Specialist Mathematics GA 3: Examination 2

GENERAL COMMENTS

The number of students who sat for the 2001 examination was 5922, 66 more than the number (5856) in 2000 (a slight increase of about 1%). Students had to answer five questions worth a total of 60 marks with each question worth from 11 to 14 marks.

Students found the examination noticeably more difficult than the 2000 paper, with about 5 fewer marks being required for the award of an equivalent grade. Whereas in 2000 about 2.9% of students scored at least 90% of the marks, in 2001 only 3% of students scored more than 85%. Only 2 students scored full marks, compared with 4 in 2000. Question 4d proved particularly difficult with only 3% of students gaining the maximum mark of 4 and the mean mark being less than 0.6.

Although some students demonstrated an excellent understanding across topics, a similar number had a limited idea of what was required. Over 40% of students scored 0 for 16 of the 27 question part; however, all but the least successful students managed to score marks on Questions 2a, 5a, 5ci and 5cii.

Many marks were lost through poor examination technique, including overlooking (or ignoring) specific directions. In particular, students lost marks by:

- not using the method specified (Questions 1a and 1bii)
- not fully answering the question (1bii, 1cii and 2ciii)
- finding an approximate value instead of the exact value (Question 2bi)
- failing to recognise dependencies between question parts (2ci, ii and iii; 4b and c)
- not proceeding as indicated by the instruction ‘hence’ (Questions 4a and 4bii)
- giving their answers in Questions 4c and 4d as quadratic expressions rather than as quadratic equations
- not showing all steps in questions where the result was given and had to be ‘shown’ (Questions 3b, 4bi, 5b and 5cii).

Many students also lost marks by not recognising the need to use their graphics calculator to evaluate the definite integral in Question 3d. Students should be alert to the fact that they are free to use their graphics calculator to evaluate *any* definite integral in the absence of a direction otherwise (such as ‘use calculus to’ or ‘hence’). Students would benefit from being more familiar with the *exact* values of the circular functions sin, cos, tan for 30º, 40º and 60º (or $\frac{\Pi}{6}, \frac{\Pi}{4}$ and $\frac{\Pi}{3}$ respectively) and being more proficient in the manipulation of surds.

Finally, as with Examination 1, it appeared that many students do not know what is meant by ‘significant figures’ (Questions 2a, 2bii, 3c and 5e). Students should understand the distinction (in general) between ‘significant figures’ and ‘decimal places’ as either may be specified in a question to indicate the (minimum) accuracy required for an answer.

SPECIFIC INFORMATION

Question 1

a. (Average mark 1.38/Available marks 2)

9.72 km

Well done. Some students ignored the direction to use the cosine rule and so forfeited their chance to earn either mark. Others did not take the square root and gave OQ^2 as their answer. The most common error was the inability to determine angle *OPQ* (135º), often indicating a lack of understanding of compass bearings.

bi. (1.20/2)

$(3\sqrt{2}+4.5)\underset{\sim}{i} - 3\sqrt{2}\,\underset{\sim}{j} \quad = \quad 8.74\underset{\sim}{i} - 4.24\,\underset{\sim}{j}$

Well done. The most common error was to overlook the direction of $\underset{\sim}{j}$ and add $3\sqrt{2}\,\underset{\sim}{j}$ instead of subtracting it.

bii. (1.03/3)

S 64.1ºE

Some students ignored the direction to use a scalar product and so received no marks for this part of the question. Many students had a correct scalar product but did not obtain a correct angle (mainly due to errors in modulus values), whereas others obtained a correct angle but did not translate it into a bearing of *Q* from *O*. The most common scalar product used was $\overrightarrow{OP}.\overrightarrow{OQ}$, giving the direction of *OQ* relative to *OP*, whereas $\overrightarrow{OQ}.\underset{\sim}{i}$ or $\overrightarrow{OQ}.(-\underset{\sim}{j})$ give the direction of *Q* from *O* directly.

ci. (0.75/2)

$9.22\,\underset{\sim}{i} - 4.24\,\underset{\sim}{j} + 0.14\,\underset{\sim}{k}$

Most students could not express the displacement up the slope ($\overrightarrow{OR}$) correctly in terms of $\underset{\sim}{i}$ and $\underset{\sim}{k}$. The most common error was to take it to be $0.5\underset{\sim}{k}$.

cii. (0.70/2)
$OR = 10.15$ and so the transmitter does *not* have sufficient range since $OR > 10$.

Most students knew how to find OR as the magnitude of $\overrightarrow{OR}$, but unfortunately few were working with a reasonable $\overrightarrow{OR}$. Some students correctly found OR to be 10.15 but concluded incorrectly that the transmitter was *within* range, or stated no conclusion at all.

Question 2

a. (0.78/1)
1.49×10^4 L/day

Very well done.

bi. (1.33/3)

$\sqrt{2}$

The factor of 10^6 often proved difficult for those students who (unnecessarily) substituted for k. Many students could not solve $4t^4 = 12 + t^4$ exactly, or failed to recognise that $\sqrt[4]{4} = \sqrt{2}$.

bii. (0.44/1)
8.84×10^4 L/day

Very well done by those students who answered part i completely correctly. Other students were able to get this second part correct because they had obtained a 'correct' approximate value for a in part i, or by using their graphics calculator to find the maximum value of $\frac{t}{12 + t^4}$ directly.

ci. (1.38/3)

$$\frac{1}{4\sqrt{3}}\text{Tan}^{-1}\left(\frac{t^2}{2\sqrt{3}}\right)$$

Reasonably well done by those students who carried through the substitution correctly. The most common errors made by these students related to mishandling of $\sqrt{12}$ and failure to express the answer in terms of t, though a significant number thought that the integral involved $\log_e$ rather than Tan^{-1}.

cii. (0.33/1)

$$V = \frac{10^6}{4\sqrt{3}}\text{Tan}^{-1}\left(\frac{t^2}{2\sqrt{3}}\right)$$

Very well done by those students proficient enough to have an expression involving Tan^{-1} as their answer to the previous part.

ciii. (0.44/2)

$$V < \frac{10^6}{4\sqrt{3}} \times \frac{\Pi}{2} = 226\,725 < 300\,000$$

Hence the newspaper report is *not* in agreement with the model.

Answered correctly by nearly half the students who had the correct answer for part ii.

Question 3

a. (0.78/2)

hyperbola with vertices (±1, 0), asymptotes $y = \pm\frac{1}{2}x$

Not well done with only 20% of students scoring both marks and just over 40% scoring 0. Many students did not recognise the rule as the equation of a hyperbola and drew the two branches using straight lines, making > and < type shapes, or only drew the upper half of the hyperbola. Some students even drew a circle or two. Most of the students who knew to draw a hyperbola either omitted the asymptotes, drew them incorrectly, or had their curves moving *away* from their asymptotes.

b. (0.88/2)

$$V = \int_0^h \pi x^2 dy = \pi\int_0^h (4y^2 + 1)dy = \pi\left[\frac{4y^3}{3} + y\right]_0^h = \pi\left(\frac{4h^3}{3} + h\right)$$

This was also not well done, with just over 30% of students scoring both marks, and over 40% scoring 0. A common incorrect answer was to obtain the required result by substituting $y = h$ into the antiderivative of $4y^2 + 1$.

c. (1.29/3)
0.00025 m/s

Just over half the students made a reasonable attempt at this question, although common errors included taking $\frac{dh}{dt}$ to be $\frac{dV}{dt}$, poor use of the chain rule, $\frac{dV}{dh}$ not found correctly, and the inability to evaluate $\frac{dh}{dt}$ accurately at h = 0.25.

d. (0.48/2)

$$t = \int_0^{0.5} \frac{\pi(1+4h^2)}{0.003 - 0.004\sqrt{h}}\,dh = 3525 \text{ s}$$

Common mistakes among those students who had a reasonably correct expression for $\frac{dh}{dt}$ included not being able express the time as a *definite* integral, and not using their graphics calculator to evaluate the definite integral.

e. (0.40/2)

$\frac{9}{16}$ m (56.25 cm)

Many students omitted this question although it was accessible to anyone who realised that it could be answered by equating the given outflow rate to the given inflow rate (and so did not depend on being able to do earlier parts of the question). A common error among those students who otherwise knew what they were doing was to go from $\sqrt{h} = \frac{3}{4}$ to $h = \frac{\sqrt{3}}{2}$.

Question 4

a. (1.95/3)

$\alpha = 3 + 4i$, $\beta = 3 - 4i$; $\alpha + \beta = 6$, $\alpha\beta = 25$

Well done, although 25% of students scored 0. Invariably, most students who attempt to solve quadratic equations by completing the square would be well advised to routinely use the formula for solving a quadratic equation when no factors are evident.

Question 4

bi. (0.74/2)

$z^2 + bz + c = (z-u)(z-v) = z^2 - (u+v)z + uv$

Hence $u + v = -b$ and $uv = c$

A common error, resulting in a score of 0, was to let u and v be the specific roots from part a. In contrast to the much simpler method of expressing the quadratic expression in factorised form and equating coefficients (see answer above), most students used the formula (or completed the square) to find expressions for u and v and then tried to simplify their resultant expressions for $u + v$ (usually successfully) and uv (much less successfully).

bii. (0.67/2)

$b = -(u+v) = -((p+qi)+(p-qi)) = -2p$, which is real since p is real;

$c = uv = (p+qi)(p-qi) = p^2 + q^2$, which is real since p and q are real.

Typical errors included finding $b = 2p$ or (more often) $c = p^2 + q$ (from writing $p^2 - (qi)^2$ as $p^2 - qi^2$), but the most common error was not to finish the answer by stating why it followed that b and c were real. Students should have realised that they could attempt this question using the results given in part i even if they were unable to do part i.

c. (0.64/2)

$z^2 - (2\sqrt{5}\,i)z - 9 = 0$

Few students saw the connection with part bi, but instead expanded $(z - 2 - \sqrt{5}\,i)(z + 2 - \sqrt{5}\,i)$. Many did this successfully but few then equated the quadratic *expression* they obtained to 0 to give a quadratic *equation*. A common error was to treat the roots as complex conjugates.

d. (0.56/4)

$z^2 + (3 - \sqrt{7}\,i)z - 3\sqrt{7}\,i = 0$ or $z^2 + (3 + \sqrt{7}\,i)z + 3\sqrt{7}\,i = 0$

Most students were unable to fathom what was required in this question. Many students did not attempt it or gave $z^2 + 3z + 4 = 0$ as their answer. Others solved correctly for u and v, but did not know what to do next. (Note: incidentally, that it is not necessary to solve for u and v — $(u + v)$ is given and $(u - v)$ can be found using $(u - v)^2 = (u + v)^2 - 4uv$, since uv is also given.) Many of the (relatively few) students who had obtained a correct quadratic *expression* gave this as their answer instead of the corresponding quadratic *equation*.

Question 5

a. (0.71/1)

$75g$ newtons vertically down, N newtons normal to the slide, F ($0.2N$) newtons up the slide.

Very well done. The most common errors were drawing the normal reaction force in the vertically up direction, and including an extra force down the slide.

b. (1.99/3)

Resolving parallel to the slide gives $N = \frac{75\sqrt{3}}{2}g$

Resolving perpendicular to the slide gives $75a = \frac{75g}{2} - F$

Using $F = \frac{1}{5}N$ to eliminate F gives the required expression for a.

Well done, particularly in comparison with similar questions in recent years.

ci. (1.47/2)

1.94 s

Very well done.

cii. (0.73/1)

$v = u + at = 0 + 3.2026 \times 1.9357 = 6.2$

Very well done.

d. (0.87/4)

2.1 m

Not well done. Most students did not appreciate the need to resolve the motion into its horizontal and vertical components. A common error was to assume straight line motion from the end of the slide, at the given angle (60º) to the vertical, with $u = 6.2$ and $a = g$.

e. (0.50/3)

661 kg m/s

Similarly, not well done. The most common answer was $75 \times 6.2 = 458.8$. A handful of students used vectors explicitly to answer Questions 5d and 5e accurately and elegantly.