Part 1: Multiple-choice questions

Question 1

If $\overrightarrow{OA} = 5\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{OB} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$, then $\overrightarrow{AB}$ is equal to

A. $9\underset{\sim}{i} - 4\underset{\sim}{j} - \underset{\sim}{k}$

B. $-\underset{\sim}{i} - 4\underset{\sim}{j} + \underset{\sim}{k}$

C. 24

D. $\underset{\sim}{i} - \underset{\sim}{k}$

E. $-\underset{\sim}{i} + \underset{\sim}{k}$

Question 2

The set of points in the complex plane defined by $|z - 1 + 2i| = 2$ is

A. the circle with centre (– 1, 2) and radius 2

B. the circle with centre (1 , – 2) and radius 2

C. the circle with centre (– 1, 2) and radius 4

D. the circle with centre (1, – 2) and radius 4

E. the line $\text{Re}(z) + \text{Im}(z) = 1$

Question 3

The implied domain of the function with rule $f(x) = -2\,\text{Sin}^{-1}(3x - 1) - 2$ is

A. $[-1, 1]$

B. $\left[-\frac{4}{3}, 0\right]$

C. $\left[0, \frac{2}{3}\right]$

D. $\left[\frac{2-\pi}{6}, \frac{2+\pi}{6}\right]$

E. $\left[\frac{1}{3}, \frac{1+\pi}{3}\right]$

Question 4

If $\tan x = \frac{2}{5}$ and $\pi < x < \frac{3\pi}{2}$, then $\sec x =$

A. $-\frac{\sqrt{29}}{5}$

B. $\frac{\sqrt{29}}{5}$

C. $-\frac{\sqrt{29}}{2}$

D. $\frac{\sqrt{29}}{2}$

E. $\frac{3}{5}$

Question 5

A polar form of $-1-i$ is

A. $2\text{cis}\left(\frac{\pi}{4}\right)$

B. $\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$

C. $\sqrt{2}\text{cis}\left(-\frac{3\pi}{4}\right)$

D. $2\text{cis}\left(-\frac{3\pi}{4}\right)$

E. $\sqrt{2}\text{cis}\left(\frac{5\pi}{4}\right)$

Question 6

An antiderivative of $\frac{3}{\sqrt{1-4x^2}}$ is

A. $-\frac{3}{2}\text{Cos}^{-1}(2x)$

B. $-3\,\text{Cos}^{-1}(2x)$

C. $3\,\text{Sin}^{-1}\left(\frac{x}{2}\right)$

D. $\frac{1}{2}\text{Sin}^{-1}(2x)$

E. $3\,\text{Sin}^{-1}(2x)$

Question 7

Using a suitable substitution $\int_2^3 \frac{4}{x^2}\log_e\left(\frac{2}{x}\right)dx$ can be expressed as

A. $-2\int_1^{\frac{2}{3}} \log_e(u)\,du$

B. $-2\int_2^3 \log_e(u)\,du$

C. $\int_2^{\frac{2}{3}} u^2\log_e(u)\,du$

D. $\int_2^3 u^2\log_e(u)\,du$

E. $-2\int_2^{\frac{2}{3}} u^2\log_e(u)\,du$

Question 8

$(1+i)^4$ can be simplified to

A. $-4\sqrt{2}$

B. -4

C. 2

D. 4

E. 16

Question 9

If $z = 2\text{cis}\left(\frac{\pi}{7}\right)$, then $\bar{z} =$

A. $\frac{1}{2}\text{cis}\left(-\frac{\pi}{7}\right)$

B. $\frac{1}{2}\text{cis}\left(\frac{\pi}{7}\right)$

C. $-2\text{cis}\left(-\frac{\pi}{7}\right)$

D. $2\text{cis}\left(-\frac{\pi}{7}\right)$

E. $-2\text{cis}\left(\frac{\pi}{7}\right)$

Question 10

The position vector, $\underset{\sim}{r}(t)$, in metres, of a particle at time t is given by $\underset{\sim}{r}(t) = 3e^{-t}\,\underset{\sim}{i} + \frac{3}{2}\cos(2t)\,\underset{\sim}{j}$. The speed of the particle, in m/s at time $t = 0$ seconds is

A. $\frac{3\sqrt{5}}{2}$

B. $3\sqrt{2}$

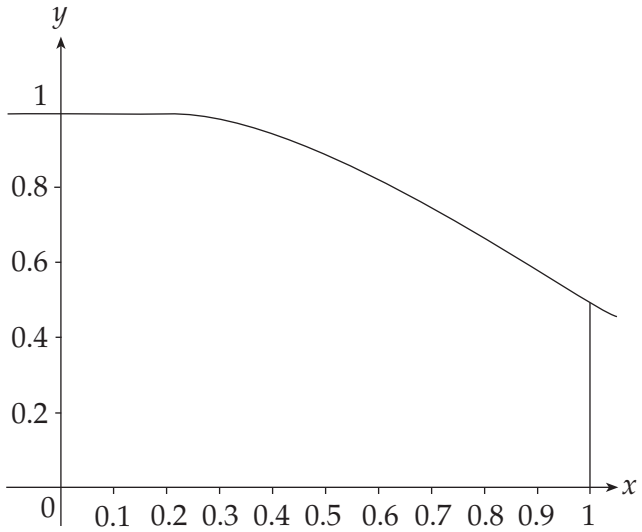
C. 9

D. 3

E. $\frac{9}{2}$

Question 11

Part of the graph of $y = \dfrac{1}{1+x^3}$ is shown below. The trapezoidal rule with two equal intervals is used to approximate the area enclosed by the curve, the line $x = 1$ and the co-ordinate axes. The value obtained is



A. $\dfrac{59}{72}$

B. $\dfrac{59}{36}$

C. $\dfrac{83}{100}$

D. $\dfrac{25}{36}$

E. $\dfrac{3}{4}$

Question 12

An antiderivative of $\dfrac{1}{x\log_e(5x)}$ is

A. $\log_e(5x)$

B. $\log_e(\log_e(5x))$

C. $\dfrac{1}{5}\log_e(\log_e(5x))$

D. $2\log_e(\log_e(5x))$

E. $\log_e(x\log_e(5x))$

Question 13

The equation of a hyperbola is given by $\frac{(x-2)^2}{9} - \frac{y^2}{36} = 1$. The equations of the asymptotes are

A. $y = \pm 2x$

B. $y = \pm 4(x-2)$

C. $y = \pm 2(x-2)$

D. $y = \pm \frac{1}{2}(x-2)$

E. $y = \pm 2(x+2)$

Question 14

The exact value of $\int_1^4 \frac{x-2}{x^2-4x+7}\,dx$ is

A. $\log_e\left(\frac{7}{4}\right)$

B. $2\log_e\left(\frac{7}{4}\right)$

C. $\log_e\left(\frac{\sqrt{7}}{2}\right)$

D. $\log_e\left(\frac{7}{2}\right)$

E. $\log_e\left(\frac{\sqrt{7}}{4}\right)$

Question 15

The angle between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ is

A. 37.4°

B. 71.2°

C. 75.6°

D. 78.5°

E. 85.8°

Question 16

When added to a quantity of water, 8 grams of a chemical dissolves at a rate equal to 25% of the amount of undissolved chemical, per hour. If l grams is the amount of dissolved chemical at time, t hours, then l satisfies the differential equation

A. $\frac{dl}{dt} = \frac{8-l}{4}$

B. $\frac{dl}{dt} = \frac{l-8}{4}$

C. $\frac{dl}{dt} = l-8$

D. $\frac{dl}{dt} = 8-l$

E. $\frac{dl}{dt} = 8-\frac{l}{4}$

Question 17

If $y = \sin(3x-1)$, then

A. $y - 9\frac{d^2y}{dx^2} = 0$

B. $y + 9\frac{d^2y}{dx^2} = 0$

C. $9y - \frac{d^2y}{dx^2} = 0$

D. $9y - 3\frac{dy}{dx} + \frac{d^2y}{dx^2} = 9\cos(3x-1)$

E. $9y + 3\frac{dy}{dx} + \frac{d^2y}{dx^2} = 9\cos(3x-1)$

Question 18

If $u = 3\text{cis}\left(\frac{5\pi}{6}\right)$ and $v = 2\text{cis}\left(\frac{3\pi}{4}\right)$, then uv (in principal value form) is equal to

A. $6\text{cis}\left(-\frac{5\pi}{12}\right)$

B. $6\text{cis}\left(\frac{19\pi}{12}\right)$

C. $6\text{cis}\left(\frac{5\pi^2}{8}\right)$

D. $5\text{cis}\left(-\frac{5\pi}{12}\right)$

E. $5\text{cis}\left(\frac{19\pi}{12}\right)$

Question 19

A tennis ball is tossed vertically with a speed of 15 m/s from the top of a building 40 metres high and then falls to the ground below. The time, in seconds, that the tennis ball is in the air is

A. 1.7

B. 2.1

C. 3.1

D. 3.2

E. 4.8

Question 20

Euler's method, with a step size of 0.2 is used to solve the differential equation $\frac{dy}{dx} = \frac{1}{x^2}$ with $y = 3$ at $x = 1$. The value obtained for y at $x = 1.4$, correct to three decimal places, is

A. 0.714

B. 3.169

C. 3.241

D. 3.286

E. 3.339

Question 21

Which one of the following vectors has magnitude 22 and is parallel to $-2\underset{\sim}{i}+6\underset{\sim}{j}-9\underset{\sim}{k}$?

A. $\frac{22}{5}(2\underset{\sim}{i}-6\underset{\sim}{j}+9\underset{\sim}{k})$

B. $\frac{5}{22}(2\underset{\sim}{i}-6\underset{\sim}{j}+9\underset{\sim}{k})$

C. $-4\underset{\sim}{i}+12\underset{\sim}{j}+18\underset{\sim}{k}$

D. $\frac{2}{11}(-2\underset{\sim}{i}+6\underset{\sim}{j}-9\underset{\sim}{k})$

E. $4\underset{\sim}{i}-12\underset{\sim}{j}+18\underset{\sim}{k}$

Question 22

If $\underset{\sim}{a}=2\underset{\sim}{i}-3\underset{\sim}{j}+4\underset{\sim}{k}$ and $\underset{\sim}{b}=\underset{\sim}{i}-2\underset{\sim}{j}$, the scalar resolute of $\underset{\sim}{a}$ parallel to $\underset{\sim}{b}$ is

A. $\frac{8}{\sqrt{29}}$

B. $\frac{8}{3}$

C. $\frac{8}{5}$

D. $\frac{8}{\sqrt{5}}$

E. 8

Question 23

A particle moves in a straight line such that its velocity is given by $v=\frac{x}{2}$. The acceleration of the particle is

A. $\frac{1}{2}$

B. $\frac{x}{2}$

C. $\frac{x}{4}$

D. $\frac{x}{8}$

E. $\frac{x^2}{4}$

Question 24

A particle moves with a constant velocity of $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$. If its initial position is $2\underset{\sim}{i} - 3\underset{\sim}{k}$, then its position vector $\underset{\sim}{r}(t)$ is equal to

A. $(2t+1)\underset{\sim}{i} - 2\underset{\sim}{j} + (1-3t)\underset{\sim}{k}$

B. $(t+2)\underset{\sim}{i} - 2t\underset{\sim}{j} + (t-3)\underset{\sim}{k}$

C. $2t\underset{\sim}{i} - 3t\underset{\sim}{k}$

D. $t\underset{\sim}{i} - 2t\underset{\sim}{j} + t\underset{\sim}{k}$

E. $3t\underset{\sim}{i} - 2t\underset{\sim}{j} - 2t\underset{\sim}{k}$

Question 25

If $y = \text{Sin}^{-1}\dfrac{2}{x}$ and $x > 2$, then $\dfrac{dy}{dx} =$

A. $\text{Cos}^{-1}\left(\dfrac{2}{x}\right)$

B. $\dfrac{2}{x\sqrt{1-x^2}}$

C. $\dfrac{-2x\log_e(x)}{\sqrt{x^2-4}}$

D. $\dfrac{-2}{\sqrt{x^2-4}}$

E. $\dfrac{-2}{x\sqrt{x^2-4}}$

Question 26

A particle starts from rest at time $t = 0$ and moves in a straight line such that its acceleration, a, at time t is given by $a = 3e^{-0.2t} + 2$. The velocity of the particle at $t = 2$, correct to two decimal places, is

A. –6.05

B. –0.40

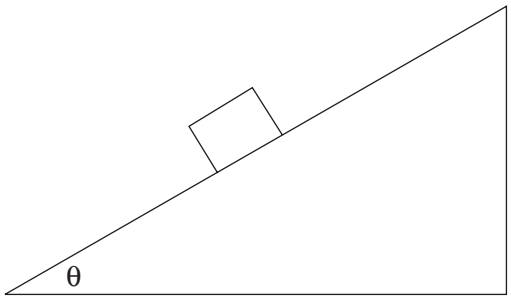
C. 3.01

D. 8.94

E. 8.95

Question 27

A body of mass m kg slides down a rough surface with a co-efficient of friction μ, inclined at an angle of θ to the horizontal. The acceleration of the body down the ramp in m/s^2 is



A. 0

B. g

C. $g\sin\theta$

D. $g\sin\theta - \mu g\cos\theta$

E. $g\cos\theta - \mu g\sin\theta$

Question 28

A box of mass 4 kg sits on the floor of a lift accelerating upwards at a magnitude of 3 m/s^2. The reaction force, in Newtons of the floor on the box, is

A. 12.0

B. 16.0

C. 27.2

D. 39.2

E. 51.2

Question 29

A body of mass 4 kg moves in a straight line to the right. Its velocity decreases from 8 m/s to 5 m/s in a time of 3 seconds. The change in momentum of the body, in kg m/s, in the direction of motion, is

A. – 4

B. –12

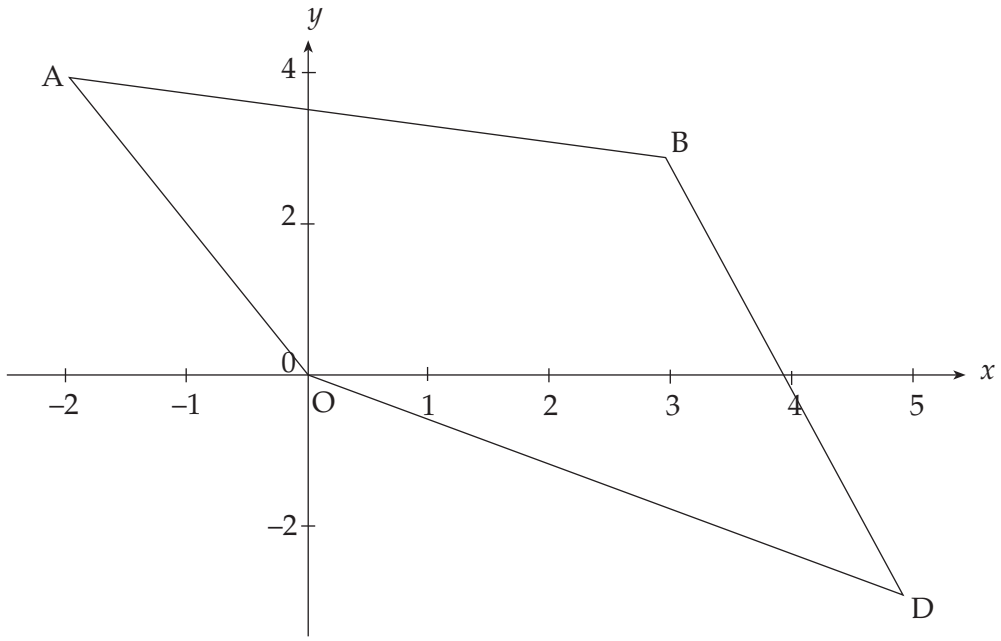
C. –24

D. 12

E. 20

Question 30

The vertices O, A, B, and D of a quadrilateral are shown below.



Given $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$, and $\overrightarrow{OD} = \underset{\sim}{d}$, in order to establish that the diagonals of the quadrilateral are perpendicular, it must be shown that

A. $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

B. $\underset{\sim}{a} \cdot \underset{\sim}{d} = 0$

C. $\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d} = 0$

D. $\left(\underset{\sim}{d} - \underset{\sim}{a}\right) \cdot \underset{\sim}{b} = 0$

E. $\left(\underset{\sim}{d} - \underset{\sim}{b}\right) \cdot \underset{\sim}{a} = 0$

PART II
SHORT ANSWER QUESTION BOOK

Question 1

a. Express $\frac{1}{9-x^2}$ in partial fraction form.

2 marks

b. Find an antiderivative of $\frac{2}{\sqrt{9-x^2}}$.

1 mark

c. The region enclosed by the graph of $y = \frac{1}{\sqrt{9 - x^2}} + 1$, the line $x = 2$ and the co-ordinate axes is rotated about the x-axis. Using calculus, find the exact value of the volume of the solid obtained.

3 marks

Question 2

Given $h'(x) = \dfrac{x}{\sqrt{1-x}}$ and $h(-3) = \dfrac{4}{3}$, show that $h(x) = -\dfrac{2}{3}(x+2)\sqrt{(1-x)}$

3 marks

Question 3

Let $z = 3\text{cis}\left(\frac{3\pi}{4}\right)$ and $w = 1 - \sqrt{3}i$

a. Find Arg(w)

1 mark

b. Use the result of part **a.** to find $\text{Arg}\left(z^2 w\right)$.

1 mark

c. Find $\frac{z}{w}$ in polar form.

1 mark

Question 4

A body of mass 2 kg is subject to two forces, one of magnitude 3 Newtons acting at a bearing of N30°E, and the other of magnitude 4 Newtons acting at a bearing of N45°W.

a. Find, correct to two decimal places, the magnitude of the resultant force.

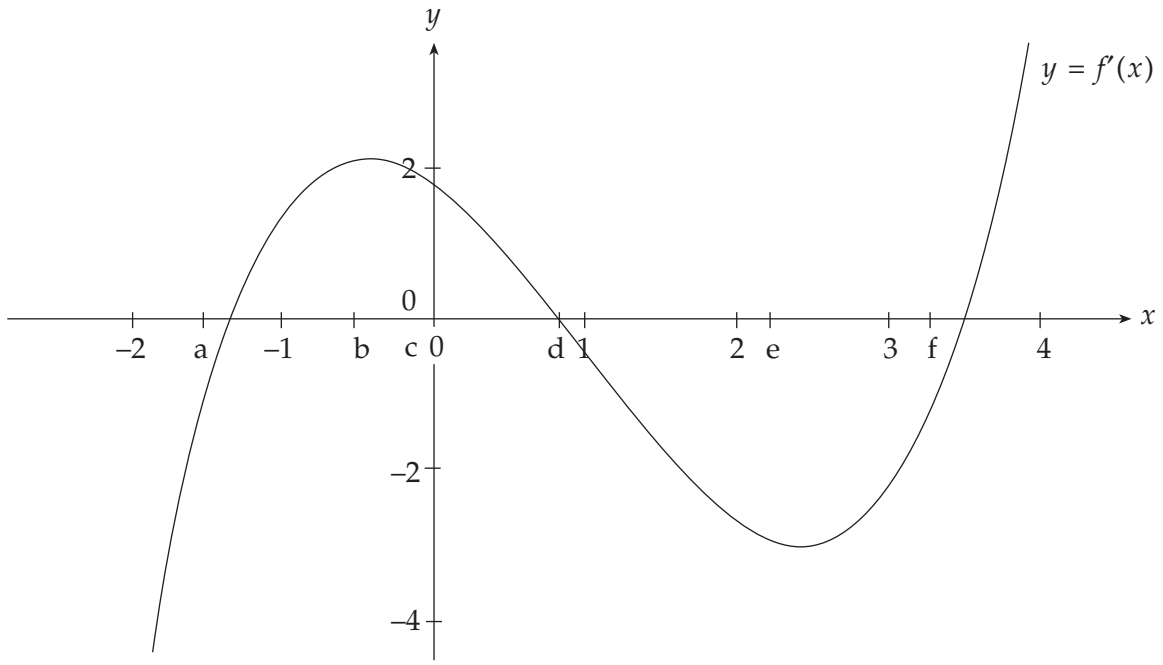
2 marks

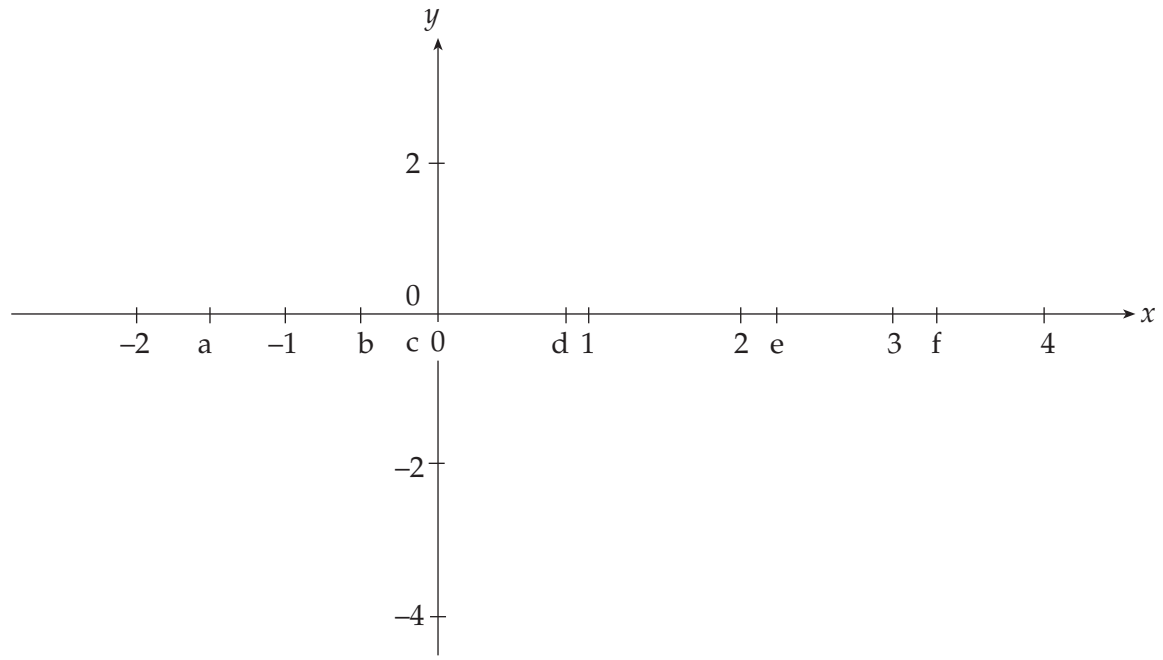
b. If $\underset{\sim}{i}$ is a unit vector in the direction of East and $\underset{\sim}{j}$ a unit vector North, express the acceleration of the body in the form $c\underset{\sim}{i} + d\underset{\sim}{j}$ correct to two decimal places.

3 marks

Question 5

The graph of $f'(x)$ is shown below. On the axes provided sketch $f(x)$.





3 marks