GENERAL COMMENTS

The number of students who sat for the 2001 examination was 5924, 68 more than the number (5856) in 2000, an increase of about 1%. As in 2000, students were required to answer 30 multiple-choice questions in Part 1 and six short answer questions worth a total of 20 marks in Part 2.

Just over 4% of students scored more than 90% of the available marks (compared to about 3% in both 1999 and 2000). Twenty-seven students scored full marks (compared with only four in 2000), the largest number for at least four years, and another 48 only lost 1 mark. Most students had sufficient time to complete the examination to the best of their ability, although 15% of students (about 5% more than in 2000) scored less than 25% of the available marks.

The wide range in quality of student work was evident in responses to the short-answer questions. The best work was accurate, logically presented, and usually clearly written. However, many students who scored reasonably well presented untidy, disorganised and barely legible work. Some students who scored few (if any) marks attempted very little, perhaps indicating poor time management, while others provided answers of considerable length that indicated significant weakness in fundamental algebraic skills.

Students are strongly advised to answer Part 2 in pen. Many students choose to write in pencil, thus making it difficult, for example, to detect decimal points or to distinguish + signs from – signs. It is not uncommon for such students to misinterpret their own work and make consequent errors.

There was a noticeable improvement in the effective use of graphics calculators compared with their use in the 2000 examination. Graphics calculators were often used to advantage in Question 1, although this was not required, and Question 2c, and occasionally in Question 3. However, the use of a graphics calculator to evaluate the integral in Question 6 was not appropriate as an **exact** answer was required. Teachers should ensure that students fully understand the implications of the instructions, included at the start of the examination paper, that apply when a question requires an exact answer or the use of calculus. Similarly, students should understand the implication of 'hence' (as in Question 2b) and its absence (as in Question 2c).

Many students did not demonstrate understanding of 'significant figures' (see comments on Question 2c). Students should understand the distinction (in general) between 'significant figures' and 'decimal places' as either may be specified in a question to indicate the (minimum) accuracy required in the answer.

SPECIFIC INFORMATION

Part 1 – Multiple-choice questions

The correct responses were:

Question	Correct Response	%	Question	Correct Response	%
1	**E**	77	16	**B**	37
2	**E**	67	17	**C**	50
3	**D**	58	18	**D**	52
4	**A**	61	19	**E**	63
5	**A**	49	20	**B**	45
6	**E**	70	21	**B**	51
7	**C**	63	22	**E**	58
8	**A**	38	23	**A**	70
9	**E**	50	24	**A**	45
10	**C**	48	25	**C**	42
11	**C**	62	26	**D**	35
12	**D**	81	27	**B**	42
13	**B**	68	28	**B**	55
14	**D**	63	29	**D**	74
15	**C**	66	30	**A**	58

The three least well answered questions were Questions 8, 16 and 26.

Question 8 can be answered readily if students recognise that the equation of a circle, centre *w* and radius *r*, in the complex plane can be written in the form $(z-w)(\overline{z}-\overline{w})=r^2$. This, however, does not seem to be well known, as only 38% of students answered correctly (A) and each of the four distractors attracted answers from between 10% and 20% of students.

Question 16 involved finding a particular solution of a differential equation as a definite integral. In general, the need for students to be able to do this directly, rather than by first finding the general solution (which is not possible if the integrand does not have an antiderivative that can be found analytically), has been repeatedly mentioned in advice to teachers since students were first assumed to have access to graphics calculators in 1998. Option A, the negative of the correct answer, was almost as popular (33%) as the correct answer (B; 37%).

Question 26 was the only question in which a distractor (B; chosen by 46% of students) proved more popular than the correct answer (D; 35%). The position vector of a particle is given and its initial *direction* of motion has to be calculated by evaluating the *velocity* vector at $t = 0$. Option B is the result obtained by evaluating the position vector at $t = 0$ and indicates a major conceptual misunderstanding. This question needs to be compared with Question 5d of Examination 2 in 2000 which asked students to calculate direction of motion and highlights the need for teachers to refer to recent Reports for Teachers when consolidating and reviewing material with students.

Four questions that were answered reasonably well deserve further investigation as they each had a distractor that was chosen by more than 25% of students. In Question 2, 29% of students chose option B indicating that they had probably entered the function rule in their graphics calculators as $(x^3+16)\div 4\times x$ and so obtained the graph of $y=\frac{x^4+16x}{4}$. An almost identical error was made by nearly 50% of the students in Question 1 of Part II of Examination 1 last year and was highlighted in the Report for Teachers.

In Question 9, 28% of students chose option B, which only gives the second quadrant 'half' of line *S*, and indicates a misconception that Arg $z = \theta$ describes a line (rather than a ray) in the complex plane.

Option C was chosen by 31% of students in Question 21 but is clearly not a parallel vector. On the other hand, it is more obvious that this vector has magnitude 6 than it is that the correct answer (B) has magnitude 6.

Option C was chosen by 27% of students in Question 28. This is the average velocity of the particle but this answer is more likely to have been obtained by *adding* the velocities and dividing by the time interval, 2 s, instead of subtracting the velocities and dividing by the time interval to get the acceleration.

Part 2 – Short-answer questions

Question 1 (Available mark 1.16/Average marks 3)

Answer: $\frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$

Over 40% of students scored no marks on this question. Some students invented formulas (such as $4\sin(x)\cos(x)$ for $\sin(4x)$, whilst others tried various substitutions for $\cos(2x)$. A common error, indicating a lack of conceptual understanding of circular functions, was to simplify $\frac{\sin(4x)}{\cos(2x)}$ to $\tan(2x)$.

Many students who substituted correctly for $\sin(4x)$ did not obtain two of the solutions, $\frac{\pi}{4}$ and $\frac{3\pi}{4}$, because they cancelled the $\cos(2x)$ term on each side of the resultant equation. This is not a valid process unless it is known that the term being cancelled is non-zero. Students should be encouraged to 'equate to 0 and factorise'. Many students used their graphics calculator to find some or all of the solutions but did not receive full marks unless they showed that the answers obtained this way were indeed exact solutions.

Question 2

a. (1.04/2)

Answer: $\frac{2}{x+3}-\frac{7}{(x+3)^2}$

Many students did not know that the correct partial fraction form was $\frac{A}{x+3}+\frac{B}{(x+3)^2}$, and tried $\frac{A}{x+3}+\frac{B}{(x+3)}$ or (less often) $\frac{A}{x+3}+\frac{Bx}{(x+3)^2}$.

About one-third of the students who had the correct form could not successfully solve for both *A* and *B*. A common error was to write $2x-1=A(x+3)^2+B(x+3)$. Finding *A* caused most difficulties since, unlike with the usual non-repeated factor case, the one value, $x=-3$, eliminates both factors giving $B = -7$, but no value for *A*. In cases where there is a repeated factor, substituting $x = 0$ is generally the simplest way to obtain an equation that will give the value of the other coefficient (A, in this case).

b. (0.40/1)

Answer: $2\log_e(x+3)+\frac{7}{x+3}$

Reasonably well done by those students who had the correct partial fraction form. Common errors were to have the sign of the second term incorrect, or (the perennial error) to give it as $-7\log_e(x+3)^2$.

c. (0.36/1)

Answer: –2.11

It was pleasing to note that many students who had experienced difficulties with parts a. or b. realised they could still answer this part by using the numerical integration capability of their graphics calculator. However, other students apparently did not recognise this opportunity and did not attempt the question. Many students who had answered parts a. and b. successfully answered this part by numerical integration anyhow. This was a wise strategy as it did not depend on their previous work being correct; alternatively, students who used their part b antiderivative to answer part c should have checked their answer in this way.

A common error was to give the answer as +2.11, indicating a lack of understanding of the distinction between evaluating an integral and using it to find an area. Most students quoted their final answer correct to three decimal places, rather than to three significant figures, suggesting a lack of familiarity with the concept of 'significant figures' (however, this was not penalised in this instance since –2.108 is correct to (at least) three significant figures).

Question 3 (1.10/3)

Answer: $1 + i$, $-1.366 + 0.366i$, $0.366 - 1.366i$

Many students answered this question, more or less successfully, by following a well-rehearsed routine of converting to polar form, finding the three cube roots using de Moivre's theorem, and converting back to Cartesian form. The most common mistakes made were to get an incorrect argument for $-2 + 2i$ (usually just a case of the wrong quadrant, a mistake easily avoided by a quick sketch plot), and to add π instead of 2π to the argument and so get answers separated by $\frac{\pi}{3}$ and $\frac{2\pi}{3}$.

The overall response to this question was, however, disappointing given its standard nature. Some students seemed to be trying to find three *cubes* of $-2 + 2i$, some seemed to think there was only one cube root, and others attempted the almost impossible by writing $(x + yi)^3 = -2 + 2i$ and trying to equate real and imaginary parts. Some students obviously used their graphics calculator to obtain $1 + i$ directly, but only a handful were then able to go on and obtain the other two cube roots correctly.

Question 4 (1.22/3)

Answer: Result follows from showing that $\overrightarrow{AD}.\overrightarrow{AB} = |\underset{\sim}{u}|^2 - |\underset{\sim}{v}|^2 = 0$

Most students made a reasonable start by correctly expressing $\overrightarrow{AD}$ and $\overrightarrow{AB}$ in terms of $\underset{\sim}{u}$ and $\underset{\sim}{v}$, although some had the expressions interchanged, but then were unable to begin, or to sustain, a mathematically correct argument.

Many students got as far as:

$$\overrightarrow{AD}.\overrightarrow{AB} = |\underset{\sim}{u}|^2 - |\underset{\sim}{v}|^2$$

but then wrote

$$|\underset{\sim}{u}|^2 - |\underset{\sim}{v}|^2 = 1 - 1 = 0, \text{ or } |\underset{\sim}{u}|^2 - |\underset{\sim}{v}|^2 = 0$$

'because the two vectors are equal' or 'because $\angle BAD$ is a right angle' (which is what had to be proved). Others correctly showed $\overrightarrow{AD}.\overrightarrow{AB} = 0$, but then gave the converse as their conclusion, ie. '$\overrightarrow{AD}.\overrightarrow{AB} = 0$ because $\angle BAD$ is a right angle'.

The misuse of vector notation was widespread, being best characterised as 'a random sprinkling' and ranging from a complete absence of tildes to their liberal overuse. Examples such as $\overrightarrow{AD} = u + v$ and $AC = CD$ so $\underset{\sim}{u} = \underset{\sim}{v}$ were common. Not withstanding the (mis)use of vector notation this year students are expected to use correct notation throughout their proofs. Correct use of vector notation can only become routine if teachers draw student attention to it throughout the year.

Question 5 (1.09/3)

Answer: $\frac{dQ}{dt} = 0.25 - \frac{3Q}{500 + 2t}; t = 0, Q = 10$

Most students got the first term correct, although some had 0.025. The second term caused considerable difficulty especially the denominator. Many students who obtained the correct differential equation, did not specify any initial condition. This may have been due to not reading the question carefully enough and missing this instruction, but others apparently did not know what was required. Specifying the initial condition should be regarded as an integral part of setting up such a differential equation. Despite the highlighted direction to the contrary, many students wasted time by trying to solve their differential equation.

Question 6 (1.81/4)

Answer: $\frac{\pi}{2}(4-\pi)$

This question was reasonably well done and there was a marked decrease in the amount of 'sloppy' notation (e.g. omission of *dx* from integrals). Common errors included writing $V=\pi\int_{0}^{\frac{\pi}{2}}(\sin x-(1-\cos x))^2\,dx$ and $(1-\cos x)^2=1-\cos^2 x$, failure to recognise the need to express $\sin^2 x$ and $\cos^2 x$ in terms of $\cos(2x)$ and errors with signs. The first mistake was avoided by students who worked with two separate integrals, though this meant they did not have the opportunity to simplify the integration by replacing $\sin^2 x-\cos^2 x$ with $\cos(2x)$.