The Mathematical Association of Victoria

2000

MATHEMATICS: SPECIALIST

Trial Examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ____

Directions to students

This examination consists of five questions.

Answer all questions.

All working and answers should be written in the spaces provided.

The marks allotted to each part of each question appear at the end of each part.

There are **60 marks** available for this task.

A formula sheet is attached.

These questions have been written and published to assist students in their preparations for the 2000 Specialist Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Board of Studies Assessing Panels. The Association gratefully acknowledges the permission of the Board to reproduce the formula sheet.

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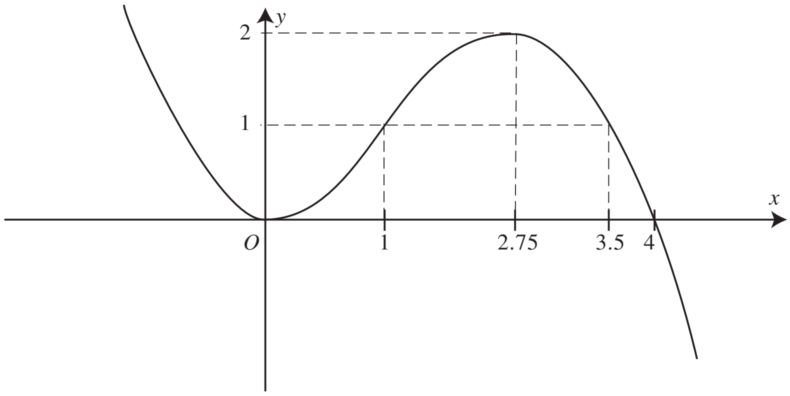
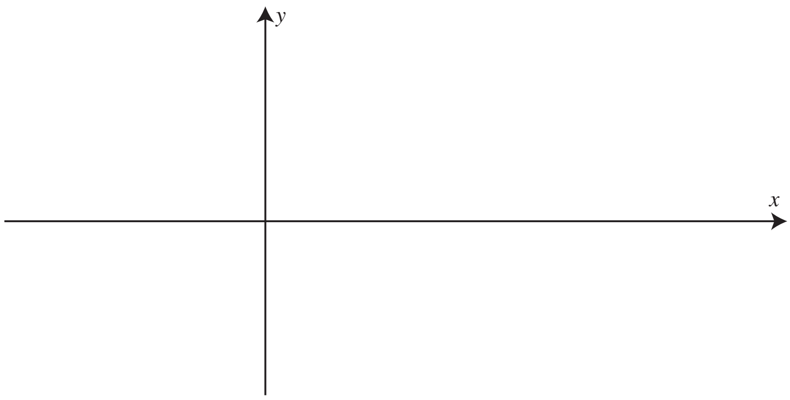
Question 1

The graph of $y_1 = f(x)$ is given below.



a. Sketch $y_2 = \dfrac{1}{f(x)}$.

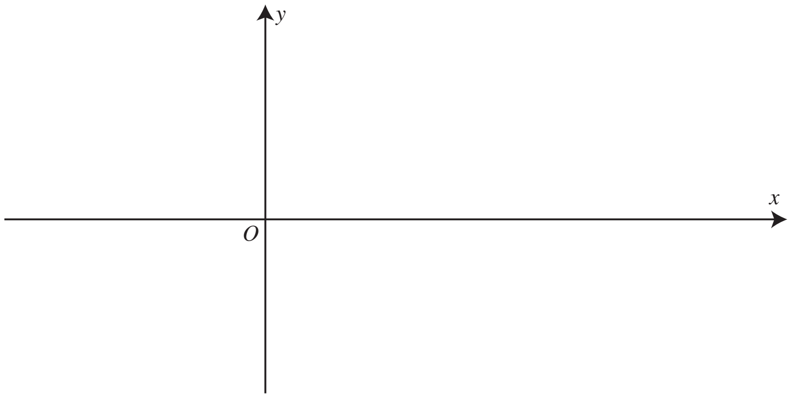


[3 marks]

b. Find, using the trapezoidal rule, an approximate area for $\displaystyle\int_{1}^{2.75} \frac{1}{f(x)}\,dx + \int_{2.75}^{3.5} \frac{1}{f(x)}\,dx$ correct to three decimal places.

[2 marks]

c. Sketch the graph of $y_3 = \int \frac{1}{f(x)}\,dx$



[3 marks]

Total: 8 marks

Question 2

Let $P(z) = z^3 + 5z^2 + 10z + 12$

a. Show that $P(-1 + \sqrt{3}i) = 0$.

[3 marks]

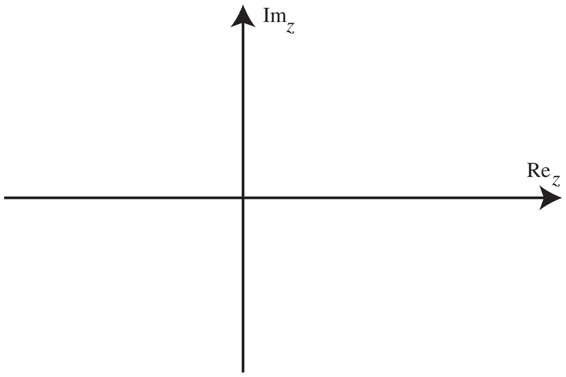
b. Hence, fully factorize $P(z)$.

[2 marks]

c. Express the solutions of $P(z) = 0$ in exact polar form.

[2 marks]

d. Plot these points on the Argand diagram.



[1 mark]

e. On the Argand diagram above, plot the region defined by

$$s = \{ z : |z| < 3 \}$$

and identify which solutions from 2(c) lie within this region.

____

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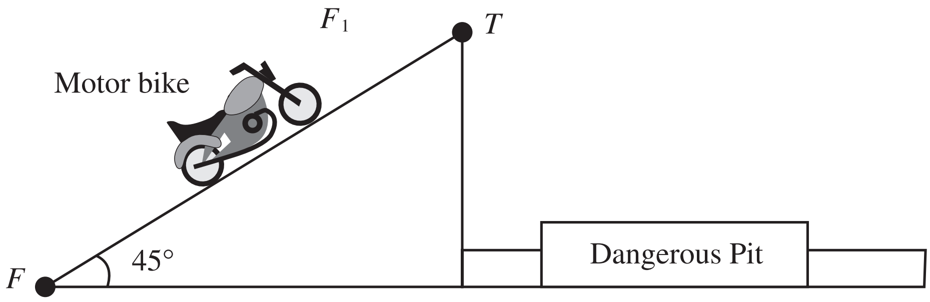
[2 marks]

Total 10 marks

Question 3

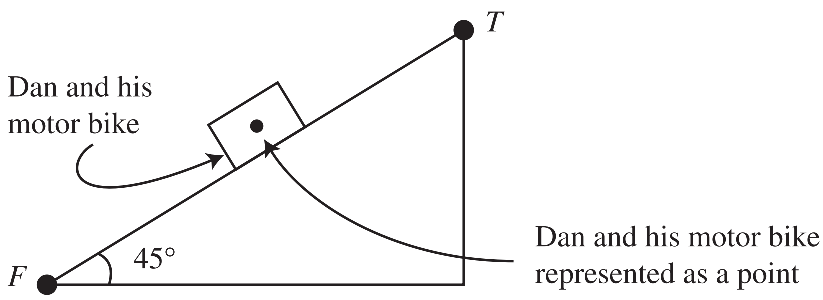
Dangerous Dan leaps across a pit after riding his motorcycle up a ramp.



He reaches a speed of 17 m/s at the point F (the bottom of the ramp) and decelerates at 2 m/s^2 as he travels up the ramp, whose length is 4 m. The combined mass of the motorcycle and Dan is 250 kg. The coefficient of friction between Dan's motorcycle and the ramp is 0.1.

Throughout this question, assume air resistance is negligible, and take the acceleration due to gravity to have magnitude g m/s^2, where g = 9.8.

a. On the diagram below, indicate all the forces on the Dan and his motorcycle as it travels up the ramp.



[1 mark]

b. What is the magnitude of the force generated by the motorcycle, to the nearest newton, as it travels up the ramp?

[3 marks]

c. What is the velocity of the motorcycle when it reaches the point T (the top of the ramp).

[2 marks]

d. At the instant after Dan leaves the ramp, his acceleration is $\ddot{r}(t) = -g\underset{\sim}{j}$.

Express the velocity of Dan and his motor cycle in the form of

$$\dot{r}(t) = \dot{x}(t)\underset{\sim}{i} + \dot{y}(t)\underset{\sim}{j},$$

where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are the horizontal and vertical components of the velocity, as he leaves the ramp, and t is time in seconds from $t = 0$.

[3 marks]

e. Find $r(t)$, and hence express the position of Dan across the pit in cartesian form.

[3 marks]

f. Sketch the graph of Dan's jump from the point he leaves the ramp until he reaches the ground.

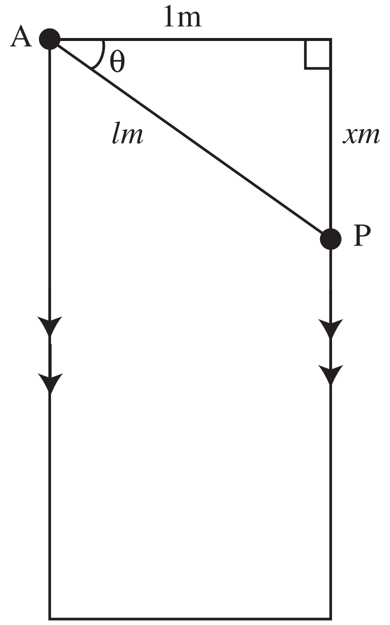
[2 marks]

g. Hence, or otherwise, find the distance from the ramp and the point at which Dan lands on the ground.

[1 mark]

Total 15 marks

Question 4



The movement of a particle (P) is controlled by a laser beam, which is located at the point A.

The particle velocity is given by $\frac{dx}{dt} = 2x$ m/s and begins to move ($t = 0$) at $x = 1$ metre.

a. Express θ in terms of x.

[1 mark]

b. Find the rate of change for the angle θ $\left(\frac{d\theta}{dt}\right)$ at $x = 1$ m.

[2 marks]

c. Show that $\dfrac{d\theta}{dt} = \sin 2\theta$.

[3 marks]

Let l represent the direct distance from A to P.

d. **i.** Express l in terms of θ.

[1 mark]

ii. Find the rate of change of l with respect to time $\left(\dfrac{dl}{dt}\right)$ in terms of θ.

[2 marks]

iii. Show that $\dfrac{dl}{dt} = \dfrac{2(l^2-1)}{l}$.

[2 marks]

iv. Express l in terms of t.

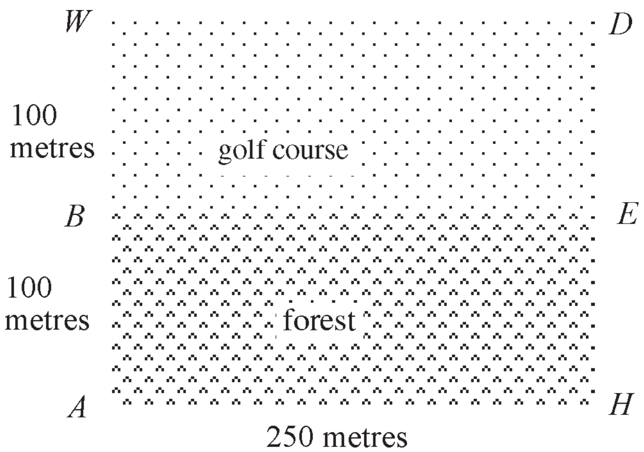
[3 marks]

Total 14 marks

Question 5

There is a narrow strip of forest 100 metres wide separating Luij's house from a nearby golf course. Luij has been down at the waterhole, which is 100 metres inside the golf course recovering golf balls that have been lost by wayward golf shots. Luij looks at his watch and realises that he is late home for lunch. He needs to find the quickest way home from the waterhole (W), across the golf course and through the forest to his home (H).

Luij can run at 5m/s across the golf course but can only travel at 3 m/s through the rough forest.



a. **i.** What distance, to the nearest metre, will Luij have to travel if he runs from W to E across the golf course and then from E to H to reach home?
(Assume the forest extends past Luij's home, beyond EH.)

ii. If he follows this route, how long, in minutes and seconds, will it take Luij to arrive home?

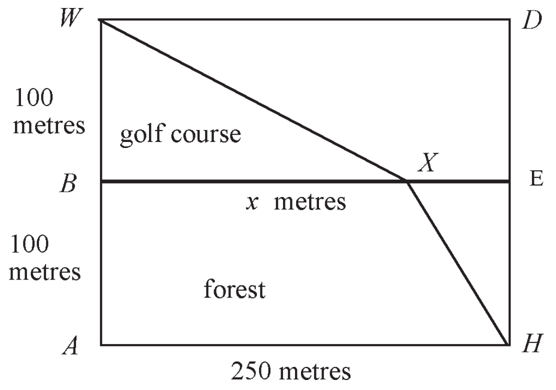
[3 marks]

b. **i.** If Luij takes the direct route home from W to H, how far, to the nearest metre will he run?

ii. How long will it take Luij if he takes this direct route?

[3 marks]

Let X be a point on BE and let the distance from B to X be x metres. Assume Luij runs directly from W to X and then directly from X to H. Let d metres be the total distance Luij travels from W to H and let t seconds be the time he takes to run home over this route.



c. **i.** Express d in terms of x.

ii. Hence express t in terms of x, stating the domain of the function.

[3 marks]

d. **i.** Use a graphics calculator to find to the nearest second, the shortest time that Luij can take to reach home.

ii. Hence, find to the nearest metre, the distance that Luij travels if he takes the quickest route home.

[4 marks]

Total: 13 marks

END OF PAPER — TOTAL MARKS 60

