Year 2002

VCE

Specialist Mathematics

Trial Examination 2

KILBAHA PTY LTD (Publishers in Education)	**TEL: (03) 9817 5374**
ABN 47 065 111 373	**FAX: (03) 9817 4334**
PO BOX 2227 KEW VIC 3101	**chemas@chemas.com**
AUSTRALIA	www.chemas.com

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigometric) functions

$\cos^2 x+\sin^2 x=1$

$1+\tan^2 x=\sec^2 x$

$\cot^2 x+1=\operatorname{cosec}^2 x$

$\sin(x+y)=\sin x\cos y+\cos x\sin y$

$\sin(x-y)=\sin x\cos y-\cos x\sin y$

$\cos(x+y)=\cos x\cos y-\sin x\sin y$

$\cos(x-y)=\cos x\cos y+\sin x\sin y$

$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$

$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$

$\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$

$\sin 2x=2\sin x\cos x$

$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$

$-\pi<\operatorname{Arg} z\le\pi$

$z_1z_2=r_1r_2\operatorname{cis}(\theta_1+\theta_2)$

$\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis} n\theta$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$, for $x > 0$
$\frac{d}{dx}(\sin ax) = a\cos ax$	$\int \sin ax\, dx = -\frac{1}{a}\cos ax + c$
$\frac{d}{dx}(\cos ax) = -a\sin ax$	$\int \cos ax\, dx = \frac{1}{a}\sin ax + c$
$\frac{d}{dx}(\tan ax) = a\sec^2 ax$	$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax + c$
$\frac{d}{dx}\left(\text{Sin}^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \text{Sin}^{-1}\frac{x}{a} + c, a > 0$
$\frac{d}{dx}\left(\text{Cos}^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \text{Cos}^{-1}\frac{x}{a} + c, a > 0$
$\frac{d}{dx}\left(\text{Tan}^{-1}x\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2 + x^2} dx = \text{Tan}^{-1}\frac{x}{a} + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET

STUDENT NUMBER **Letter**

Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2002

SPECIALIST MATHEMATICS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*
5	5

Directions to students

Materials

Question and answer book of 11 pages.
Working space is provided throughout the book.
There is a detachable sheet of miscellaneous formula supplied.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the book during reading time.
Please ensure that your **student number** in the space provided on the front cover of this book.
Answer **all** questions
The marks allotted to each part of each question are indicated at the end of the part.
There is a total of 60 marks available for the examination.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , e, surds or fractions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.
All written responses should be in English.

Question 1

The flight of a butterfly is given by the parametric equation $x = \sin t, y = \cos 2t, t \geq 0$

a. Find the vector equation of the path of the butterfly.

1 mark

b. Find the Cartesian equation of the path of the butterfly.

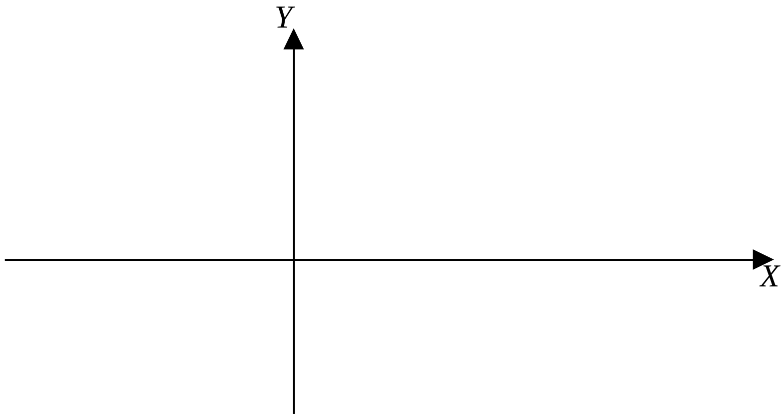
1 mark

c. Give the domain and range of this path of the butterfly.

2 marks

Question 1 (continued)

d. On the axes below, sketch the curve traced out by the butterfly, showing all relevant points.



4 marks

e. When $t = \frac{\pi}{4}$ the butterfly sees a flower at the point (0.8,–0.2). What is the position vector of the butterfly at this time?

1 mark

Question 1 (continued)

f. When the butterfly sees the flower, it flies off on a tangent to its original path.

i . Find the vector $\overrightarrow{AF}$ where A is the point on the path where the butterfly changes course and F is the position of the flower.

2 marks

ii. Find a vector which represents the butterfly's new path of travel.

2 marks

g. Will the butterfly reach the flower if it maintains this course?
Give a reason for your answer.

2 marks

(Total = 15 marks)

Question 2

A body of mass m kg is projected vertically upwards from the surface of the earth with a velocity, u. The acceleration of a particle in space is $\vec{a} = \frac{k}{x^2}$ towards the centre of the earth where k is a constant and x is the distance of the body from the centre of the earth.
If the acceleration at the earth's surface is g and the radius of the earth is R ,

a. Find the value of k in terms of R and g.

1 mark

b. Taking the upward direction from the earth as positive, find the force acting on the body.

1 mark

c. Show that the velocity, v, of the body at any time, t, can be given by the equation

$$v^2 = u^2 - 2gR^2\left(\frac{1}{R} - \frac{1}{x}\right)$$

5 marks

Question 2 (continued)

d. If $u^2 = 2gR$, find t in terms of x

5 marks

e. What is the minimum velocity of projection, u, in km/sec that the body needs so that it never returns to earth. Take the radius of the earth, $R = 6.4 \times 10^6$ m.
Give your answer to the nearest integer.

3 marks

(Total = 15 marks)

Question 3

a. Ann, Ben and Chris are standing on three points A, B and C which have position vectors $2\hat{i} + \hat{j}$, $3\hat{i} + 3\hat{j}$ and $5\hat{i} + 2\hat{j}$ respectively. David is asked to stand in a fourth position so that the four friends form a square. Find the position vector for the point where David stands.

3 marks

Question 3 (continued)

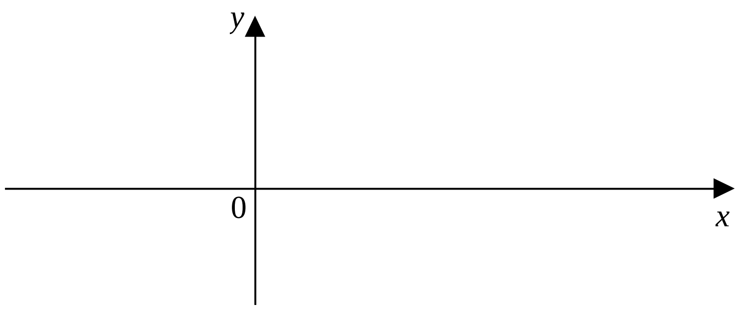
b. Let A and B be endpoints of a diameter of a circle. Let C be a third point on the circumference of the circle. Use a vector method to prove that $\angle ACB = 90^0$

4 marks

(Total = 7 marks)

Question 4

a. Sketch the curve $x^2 = 10y$ on the axes below.



1 mark

b. The part of the curve from $y = 0$ to $y = h$ is rotated about the Y axis. Show that the volume of revolution is given by $V = 5\pi h^2$

2 marks

c. If the volume thus generated represents a pool which is filled with water to a depth of h metres, show that the surface area of the water, S, is equal to $10\pi h$

2 marks

Question 4 (continued)

d. If water evaporates from the pool according to the rule $\frac{dV}{dt} = -0.002S$, find the rate of change of height with respect to time, where time is measured in hours.

3 marks

e. Initially, the pool contains 80π m^3 of water.
How long will it take for the pool to empty by evaporation?

4 marks

(Total = 12 marks)

Question 5

A boat of mass 5 kg, which is initially at rest, is pushed in the direction of motion by a constant force of 20N exerted by its motor. The water resists the motion with a backwards force whose magnitude is always half the speed, v.

a. Draw a diagram showing the forces acting on the boat.

1 mark

b. Find the resultant force in the direction of motion in terms of v.

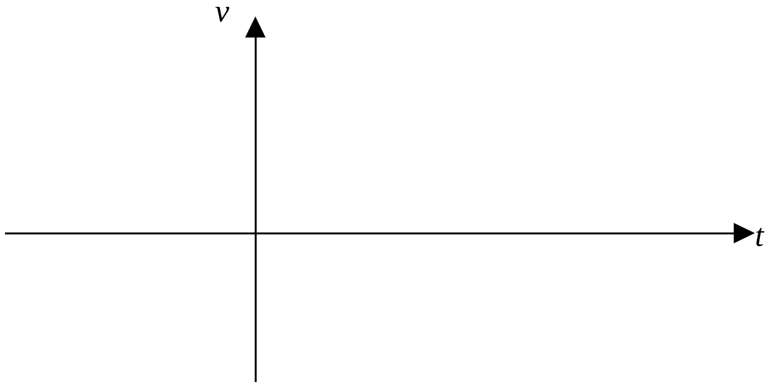
1 mark

c. Find the speed of the boat at any time, t.

4 marks

Question 5 (continued)

d. Sketch the graph of v against t on the axes below.



2 marks

e. How fast can the boat go?

1 mark

f. How far does the boat travel in the first 60 seconds? Give your answer in km. to one decimal place.

2 marks

(Total = 11 marks)

END OF QUESTION AND ANSWER BOOK

KILBAHA PTY LTD (Publishers in Education)	**TEL: (03) 9817 5374**
ABN 47 065 111 373	**FAX: (03) 9817 4334**
PO BOX 2227 KEW VIC 3101	**chemas@chemas.com**
AUSTRALIA	www.chemas.com