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SPECIALIST MATHS TRIAL EXAM 1
2001
SOLUTIONS

Part I – Multiple-choice answers

1.	**C**	**7.**	**D**	**13.**	**A**	**19.**	**D**	**25.**	**A**
2.	**C**	**8.**	**D**	**14.**	**E**	**20.**	**C**	**26.**	**A**
3.	**E**	**9.**	**E**	**15.**	**E**	**21.**	**D**	**27.**	**E**
4.	**E**	**10.**	**D**	**16.**	**B**	**22.**	**B**	**28.**	**E**
5.	**B**	**11.**	**B**	**17.**	**D**	**23.**	**B**	**29.**	**C**
6.	**A**	**12.**	**B**	**18.**	**B**	**24.**	**C**	**30.**	**C**

Part I- Multiple-choice solutions

Question 1

Now, $y=\dfrac{3-2x^2}{x}$

$=\dfrac{3}{x}-2x$

The asymptotes are given by $x=0$ and $y=-2x$.
The answer is C.

Question 2

The hyperbola $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$

has centre $(h,\ k)$ and asymptotes $y-k=\pm\dfrac{b}{a}(x-h)$.

So the centre of our hyperbola is given by $(3,\ -2)$

The asymptotes are given by $y+2=\pm\dfrac{3}{1}(x-3)$

$y+2=3x-9$ or $y+2=-3x+9$

So, $y=3x-11$ or $y=-3x+7$

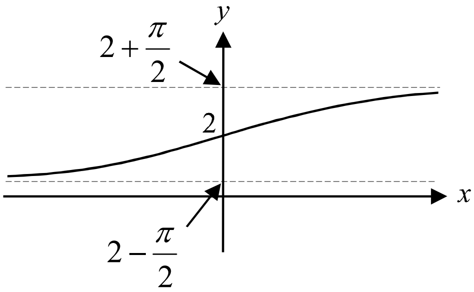
The answer is C.

Question 3

Sketch the graph of the function

$f(x)=2+\text{Tan}^{-1}\dfrac{x}{2}$

The domain is R. The answer is E.



Question 4

$$y = 2\mathrm{Cos}^{-1}(3x)$$

$$= 2\mathrm{Cos}^{-1}u \qquad \text{where } u = 3x \text{ and so } \frac{du}{dx} = 3$$

So, $\dfrac{dy}{du} = 2 \times \dfrac{-1}{\sqrt{1-u^2}}$

Now $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

So, $\dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-9x^2}} \times 3$

$$= \frac{-6}{\sqrt{1-9x^2}}$$

The answer is E.

Question 5

In Cartesian form $z = 1 + \sqrt{3}i$

$r = \sqrt{1^2 + (\sqrt{3})^2}$

$= 2$

$\theta = \tan^{-1}\dfrac{\sqrt{3}}{1}$

$= \dfrac{\pi}{3}$ since z is in the first quadrant

So, $z = 2\mathrm{cis}(\dfrac{\pi}{3})$

The answer is B.

Question 6

Now, $\dfrac{1}{\bar{v}-2} = \dfrac{1}{3+2i-2}$

$$= \frac{1}{1+2i}$$

$$= \frac{1}{1+2i} \times \frac{1-2i}{1-2i}$$

$$= \frac{1-2i}{5}$$

$$= \frac{1}{5} - \frac{2}{5}i$$

The answer is A.

Question 7

$$w = 1 - i$$

Now, $r = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$$\theta = \tan^{-1}\frac{-1}{1}$$

$$= -\frac{\pi}{4} \text{ since } w \text{ is in the fourth quadrant}$$

So, $w^{10} = (\sqrt{2}\text{cis}(-\frac{\pi}{4}))^{10}$

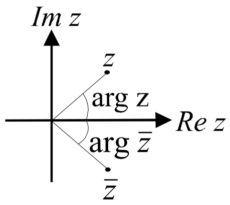
$$= 32\text{cis}(-\frac{10\pi}{4})$$

$$= 32\text{cis}(-\frac{\pi}{2})$$

The answer is D.

Question 8

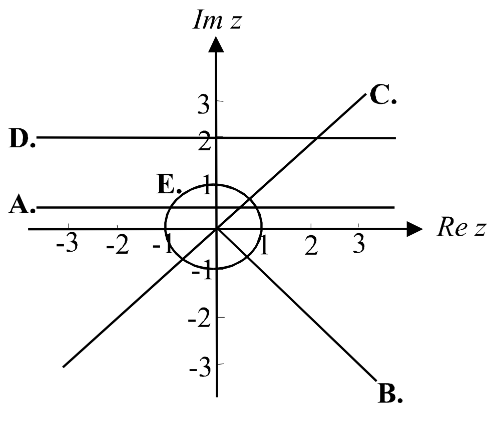
The complex number z and its conjugate $\bar{z}$ are reflections of one another in the real axis. Hence, $\arg z = -\arg(\bar{z})$.



The answer is D.

Question 9

The graphs of all the options are shown.
Only option E provides a curve.
The answer is E.



Question 10

$$\int \frac{9}{9+x^2}\,dx = 3\int \frac{3}{9+x^2}\,dx$$

$$= 3\text{Tan}^{-1}\frac{x}{3} \quad \text{where } c = 0$$

The answer is D.

Question 11

$$\int 6x^2\sqrt{x^3+1}\,dx = \int 2\times\frac{du}{dx}\sqrt{u}\,dx \qquad \text{where } u = x^3+1 \text{ and hence } \frac{du}{dx} = 3x^2$$

$$= 2\int u^{\frac{1}{2}}du$$

$$= 2\times u^{\frac{3}{2}}\times\frac{2}{3}+c$$

$$= \frac{4(x^3+1)^{\frac{3}{2}}}{3}+c$$

The answer is B.

Question 12

$$\int_{-2}^{0} x\sqrt{x+2}\,dx = \int_{0}^{2}(u-2)u^{\frac{1}{2}}\frac{du}{dx}dx \qquad \text{where } u = x+2 \text{ and } \frac{du}{dx} = 1 \text{ and } x = u-2$$

$$= \int_{0}^{2}(u^{\frac{3}{2}}-2u^{\frac{1}{2}})du \qquad \text{So, if } x = 0,\ u = 2 \text{ and if } x = -2,\ u = 0$$

$$= \left[\frac{2u^{\frac{5}{2}}}{5}-\frac{4u^{\frac{3}{2}}}{3}\right]_0^2$$

The answer is B.

Question 13

Now, $f'(x) = \dfrac{\sin^4 x}{\sec x}$

So, $f(x) = \displaystyle\int\frac{\sin^4 x}{\sec x}dx$

$$= \int(\sin^4 x)\cos x\,dx \qquad \text{Let } u = \sin x \text{ and so } \frac{du}{dx} = \cos x$$

$$= \int u^4\frac{du}{dx}dx$$

$$= \int u^4 du$$

$$= \frac{u^5}{5}+c$$

$$= \frac{\sin^5 x}{5}+c$$

Since $f(\pi) = 0$,

we have $0 = \dfrac{\sin^5 \pi}{5}+c$

So, $c = 0$

So, $f(x) = \dfrac{\sin^5 x}{5}$

The answer is A.

Question 14
We cannot evaluate this definite integral analytically but we can use a graphics calculator to do so. We obtain 1.169229. Correct to 2 decimal places, we have 1.17
The answer is E.

Question 15

The trapezoidal rule is given by $\int_a^b f(x)\,dx \approx \frac{1}{2}(b-a)\{f(a)+f(b)\}$

So, for 2 equal intervals we have $\int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx$

$$= \frac{1}{2}\times 1(f(2)+f(3)) + \frac{1}{2}\times 1(f(3)+f(4))$$
$$= \frac{1}{2}(f(2)+2f(3)+f(4))$$
$$= \frac{1}{2}(\frac{1}{3}+2\times\frac{1}{4}+\frac{1}{5})$$
$$= \frac{31}{60}$$

The answer is E.

Question 16

$\frac{dy}{dx} = 2e^{2x}$ and $x = \log_e t$ So, $\frac{dx}{dt} = \frac{1}{t}$

Now $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$

So, $2e^{2x} = \frac{dy}{dt}t$

So, $\frac{dy}{dt} = \frac{2e^{2x}}{t}$ Now, if $x = \log_e t$, then $e^x = t$ and so $e^{2x} = t^2$

So, $\frac{dy}{dt} = \frac{2t^2}{t}$

$= 2t$

So, $y = \int 2t\,dt$

$= \frac{2t^2}{2} + c$ Now, when $t = 1$, $y = 2$,

So, $2 = \frac{2}{2} + c$

$c = 1$

So, $y(t) = t^2 + 1$
The answer is B.

Question 17

Now if $\dfrac{dy}{dx} = f(x), \quad x_0 = a, \quad y_0 = b$

$x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

Now, $x_0 = 1, \quad y_0 = \dfrac{1}{2}$ and $\dfrac{dy}{dx} = f(x) = 5x + 1$

So, $x_1 = 1 + 0.1$ and $y_1 = \dfrac{1}{2} + 0.1 \times (5 \times 1 + 1)$

$= 1.1 \qquad = 1.1$

$x_2 = 1.1 + 0.1$ and $y_2 = 1.1 + 0.1 \times (5 \times 1.1 + 1)$

$= 1.2 \qquad = 1.75$

$x_3 = 1.2 + 0.1$ and $y_3 = 1.75 + 0.1 \times (5 \times 1.2 + 1)$

$= 1.3 \qquad = 2.45$

The answer is D.

Question 18

For $x < 0$, the graph of *f(x)* has a positive, decreasing gradient. All options show this.

At $x = 0$, the gradient of the graph of *f(x)* is a minimum, but it does not equal zero, that is, there is not a stationary point there. Rather, there is a non-stationary point of inflection. This rules out options A and C.

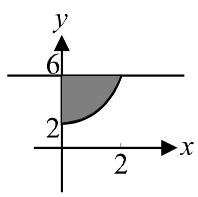
At $x = a$ the gradient is at a maximum and thereafter decreases whilst remaining positive. This rules out option D.

At $x = b$ the gradient of *f(x)* is zero and thereafter becomes negative. This rules out option E but option B shows this.

The answer is B.

Question 19

Do a quick sketch.



$$\text{Volume required} = \pi \int_0^2 \left\{36 - (x^2 + 2)^2\right\} dx$$

$$= \pi \int_0^2 (36 - x^4 - 4x^2 - 4) dx$$

$$= \pi \int_0^2 (-x^4 - 4x^2 + 32) dx$$

The answer is D.

Question 20

The displacement of the particle is the signed area or the value of the definite integral. The distance travelled would be given by option B, but we require displacement.

$$\text{Displacement } = \int_0^{10} \left(\frac{-t^2}{10}+10\right)dt + \int_{10}^{15} (-t+10)\,dt$$

$$= \int_0^{10} \left(\frac{-t^2}{10}+10\right)dt - \int_{10}^{15} (t-10)\,dt$$

The answer is C.

Question 21

We have $\overrightarrow{OP} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$

and $\overrightarrow{OQ} = \underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$

So, $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$$= -2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k} + \underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$$

$$= -\underset{\sim}{i} + 3\underset{\sim}{j} - 8\underset{\sim}{k}$$

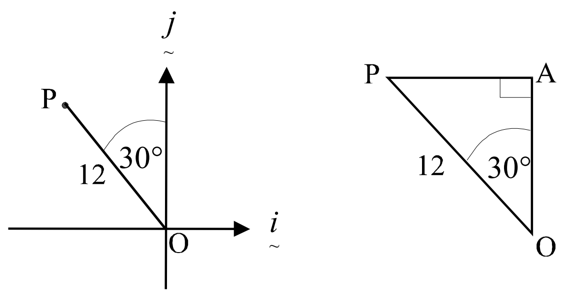
$$\left|\overrightarrow{PQ}\right| = \sqrt{1+9+64}$$

$$= \sqrt{74}$$

The answer is D.

Question 22

Do a quick sketch.



Now, in ΔAPO, $\cos 30° = \dfrac{AO}{12}$

$$AO = 12 \times \frac{\sqrt{3}}{2}$$

$$= 6\sqrt{3}$$

Also, $\sin 30° = \dfrac{AP}{12}$

$$AP = 6$$

So, $\overrightarrow{OP} = -6\underset{\sim}{i} + 6\sqrt{3}\,\underset{\sim}{j}$

The answer is B.

Question 23

Let $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - 6\underset{\sim}{k}$

$$\left|\underset{\sim}{a}\right| = \sqrt{9+4+36}$$
$$= \sqrt{49}$$
$$= 7$$

Let $\underset{\sim}{b} = \underset{\sim}{i} + \sqrt{2}\,\underset{\sim}{j} - 4\underset{\sim}{k}$

$$\left|\underset{\sim}{b}\right| = \sqrt{1+2+16}$$
$$= \sqrt{19}$$

Now, $\underset{\sim}{a}.\underset{\sim}{b} = \left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta$

So, $\cos\theta = \dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|}$

$$= \frac{3\times 1 + 2\times\sqrt{2} - 6\times -4}{7\times\sqrt{19}}$$
$$= \frac{27+2\sqrt{2}}{7\sqrt{19}}$$
$$\theta = 12°9'$$

The answer is B.

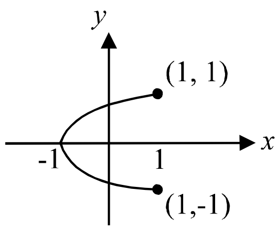
Question 24

The component of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is given by

$$\underset{\sim}{a} - (\underset{\sim}{a}.\underset{\sim}{\hat{b}})\underset{\sim}{\hat{b}}$$
$$= 2\underset{\sim}{i} + 6\underset{\sim}{j} - 7\underset{\sim}{k} - \left\{(2\underset{\sim}{i} + 6\underset{\sim}{j} - 7\underset{\sim}{k}).\frac{1}{3}(-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k})\right\}\underset{\sim}{\hat{b}} \quad \text{since } \left|\underset{\sim}{b}\right| = \sqrt{1+4+4} = 3$$
$$= 2\underset{\sim}{i} + 6\underset{\sim}{j} - 7\underset{\sim}{k} - \left(\frac{-2+12+14}{3}\right)\underset{\sim}{\hat{b}}$$
$$= 2\underset{\sim}{i} + 6\underset{\sim}{j} - 7\underset{\sim}{k} - \frac{8}{3}(-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k})$$
$$= 2\underset{\sim}{i} + 6\underset{\sim}{j} - 7\underset{\sim}{k} + \frac{8}{3}\underset{\sim}{i} - \frac{16}{3}\underset{\sim}{j} + \frac{16}{3}\underset{\sim}{k}$$
$$= \frac{14}{3}\underset{\sim}{i} + \frac{2}{3}\underset{\sim}{j} - \frac{5}{3}\underset{\sim}{k}$$
$$= \frac{1}{3}(14\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k})$$

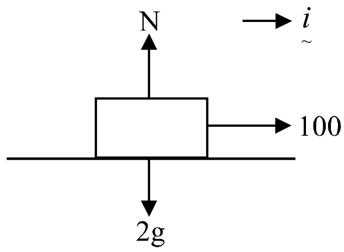
The answer is C.

Question 25
Now, $x = \cos(2t)$ and $y = \cos t$. Since $0 \le t \le 2\pi$, $x \in [-1,1]$ and y $\in$ [-1,1]
This can be graphed in parametric mode on a graphics calculator. Alternatively:
When $t = 0$, $x = 1$ and $y = 1$ and so we have $(1,1)$
When $t = \pi$, $x = 1$ and $y = -1$ and so we have $(1,-1)$
When $t = 2\pi$, $x = 1$ and $y = 1$ and so we have $(1,1)$
Now, $\cos(2t) = 2\cos^2 t - 1$ and since $x = \cos(2t)$
We have , $x = 2\cos^2 t - 1$
so $x = 2y^2 - 1$
so, $y^2 = \dfrac{x+1}{2}$
$y = \pm\sqrt{\dfrac{x+1}{2}}$



The path is parabolic. The endpoints are $(1,1)$ and $(1,-1)$.
The answer is A.

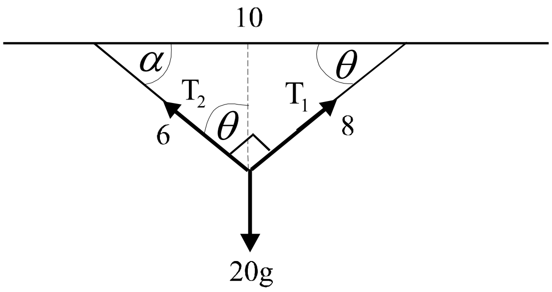
Question 26
From the diagram, we have
$\underset{\sim}{R} = 100\underset{\sim}{i}$
$\underset{\sim}{R} = ma\underset{\sim}{i}$
So, $2a = 100$
$a = 50\text{m/s}^2$



Now, the acceleration is constant and so
We have $s = ut + \dfrac{1}{2}at^2$
Where $u = 0$, $s = 10$ and $a = 50$
So, $10 = 0 + \dfrac{1}{2} \times 50t^2$
$t = \dfrac{\sqrt{10}}{5}$ seconds
The answer is A.

Question 27
The change in momentum is equal to the inertial mass times the change in velocity.
So, we have $2 \times 15 - 2 \times 12 = 6$ kg m/s
The answer is E.

Question 28
Do a quick sketch.



Resolving horizontally, we have
$T_1 \cos\theta = T_2 \sin\theta$ ……….(A)
Resolving vertically, we have
$T_1 \sin\theta + T_2 \cos\theta = 20g$ ……..(B)
Also, $\cos\theta = \dfrac{4}{5}$ and $\sin\theta = \dfrac{3}{5}$

So, (A) becomes $\frac{4}{5}T_1 = \frac{3}{5}T_2$

$$T_1 = \frac{3}{4}T_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(C)$$

In (B), we have $\frac{3}{4}T_2 \times \frac{3}{5} + T_2 \times \frac{4}{5} = 20g$

So, $\frac{9}{20}T_2 + \frac{4}{5}T_2 = 20g$

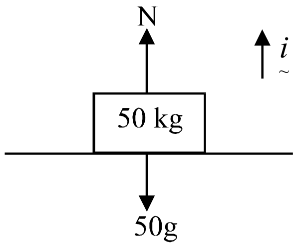
$$\frac{5}{4}T_2 = 20g$$

$$T_2 = 16g$$

The answer is E.

Question 29

Do a quick sketch.



Now, $\underset{\sim}{R} = m\underset{\sim}{a}$

$= 50a\underset{\sim}{i}$

Also, $\underset{\sim}{R} = (N - 50g)\underset{\sim}{i}$

So, $50a = N - 50g$

$a = \frac{N - 50g}{50}$ Now, since $N = 75g$

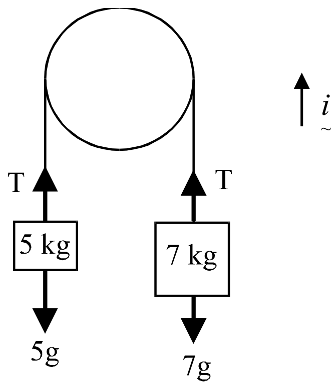
we have $a = \frac{75g - 50g}{50}$

$= \frac{25g}{50}$

So, $a = \frac{g}{2}$

The answer is C.

Question 30



Mark in the forces on the diagram.
Resolving around the 7 kg weight, we have

$\underset{\sim}{R} = m\underset{\sim}{a}$

$= 7a \times -\underset{\sim}{i}$

$= -7a\underset{\sim}{i}$

So, $\underset{\sim}{R} = (T - 7g)\underset{\sim}{i}$

$T - 7g = -7a$

$a = \frac{7g - T}{7}$

The answer is C.

Part II – Short answer questions

Question 1

$$y = \frac{1}{2x^2 - 9x - 5}$$
$$= \frac{1}{(2x+1)(x-5)}$$

The graph has an asymptote when $2x+1=0$, that is $x = -\frac{1}{2}$ and another when $x-5=0$, that is $x=5$. **(1 mark)**

Since $y = (2x^2 - 9x - 5)^{-1}$

$$\frac{dy}{dx} = -1(2x^2 - 9x - 5)^{-2} \times (4x - 9)$$
$$= \frac{9-4x}{(2x^2-9x-5)^2}$$

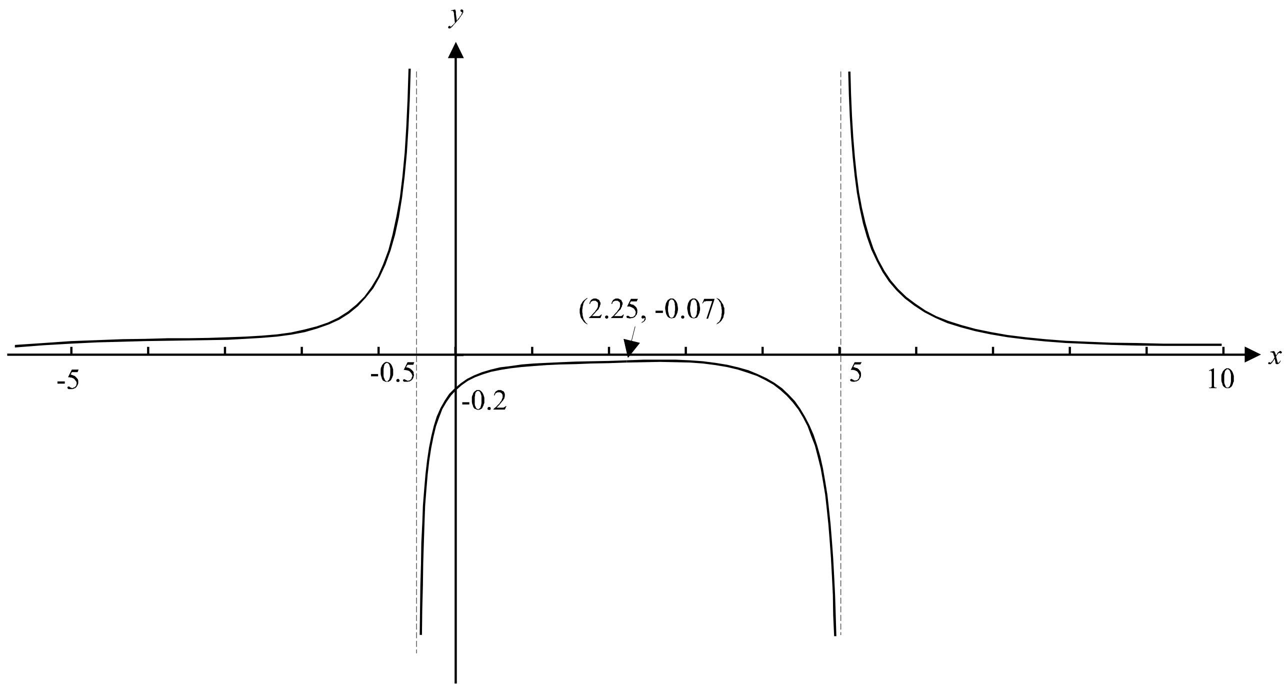
When $\frac{dy}{dx} = 0$, we have $9 - 4x = 0$

$$x = 2.25$$

So, $y = -0.07$ (correct to 2 decimal places)

So, there is a maximum turning point at $(2.25, -0.07)$ **(1 mark)**

There are no x-intercepts since when $y = 0$, we have $0 = \frac{1}{2x^2 - 9x - 5}$ which has no solutions. The y-intercept occurs at $(0, -0.2)$. **(1 mark)**



(1 mark)

Question 2

To show that we have a square, it is sufficient to show that we have 4 equal sidelengths and one internal right angle.

Now $\overrightarrow{ZA} = \overrightarrow{XB} = \overrightarrow{DZ} = \overrightarrow{CX} = \underset{\sim}{a}$

and $\overrightarrow{AW} = \overrightarrow{WB} = \overrightarrow{DY} = \overrightarrow{YC} = \underset{\sim}{b}$

Also, $\overrightarrow{ZW} = \underset{\sim}{a} + \underset{\sim}{b}$

And $\overrightarrow{YX} = \overrightarrow{YC} + \overrightarrow{CX}$

$= \underset{\sim}{b} + \underset{\sim}{a}$

$= \overrightarrow{ZW}$

So, $\overrightarrow{ZW}$ and $\overrightarrow{YX}$ are parallel and the same length.

Similarly, $\overrightarrow{WX} = \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{ZY} = -\underset{\sim}{a} + \underset{\sim}{b}$

$= \overrightarrow{WX}$

So, $\overrightarrow{WX}$ and $\overrightarrow{ZY}$ are parallel and the same length. **(1 mark)**

Now, $\left|\overrightarrow{ZY}\right| = \sqrt{\left|-\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2}$

$= \sqrt{\left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2}$

and $\left|\overrightarrow{ZW}\right| = \sqrt{\left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2}$

So, $\left|\overrightarrow{ZY}\right| = \left|\overrightarrow{WX}\right| = \left|\overrightarrow{ZW}\right| = \left|\overrightarrow{YX}\right|$

So the sides of the quadrilateral WXYZ are equal in length. **(1 mark)**

Also, $\overrightarrow{ZW}.\overrightarrow{WX} = (\underset{\sim}{a} + \underset{\sim}{b}).(-\underset{\sim}{a} + \underset{\sim}{b})$

$= -\underset{\sim}{a}.\underset{\sim}{a} + \underset{\sim}{a}.\underset{\sim}{b} - \underset{\sim}{a}.\underset{\sim}{b} + \underset{\sim}{b}.\underset{\sim}{b}$

$= -\left|\underset{\sim}{a}\right|\left|\underset{\sim}{a}\right|\cos 0 + \left|\underset{\sim}{b}\right|\left|\underset{\sim}{b}\right|\cos 0$

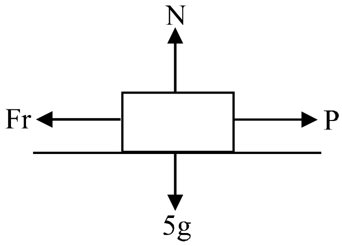
$= -\left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2$

$= 0 \qquad \text{since } \left|\underset{\sim}{a}\right| = \left|\underset{\sim}{b}\right|$

So, $\overrightarrow{ZW}$ is perpendicular to $\overrightarrow{WX}$.

So we have four equal sidelengths and one internal angle of $90°$. Therefore WXYZ is a square. **(1 mark)**

Question 3



Now, from the diagram, we have, $N = 5g$.
Also, $P = Fr$ **(1 mark)**
Now, when the toy box is on the point of moving,

$$Fr = \mu N \quad \textbf{(1 mark)}$$
$$= \frac{2}{g} \times 5g$$
$$= 10 \text{ Newton}$$

Since $P = 8$ Newton, the toy box is not on the point of moving and hence the child will not move it by applying this force. **(1 mark)**

Question 4

a. Let $p(z) = z^4 + 2z^3 + 2kz^2 + 8z + 40 = 0$
If $2i$ is a solution to the equation then $p(2i) = 0$
So, $p(2i) = 16 + 2 \times 8 \times -i + 2k \times -4 + 16i + 40 = 0$

$$16 - 16i - 8k + 16i + 40 = 0$$
$$56 = 8k$$
$$k = 7 \quad \textbf{(1 mark)}$$

b. Since $p(z)$ has real coefficients we can use the conjugate root theorem.
If $2i$ is a solution of $p(z)$ then $z - 2i$ is a factor and according to the conjugate root theorem, so is $z + 2i$.
Now, $(z - 2i)(z + 2i) = z^2 + 4$ **(1 mark)**

$$\begin{array}{r|l} & z^2 + 2z + 10 \\ \hline z^2 + 4 & z^4 + 2z^3 + 14z^2 + 8z + 40 \\ & \underline{z^4 \qquad\quad + 4z^2} \\ & \qquad 2z^3 + 10z^2 + 8z \\ & \qquad \underline{2z^3 \qquad\quad + 8z} \\ & \qquad\qquad 10z^2 \qquad + 40 \\ & \qquad\qquad \underline{10z^2 \qquad + 40} \end{array}$$

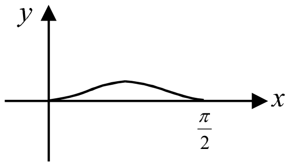
So, $p(z) = (z^2 + 4)(z^2 + 2z + 10)$

$$= (z^2 - 4i^2)((z^2 + 2z + 1) - 1 + 10)$$
$$= (z - 2i)(z + 2i)((z + 1)^2 - 9i^2)$$
$$= (z - 2i)(z + 2i)(z + 1 - 3i)(z + 1 + 3i)$$

Solutions to $p(z) = 0$ are $\pm 2i, -1 \pm 3i$ **(1 mark)**

Question 5
Do a quick sketch to see how the region described is in relation to the *x*-axis.

So, area required $= \int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx$ **(1 mark)**



$= \int_0^{\frac{\pi}{2}} (\sin^2 x)(\cos^2 x)(\cos x)\, dx$

$= \int_0^{\frac{\pi}{2}} \sin^2 x (1 - \sin^2 x) \cos x \, dx$ Now, let $u = \sin x$ and so $\dfrac{du}{dx} = \cos x$

$= \int_0^1 u^2 (1 - u^2) \dfrac{du}{dx} dx$ so if $x = \dfrac{\pi}{2}$, $u = 1$ and if $x = 0$, $u = 0$

$= \int_0^1 (u^2 - u^4) du$ **(1 mark)**

$= \left[\dfrac{u^3}{3} - \dfrac{u^5}{5} \right]_0^1$

$= (\dfrac{1}{3} - \dfrac{1}{5}) - 0$

$= \dfrac{2}{15}$ square units **(1 mark)**

Question 6

a. Now, $a = g - 0.1v^2$ so,

$$v \frac{dv}{dx} = g - 0.1v^2$$

$$\frac{dv}{dx} = \frac{g}{v} - \frac{v}{10}$$

$$= \frac{10g - v^2}{10v}$$

so, $\dfrac{dx}{dv} = \dfrac{10v}{10g - v^2}$ as required **(1 mark)**

b. $\int \dfrac{dx}{dv} dv = \int \dfrac{10v}{10g - v^2} dv$

$= \dfrac{10}{-2} \int \dfrac{-2v}{10g - v^2} dv$

$x = -5 \log_e (10g - v^2) + c$ **(1 mark)**

Now when $x = 0$, $v = 0$, that is, the particle is dropped from rest when $x = 0$

So, $0 = -5 \log_e (10g) + c$

$c = 5 \log_e (10g)$ **(1 mark)**

So, $x = -5 \log_e (10g - v^2) + 5 \log_e (10g)$

$x = 5 \log_e \dfrac{10g}{10g - v^2}$

Now, when $v = 1$, $x = 0.051$ correct to 2 significant figures. **(1 mark)**