Question 1 C

A constant over the squared factor, plus a constant over this factor unsquared plus a constant over the other factor.

Question 2 E

$$Z^{-1} = \frac{1}{Z} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i}$$

$$= \frac{4+3i}{16+9}$$

$$= \frac{4+3i}{25}$$

$$Z^{-1} = \frac{4}{25} + \frac{3}{25}i$$

Question 3 A

$$Z^4 - Z^2 - 20 = (Z^2+4)(Z^2-5)$$

$$Z^4 - Z^2 - 20 = (Z-2i)(Z+2i)(Z-\sqrt{5})(Z+\sqrt{5})$$

$\therefore$ a factor is $Z + 2i$

Question 4 B

Represents all points which are less than or equal to 2 units from $Z = 2 - i$

Question 5 D

$$3(\operatorname{Re} Z)^2 + 6(\operatorname{Im} Z)^2 = 6$$

If $Z = x + iy$

$$3x^2 + 6y^2 = 6$$

$$\frac{x^2}{2} + \frac{y^2}{1} = 1$$

ellipse, $a = \sqrt{2}, b = 1$

Question 6 E

$$y = \frac{x^3 - 32}{x^2}$$

$$\begin{array}{r|l} & x \\ x^2 & x^3 - 32 \\ & x^3 \\ \hline & -32 \end{array}$$

$$y = x - \frac{32}{x^2}$$

$$\frac{dy}{dx} = 1 + 64x^{-3} = 0 \text{ for T.P.}$$

$$\frac{64}{x^3} = -1$$

$$x^3 = -64$$

$$x = -4$$

When $x < -4$ $\frac{dy}{dx} > 0$

When $x > -4$ $\frac{dy}{dx} < 0$

$\therefore$ local maximum when $x = -4$

Question 7 B

$$\int \frac{2x}{\sqrt{x^2+6}}dx$$

Let $u = x^2 + 6$

$du = 2xdx$

$$\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}} du$$

$$= 2u^{\frac{1}{2}} + c = 2\sqrt{x^2+6} + c$$

An antiderivative is $2\sqrt{x^2+6}$

Question 8 A

$$\int_0^{\frac{\pi}{3}} \sin^2(5x)dx = \frac{1}{2}\int_0^{\frac{\pi}{3}}(1-\cos 10x)dx$$

$$= \frac{1}{2}[x - \frac{1}{10}\sin 10x]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{2}[\frac{\pi}{3} - \frac{1}{10}\sin\frac{10\pi}{3}] - [0]$$

$$= \frac{\pi}{6} - \frac{1}{20}\sin\frac{10\pi}{3}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{40}$$

$$= \frac{20\pi + 3\sqrt{3}}{120}$$

Question 9 C

$$V = \int_0^1 \pi y^2 dx$$

$$V = \pi\int_0^1 \frac{1}{4}(e^x + e^{-x})^2 dx$$

$$V = \frac{\pi}{4}\int_0^1 (e^{2x} + 2 + e^{-2x})dx$$

$$V = \frac{\pi}{4}[\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x}]$$

$$V = \frac{\pi}{4}[\frac{1}{2}e^2 + 2 - \frac{1}{2}e^{-2}] - \frac{\pi}{4}[\frac{1}{2} - \frac{1}{2}]$$

$$V = 4.4$$

Question 10 E

$y = \cos^{-1}(2x-1)$

Let $u = 2x - 1$

$$\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-u^2}} \times 2$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1-(2x-1)^2}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1-[4x^2-4x+1]}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1-4x^2+4x-1}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{-4x^2+4x}}$$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{-4x(x-1)}}$$

$$\frac{dy}{dx} = \frac{-2}{2\sqrt{(x-x^2)}} = \frac{-1}{\sqrt{(x-x^2)}}$$

Question 11 B

$-1 \le 4x + 1 \le 1$

$-2 \le 4x \le 0$

$-\frac{1}{2} \le x \le 0$

Question 12 C

$f(x) = x \log_e x$

$\Delta x = \frac{1}{2}$

$\int_1^3 x \log_e x dx$

$\approx \frac{1}{4}(1\log_e 1 + 2 \times \frac{3}{2}\log_e \frac{3}{2} + 2 \times 2\log_e 2 + 2 \times \frac{5}{2}\log_e \frac{5}{2} + 3\log_e 3)$

$= \frac{1}{4}\left(0 + 1.216 + 2.773 + 4.581 + 3.296\right)$

$= 2.97$ to 2 decimal places.

Question 13 A

$y = \sin(\log_e x)$

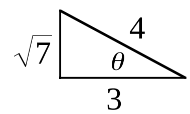
$\frac{dy}{dx} = \cos u \times \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{x}\cos(\log_e x)$

$\frac{d^2y}{dx^2} = \frac{1}{x} \times -\sin(\log_e x) \times \frac{1}{x} + \cos(\log_e x)(-\frac{1}{x^2})$

$\frac{d^2y}{dx^2} = \frac{-[\sin(\log_e x) + \cos(\log_e x)]}{x^2}$

Question 14 E



θ is in the 4th quadrant $\therefore \sin\theta$ is negative

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 2 \times \frac{-\sqrt{7}}{4} \times \frac{3}{4}$

$\sin 2\theta = \frac{-3\sqrt{7}}{8}$

Question 15 D

At $x = 4$ gradient of $f(x) = 0$. Passes from positive to positive gradient either side of $x = 4$
$\therefore$ stationary point of inflexion.
At $x = 0$ gradient of $f(x) = 0$. Passes from positive to negative gradient either side of $x = 0$
$\therefore$ local maximum.
At $x = 1$ gradient of $f(x) \neq 0$ $\therefore$ not a turning point.
At $x = 2$ gradient of $f(x) = 0$. Passes from negative to positive gradient either side of $x = 2$
$\therefore$ local minimum.
At $x = 3$ gradient of $f(x) > 0$.

Question 16 D

$$(3\hat{i} - 2\hat{j} + \hat{k}) \bullet (\hat{i} + 2\hat{j} + 2\hat{k}) = 3 - 4 + 2 = 1$$

$$\vec{a} \bullet \vec{b} = |\vec{a}||\vec{b}|\cos\theta = 1$$

$$3\sqrt{14}\cos\theta = 1$$

$$\cos\theta = \frac{1}{3\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}}$$

$$\cos\theta = \frac{\sqrt{14}}{42}$$

Question 17 A

$$a = -6v$$

$$\frac{dv}{dt} = -6v$$

$$\frac{dt}{dv} = -\frac{1}{6v}$$

$$t = -\frac{1}{6}\int \frac{dv}{v}$$

$t = -\frac{1}{6}\log_e v + c$ where c is a constant

$$\log_e v = -6t + c_1$$

When $t = 0, v = 3$

$$\therefore c_1 = \log_e 3$$

$$\log_e v = -6t + \log_e 3$$

$$\log_e \frac{v}{3} = -6t$$

$$e^{-6t} = \frac{v}{3}$$

$$v = 3e^{-6t}$$

Question 18 C

Overtake occurs when car A and car B have gone the same distance at the same time, t .

car A : $s = 25t$

car B : $s = ut + \frac{1}{2}at^2$

$$a = \frac{5}{3}$$

$$\Rightarrow s = 0 + \frac{1}{2} \times \frac{5}{3} \times t^2$$

$$\Rightarrow s = \frac{5}{6}t^2$$

$$\Rightarrow \frac{5}{6}t^2 = 25t$$

$$\Rightarrow t^2 = 30t$$

$$\Rightarrow t^2 - 30t = 0$$

$$\Rightarrow t(t - 30) = 0$$

$\Rightarrow t = 0$ or 30

But $t > 0$

$\therefore t = 30$ sec

Question 19 C

$3\vec{a} = 9\hat{i} - 6\hat{j} + 12\hat{k}$

$2\vec{b} = 4\hat{i} - 6\hat{j} - 2\hat{k}$

$3\vec{a} - 2\vec{b} = 9\hat{i} - 6\hat{j} + 12\hat{k} - 4\hat{i} + 6\hat{j} + 2\hat{k}$

$3\vec{a} - 2\vec{b} = 5\hat{i} + 14\hat{k}$

Question 20 A

$(\vec{a} \bullet \hat{b})\hat{b}$ is the component of $\vec{a}$ parallel to $\vec{b}$

$|\vec{b}| = \sqrt{9+4} = \sqrt{13}$

$(\vec{a} \bullet \hat{b})\hat{b} = \left[(6\hat{i} + 12\hat{j}) \bullet \frac{1}{\sqrt{13}}(-3\hat{i} + 2\hat{j})\right]\hat{b}$

$(\vec{a} \bullet \hat{b})\hat{b} = \frac{1}{\sqrt{13}}(-18 + 24)\hat{b}$

$(\vec{a} \bullet \hat{b})\hat{b} = \frac{6}{\sqrt{13}}\hat{b}$

$(\vec{a} \bullet \hat{b})\hat{b} = \frac{6}{\sqrt{13}} \times \frac{1}{\sqrt{13}}(-3\hat{i} + 2\hat{j})$

$(\vec{a} \bullet \hat{b})\hat{b} = \frac{6}{13}(-3\hat{i} + 2\hat{j}) = -\frac{6}{13}(3\hat{i} - 2\hat{j})$

Question 21 E

$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$

$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = 2x \times 3 = 6x$

$x = 3t + c$

$\frac{dy}{dt} = 6(3t + c) = 18t + c_1$

$\frac{d^2y}{dt^2} = 18$

Question 22 E

Either $\vec{a} = 0$ or $\frac{d\vec{a}}{dt} = 0$ or $\theta = 90^0$

If $\theta = 90^0$ then $\vec{a}$ is perpendicular to $\frac{d\vec{a}}{dt}$

This means that the tangent is perpendicular to the motion

Hence, $\vec{a}$ moves in a circle.

If $\frac{d\vec{a}}{dt} = 0$ then $\vec{a}$ is a constant vector.

Question 23 D

$x - 1 = \cos t$

$(x-1)^2 = \cos^2 t$

$\frac{y}{3} = \sin t$

$\frac{y^2}{9} = \sin^2 t$

$(x-1)^2 + \frac{y^2}{9} = \sin^2 t + \cos^2 t$

$\frac{(x-1)^2}{1} + \frac{y^2}{9} = 1$

$\Rightarrow$ ellipse with centre (1,0),

with semi major axis = 1

and semi minor axis = 3

Question 24 E

$a = -6$

$u = 10$

$v = 0$

$s = ?$

$v^2 = u^2 + 2as$

$0 = 100 - 12s$

$100 = 12s$

$s = \frac{100}{12}$

$s = 8.3m$

Question 25 B

Distance $= \sqrt{\vec{r} \bullet \vec{r}}$

$= \sqrt{(\cos 2t\hat{i} - \sin 2t\hat{j}) \bullet (\cos 2t\hat{i} - \sin 2tj)}$

$= \sqrt{\cos^2 2t + \sin^2 2t}$

$= 1$ (for all values of t)

Question 26 A

$\vec{F} = m\vec{a}$

$3\vec{a} = 3\hat{j} - 6\hat{k}$

$\vec{a} = \hat{j} - 2\hat{k}$

$\frac{d\vec{v}}{dt} = \hat{j} - 2\hat{k}$

$\vec{v} = t\hat{j} - 2t\hat{k} + c$

When $t = 0, v = 0$

$\therefore c = 0$

$\vec{v} = t\hat{j} - 2t\hat{k}$

$\vec{x} = \frac{t^2}{2}\hat{j} - t^2\hat{k} + c_1$

When $t = 0, \vec{x} = 3\hat{i} + \hat{j} - \hat{k}$

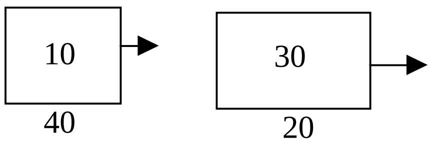
So $c_1 = 3\hat{i} + \hat{j} - \hat{k}$

So $\vec{x} = 3\hat{i} + (1 + \frac{t^2}{2})\hat{j} - (1 + t^2)\hat{k}$

When $t = 2$

$\vec{x} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

Question 27 B



$\vec{p}_{\text{before}} = \vec{p}_{after}$

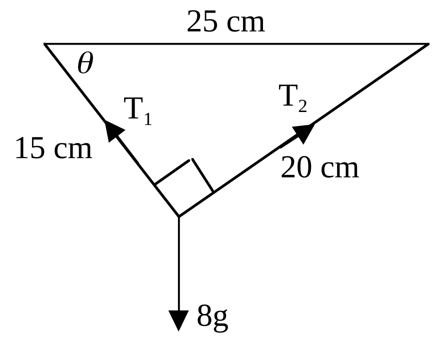
$10 \times 40 + 30 \times 20 = 40v$

$400 + 600 = 40v$

$\frac{1000}{40} = v$

$v = 25$ km/hr

Question 28 D



15:20:25 = 3:4:5 so strings form a right angle with each other.

$\sin\theta = \frac{20}{25} = \frac{4}{5}$

$\cos\theta = \frac{15}{25} = \frac{3}{5}$

Horizontal Equilibrium $T_1 \cos\theta = T_2 \sin\theta$

$\frac{3}{5}T_1 = \frac{4}{5}T_2$ so $T_2 = \frac{3}{4}T_1$

Vertical Equilibrium

$T_1 \sin\theta + T_2 \cos\theta = 8g$

$\frac{4}{5}T_1 + \frac{3}{4}T_1 \times \frac{3}{5} = 8g$

$\frac{4}{5}T_1 + \frac{9}{20}T_1 = 8g$

$\frac{25}{20}T_1 = 8g$

$T_1 = 8g \times \frac{20}{25} = 62.72N$

Question 29 B

$$\int_1^{e^2} \frac{\log_e x}{x}\,dx$$

Let $u = \log_e x$

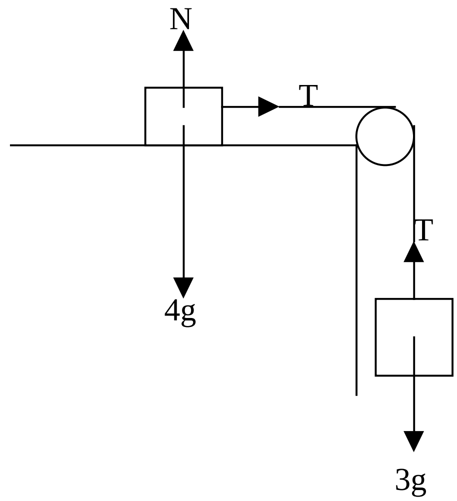
$du = \frac{1}{x}dx$

When $x = 1, u = \log_e 1 = 0$

When $x = e^2, u = \log_e e^2 = 2$

$$\int_0^2 u\,du = \left[\frac{u^2}{2}\right]_0^2 = 2$$

Question 30 B



For 4 kg mass

$\vec{R} = m\vec{a}$

$\Rightarrow \vec{T} = 4\vec{a}(1)$

For 3 kg mass

$\vec{R} = m\vec{a}$

$3g - \vec{T} = 3\vec{a}(2)$

Substituting (1) in (2)

$3g - 4\vec{a} = 3\vec{a}$

$7\vec{a} = 3g$

$\vec{a} = \frac{3 \times 9.8}{7}$

$\vec{a} = 4.2m/\sec^2$

END OF PART I
MULTIPLE CHOICE QUESTION BOOK

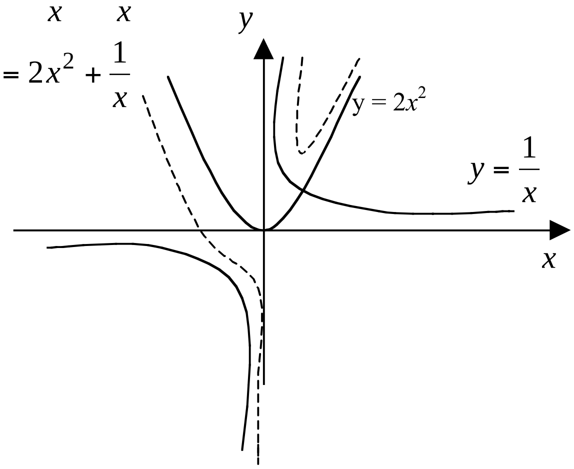
KILBAHA PTY LTD (Publishers in Education) **PO BOX 2227 KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com

Question 1

a.

$y = \dfrac{2x^3}{x} + \dfrac{1}{x}$

$y = 2x^2 + \dfrac{1}{x}$



\- - - - - - - - - - - - - - - - is the graph of $y = 2x^2 + \dfrac{1}{x}$

(1 mark)

b.

$\dfrac{dy}{dx} = 4x - \dfrac{1}{x^2} = 0$ for minimum

$4x = \dfrac{1}{x^2}$

$4x^3 = 1$

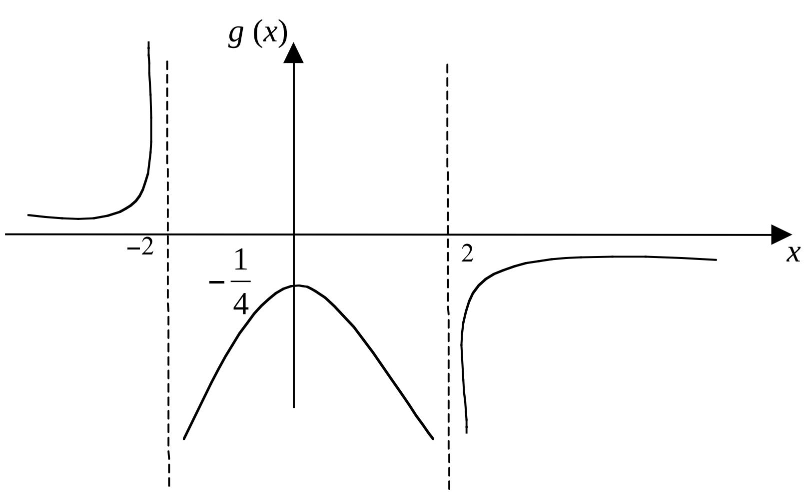
$x^3 = \dfrac{1}{4}$

$x = \dfrac{1}{\sqrt[3]{4}}$ (1 mark)

$y = \dfrac{\frac{1}{2} + 1}{\frac{1}{\sqrt[3]{4}}} = \sqrt[3]{4}\left(\dfrac{3}{2}\right) = \dfrac{3\sqrt[3]{4}}{2}$ (1 mark)

$\left(\dfrac{1}{\sqrt[3]{4}}, \dfrac{3\sqrt[3]{4}}{2}\right)$

c.



(1 mark)

Question 2

$\tan(x + \frac{\pi}{3}) = \pm\frac{1}{\sqrt{3}} \quad \frac{\pi}{3} \le x \le \frac{7\pi}{3}$ (1 mark)

$x + \frac{\pi}{3} = \cancel{\frac{\pi}{6}}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$ (1 mark)

$x = \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$

$x = \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ (1 mark)

Question 3

$Z = 2\sqrt{2}(\cos 45^0 + i\sin 45^0)$

$Z^2 = 8(\cos 90^0 + i\sin 90^0)$ (1 mark)

$W = 2\sqrt{2}(\cos 135^0 + i\sin 135^0)$

$W^5 = 128\sqrt{2}(\cos 675^0 + i\sin 675^0)$

$W^5 = 128\sqrt{2}(\cos 315^0 + i\sin 315^0)$ (1 mark)

$\frac{Z^2}{W^5} = \frac{8}{128\sqrt{2}}\left[\cos(90-315)^0 + i\sin(90-315)^0\right]$

$\frac{Z^2}{W^5} = \frac{1}{16\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\left[\cos(-225)^0 + i\sin(-225)^0\right]$

$\frac{Z^2}{W^5} = \frac{\sqrt{2}}{32}\left[\cos 225^0 - i\sin 225^0\right]$ (1 mark)

$\frac{Z^2}{W^5} = \frac{\sqrt{2}}{32}\left[-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right]$

$\frac{Z^2}{W^5} = -\frac{1}{32} + \frac{1}{32}i$ (1 mark)

Question 4

a.

$\frac{dy}{dx} = \frac{1}{2}e^x[\cos x - \sin x] + [\sin x + \cos x]\frac{1}{2}e^x$

$\frac{dy}{dx} = \frac{1}{2}e^x[\cos x - \sin x + \sin x + \cos x]$

$\frac{dy}{dx} = \frac{1}{2}e^x \times 2\cos x = e^x \cos x$

(1 mark)

b.

$\int(7e^x \cos x)dx - \int 7dx + \int\frac{1}{\sqrt{e^2 - (2x)^2}}dx$

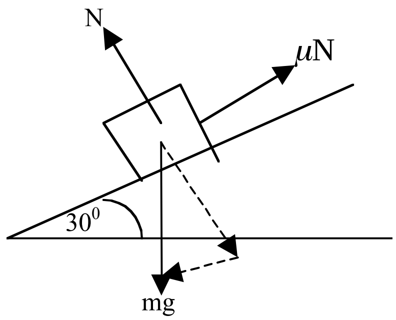
$= \frac{7}{2}e^x(\sin x + \cos x) - 7x + \frac{1}{2}\int\frac{1}{\sqrt{\frac{e^2}{4} - x^2}}dx$

(1 mark)

$= \frac{7}{2}e^x(\sin x + \cos x) - 7x + \frac{1}{2}\sin^{-1}\left(\frac{2x}{e}\right) + c$

where c is a constant (1 mark)

Question 5



Resolving forces parallel to plane:

Resultant force = $mg\sin 30^0 - \mu N$

$2m = mg\sin 30^0 - \mu N$ (1) (1 mark)

Resolving forces perpendicular to the plane

$N - mg\cos 30^0 = 0$

$N = mg\cos 30^0$ (2) (1 mark)

Substituting (2) in (1)

$2\cancel{m} = \cancel{m}g\sin 30^0 - \mu \times \cancel{m}g\cos 30^0$

$2 = 4.9 - \mu \times 8.487$

$8.487\mu = 2.9$

$\mu = 0.34$ to 2 decimal places (1 mark)

Question 6

Let $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{u} \bullet \vec{a} = 0$

$2x + y + 3z = 0$ (1) (1 mark)

$\vec{u} \bullet \vec{b} = 0$

$x - 2y - z = 0$ (2) (1 mark)

$(2) \times 2 \to 2x - 4y - 2z = 0$ (2a)

$2x + y + 3z = 0$ (1)

$\Rightarrow -5y - 5z = 0$

$\Rightarrow -5y = 5z$

$\Rightarrow y = -z$

Substituting in (1)

$2x - z + 3z = 0$

$2x = -2z$

$x = -z$

$\therefore \vec{u} = -z\hat{i} - z\hat{j} + z\hat{k}$

$\Rightarrow \vec{u} = z(-\hat{i} - \hat{j} + \hat{k})$

But z is a constant

So $\mu = c(-\hat{i} - \hat{j} + \hat{k})$ (1 mark)

END OF SUGGESTED SOLUTIONS
2002 Specialist Mathematics Trial Examination 1 Part II

KILBAHA PTY LTD (Publishers in Education)	**TEL: (03) 9817 5374**
ABN 47 065 111 373	**FAX: (03) 9817 4334**
PO BOX 2227 KEW VIC 3101	**chemas@chemas.com**
AUSTRALIA	www.chemas.com