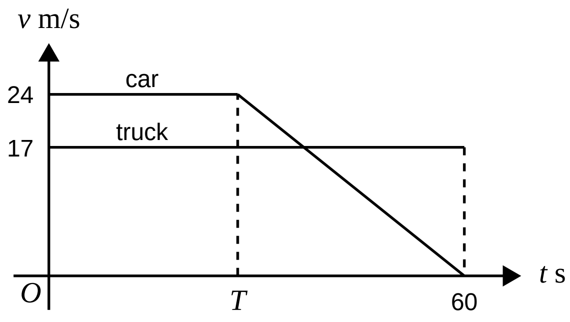
PART II – continued
TURN OVER

Question 2

A car travelling at 24 m/s overtakes a truck travelling at a constant speed of 17 m/s along a straight road. T seconds later, the car decelerates uniformly to rest.

The truck continues at constant speed and it passes the car at the instant the car comes to a stop. This is exactly 60 seconds after the car had passed the truck.

The velocity-time graph representing this situation is shown below.



Find T.

3 marks

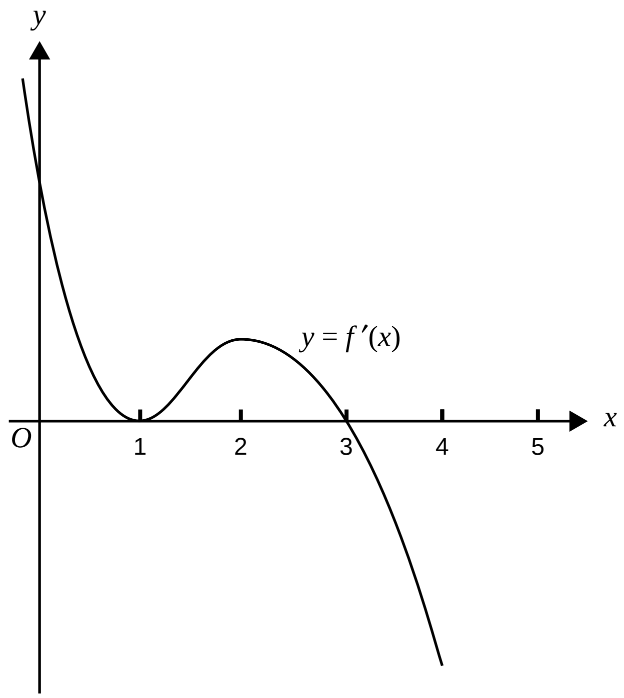
Question 3

Find an antiderivative of $\dfrac{\sin\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)}$.

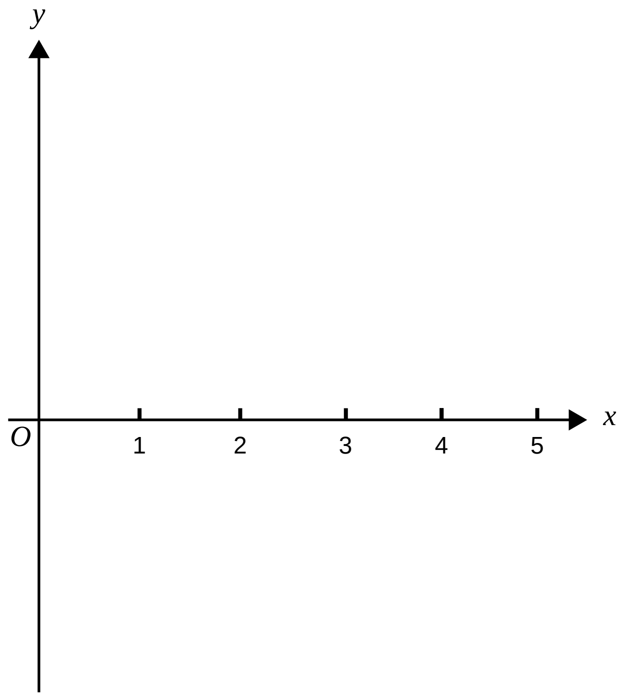
2 marks

Working space

Question 4



The graph of the derivative of a function *f* is shown above.
On the axes below, draw **two** graphs each of which could be the graph of the function *f*.
In each case, show clearly any stationary points.



4 marks

Question 5

Let $\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} + a\underset{\sim}{j} + \underset{\sim}{k}$, where $a \in R$. The angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is 60°.

Find the exact value of a.

4 marks

Working space

Question 6

A particle travels in a straight line with velocity v m/s at time t s. The acceleration of the particle, a m/s^2, is given by $a = -2 + \sqrt{v^2 + 5}$.

Find, correct to two significant figures, the time taken in seconds for the speed of the particle to increase from 3 m/s to 10 m/s.

3 marks

END OF PART II QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
hyperbola:	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$

$\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$

$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$

$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$

$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$

$-\pi<\operatorname{Arg} z\le\pi$

$z_1z_2=r_1r_2\operatorname{cis}(\theta_1+\theta_2)$

$\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$ | $\int \frac{1}{x} dx = \log_e(x) + c$, for $x > 0$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$ | $\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$ | $\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$ | $\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\text{Sin}^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ | $\int \frac{1}{\sqrt{a^2-x^2}} dx = \text{Sin}^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}\left(\text{Cos}^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$ | $\int \frac{-1}{\sqrt{a^2-x^2}} dx = \text{Cos}^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}\left(\text{Tan}^{-1}(x)\right) = \frac{1}{1+x^2}$ | $\int \frac{a}{a^2+x^2} dx = \text{Tan}^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)\big(f(a) + f(b)\big)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h\, f(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET