The Mathematical Association of Victoria

2000

MATHEMATICS: SPECIALIST

Trial Examination 1

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Directions to students

This examination has two parts:
Part I (multiple-choice questions) and **Part II** (short-answer questions).

Answer all questions in **Part I** on the multiple-choice answer sheet provided. There are **30 marks** available for this part.

Part II consists of six questions. Answers all questions in **Part II** in the spaces provided. There are **21 marks** available for this part.

There are **51 marks** available for this task.

A formula sheet is attached.

These questions have been written and published to assist students in their preparations for the 2000 Specialist Mathematics Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Board of Studies Assessing Panels. The Association gratefully acknowledges the permission of the Board to reproduce the formula sheet.

Published by The Mathematical Association of Victoria
"Cliveden", 61 Blyth Street, Brunswick, 3056
Phone: (03) 9380 2399 Fax: (03) 9389 0399
E-mail: office@mav.vic.edu.au website: http://www.mav.vic.edu.au

Multiple-Choice Answer Sheet

Student's Name: _____

Cross through the letter that corresponds to each answer.

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**
23.	**A**	**B**	**C**	**D**	**E**
24.	**A**	**B**	**C**	**D**	**E**
25.	**A**	**B**	**C**	**D**	**E**
26.	**A**	**B**	**C**	**D**	**E**
27.	**A**	**B**	**C**	**D**	**E**
28.	**A**	**B**	**C**	**D**	**E**
29.	**A**	**B**	**C**	**D**	**E**
30.	**A**	**B**	**C**	**D**	**E**

Part I (Multiple-choice questions)

Question 1

If $u = 3\text{cis}\dfrac{3\pi}{4}$ and $v = 2\text{cis}\dfrac{-\pi}{3}$ then u^2v equals

A. $6\text{cis}\dfrac{5\pi}{12}$

B. $9\text{cis}\dfrac{9\pi}{16}$

C. $\dfrac{3}{2}\text{cis}\dfrac{13\pi}{12}$

D. $18\text{cis}\dfrac{7\pi}{6}$

E. $18\text{cis}\dfrac{11\pi}{48}$

Question 2

If $y = \text{Tan}^{-1}\left(\dfrac{2}{x}\right)$, then $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{-2}{x^2+4}$

B. $\dfrac{\frac{1}{2}}{\frac{1}{4}+x^2}$

C. $\dfrac{2}{4+x^2}$

D. $\dfrac{2x^{-2}}{4+x^2}$

E. $\dfrac{2x^{-2}}{\frac{1}{4}+x^2}$

Question 3

If $w = 1+i$ and $z = w^5$ then Arg z is equal to

A. $\dfrac{\pi}{4}$

B. $\dfrac{5\pi}{4}$

C. $\dfrac{-3\pi}{4}$

D. $\dfrac{3\pi}{4}$

E. $4\sqrt{2}$

Question 4

Let $\underset{\sim}{a} = -3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$. $\left(\underset{\sim}{a}.\underset{\sim}{b}\right)\hat{\underset{\sim}{b}}$ is

A. -4

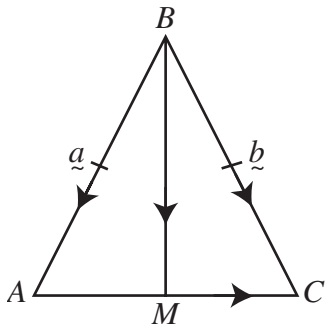
B. $-4(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k})$

C. $\dfrac{2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}}{3}$

D. $\dfrac{-8\underset{\sim}{i} + 8\underset{\sim}{j} - 4\underset{\sim}{k}}{3}$

E. $\dfrac{-8\underset{\sim}{i} - 8\underset{\sim}{j} + 4\underset{\sim}{k}}{\sqrt{26}}$

Question 5



For the triangle *ABC*, *M* is the mid-point of *AC*. The triangle would be an isosceles if

A. $\frac{1}{2}\left(\underset{\sim}{b} + \underset{\sim}{a}\right) \bullet \left(\underset{\sim}{b} - \underset{\sim}{a}\right) = 0$

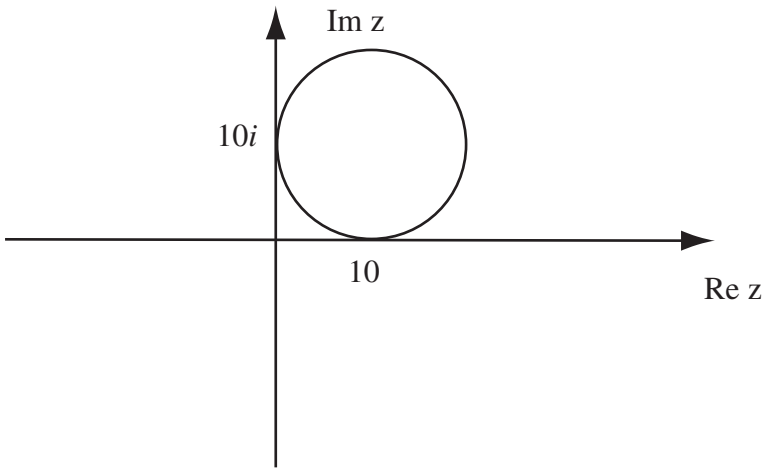
B. $\underset{\sim}{a} \bullet \underset{\sim}{b} = 0$

C. $\frac{1}{2}\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \bullet \left(\underset{\sim}{b} - \underset{\sim}{a}\right) = 0$

D. $\underset{\sim}{a} - \underset{\sim}{b} = 0$

E. $\frac{1}{2}\left(\underset{\sim}{b} + \underset{\sim}{a}\right) \bullet \left(\underset{\sim}{b} + \underset{\sim}{a}\right) = 0$

Question 6



The circle shown above is specified by

A. $\{z:|z+10+10i|=10\}$

B. $\{z:|z-10-10i|=100\}$

C. $\{z:|z-10|=100\}$

D. $\{z:|z-10-10i|=10i\}$

E. $\{z:|z-10-10i|=10\}$

Question 7

An antiderivative of $\dfrac{-16}{\sqrt{1-4x^2}}$ is

A. $-16\log_e\sqrt{1-4x^2}$

B. $8\text{Cos}^{-1}(2x)$

C. $16\text{Cos}^{-1}(2x)$

D. $\text{Cos}^{-1}(2x)$

E. $\text{Sin}^{-1}(2x)$

Question 8

The asymptotes of the hyperbola with equation $4(x-1)^2 - 9y^2 = 36$ are

A. $y = \pm\frac{2}{3}x$

B. $y = \pm\frac{9}{4}(x+1)$

C. $y = \pm\frac{2}{3}(x-1)$

D. $y = \pm\frac{3}{2}(x-1)$

E. $y = \pm\frac{3}{2}(x-1)+36$

Question 9

When the rational expression $\frac{5x+6}{x^2+6x+9}$ is expressed as partial fractions they are

A. $\frac{5x}{x+3}+\frac{6}{x+3}$

B. $\frac{5}{x+3}-\frac{9}{(x+3)^2}$

C. $\frac{3}{(x+3)^2}+\frac{2x}{x+3}$

D. $\frac{9}{(x+3)^2}+\frac{5}{x+3}$

E. $\frac{2}{9}+\frac{3}{x+3}$

Question 10

The position of a particle is given by $\underset{\sim}{r}(t) = 2\cos 2t\,\underset{\sim}{i} + 3\sin 2t\,\underset{\sim}{j}$. The cartesian equation of its path is

A. $4x^2 + 9y^2 = 36$

B. $9x^2 + 4y^2 = 36$

C. $18x^2 + 8y^2 = 36$

D. $x^2 + y^2 = 36$

E. $y = 36 + 16x^2$

Question 11

A particle, initially at rest, has acceleration $\ddot{\underset{\sim}{r}}(t) = 2\underset{\sim}{i} + 6t\,\underset{\sim}{j} + \underset{\sim}{k}$. If $\underset{\sim}{r}(0) = \underset{\sim}{i} + 3\,\underset{\sim}{j}$, then $\underset{\sim}{r}(t)$ is equal to

A. $(t^2+1)\underset{\sim}{i} + (t^3-3)\underset{\sim}{j} + \frac{t^2}{2}\underset{\sim}{k}$

B. $t^2\underset{\sim}{i} + t^3\underset{\sim}{j} + \frac{t^2}{2}\underset{\sim}{k}$

C. $(t^2-1)\underset{\sim}{i} + (t^3-3)\underset{\sim}{j} + \frac{t^2}{2}\underset{\sim}{k}$

D. $(t^2+1)\underset{\sim}{i} + (t^3+3)\underset{\sim}{j} + \frac{t^2}{2}\underset{\sim}{k}$

E. $(t^2+1)\underset{\sim}{i} + (t^3-3)\underset{\sim}{j} + t\,\underset{\sim}{k}$

Question 12

The velocity of a particle, at time t, is given by $\dot{\underset{\sim}{r}}(t) = \cos\frac{\pi t}{6}\underset{\sim}{i} + 3\sin\frac{\pi t}{6}\underset{\sim}{j}$.

The magnitude of its acceleration at $t = 6$ is

A. $\sqrt{10}$

B. $\frac{\pi}{2}$

C. $\pi\sqrt{\frac{1}{4} + \frac{1}{36}}$

D. $\frac{-\pi}{6}$

E. $\frac{\pi}{6}$

Question 13

Let $\underset{\sim}{v} = \underset{\sim}{i} + 2\,\underset{\sim}{j} + \underset{\sim}{k}$ and $w = 2\,\underset{\sim}{i} - \underset{\sim}{k}$. The magnitude of the vector $\underset{\sim}{v} + 2\,\underset{\sim}{w}$ is

A. 30

B. $\sqrt{11}$

C. 11

D. $\sqrt{30}$

E. $\sqrt{38}$

Question 14

Which one of the following differential equations is satisfied by $y = \cos 3x + \sin 3x$?

A. $\frac{dy}{dx} = 3y$

B. $\frac{dy}{dx} = -3y$

C. $\frac{d^2y}{dx^2} = 9y^2$

D. $\frac{d^2y}{dx^2} = 9y$

E. $\frac{d^2y}{dx^2} = -9y$

Question 15

An antiderivative of $\cos^3 x$ is

A. $\sin x - \frac{\sin^3 x}{3}$

B. $-\frac{\cos^4 x}{4}$

C. $\sin x + \frac{\sin^3 x}{3}$

D. $-\frac{\sin^4 x}{4}$

E. $-\sin x - \frac{\cos^3 x}{3}$

Question 16

If $y = \tan(\log_e 3x)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{x}\sec^2(\log_e 3x)$

B. $\frac{3}{x}\sec^2(\log_e 3x) + \frac{1}{x}\tan x$

C. $\sec^2(\log_e 3x) + \frac{1}{x}\tan x$

D. $\sec^2(\log_e 3x)$

E. $3\tan(\log_e 3x)\sec^2(\log_e 3x)$

Question 17

With a suitable substitution, $\int_0^{\frac{\pi}{2}} \cos 3x \, e^{\sin 3x} \, dx$ can be expressed as

A. $\frac{1}{3}\int_0^{\frac{\pi}{2}} e^u \text{du}$

B. $3\int_0^{-1} e^u \text{du}$

C. $-\frac{1}{3}\int_0^{-1} e^u \text{du}$

D. $-\frac{1}{3}\int_{-1}^{0} e^u \text{du}$

E. $-\frac{1}{3}\int_0^{\frac{\pi}{2}} e^u \text{du}$

Question 18

Let $\text{cosec}x = \sqrt{3}$, $\frac{\pi}{2} \leq x < \pi$. The value of $\tan x$ is

A. $-\frac{2}{\sqrt{2}}$

B. $\sqrt{2}$

C. $\frac{1}{\sqrt{3}}$

D. $-\frac{\sqrt{2}}{2}$

E. $-\sqrt{2}$

Question 19

The implied domain of the function with rule $f(x) = \text{Sin}^{-1}(x - a)$ is

A. $[-1 - a, 1 - a]$

B. $[a - 1, a + 1]$

C. $[-\frac{\pi}{2}, \frac{\pi}{2}]$

D. $[-a, \frac{\pi}{2} - a]$

E. $[a, \frac{\pi}{2 + a}]$

Question 20

An antiderivative of $\frac{3}{3+4x^2}$ is

A. $\frac{3}{4}\text{Tan}^{-1}\left(\frac{2x}{3}\right)$

B. $\frac{3}{4}\text{Tan}^{-1}\left(\frac{\sqrt{3}x}{2}\right)$

C. $\frac{\sqrt{3}}{2}\text{Tan}^{-1}\left(\frac{\sqrt{3}x}{2}\right)$

D. $3\text{Tan}^{-1}(2x)$

E. $\frac{\sqrt{3}}{2}\text{Tan}^{-1}\left(\frac{2x}{\sqrt{3}}\right)$

Question 21

If $V=\frac{4}{3}\pi r^3$ and $\frac{dV}{dt}=4$, then $\frac{dr}{dt}$ is equal to

A. $16\pi r^2$

B. $\frac{1}{\pi r^2}$

C. πr^2

D. $\frac{4}{\pi r^2}$

E. $\frac{1}{4\pi r^2}$

Question 22

A particle moves in a straight line so that its position at time t is given by $x=13-4\sin 2t,$ where $t\geq 0.$ The initial velocity of the particle is

A. 5

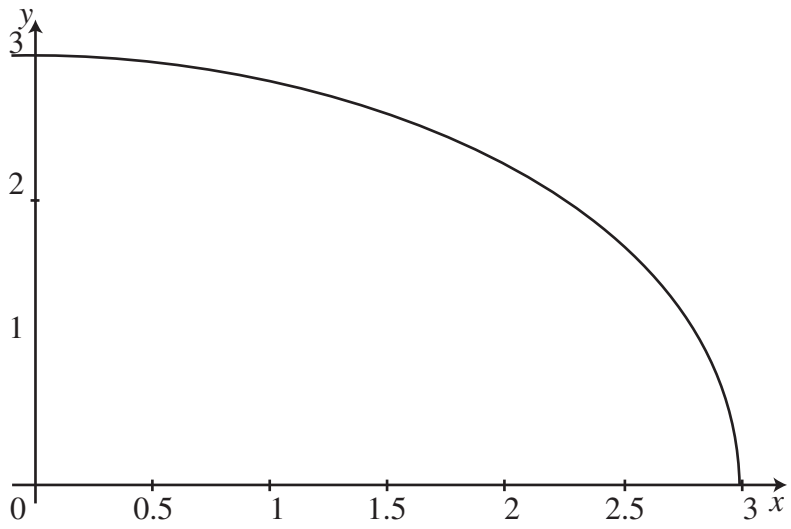
B. 8

C. -8

D. 13

E. 21

Question 23



Part of the graph of $y = \sqrt{9 - x^2}$ is shown above. If the trapezoidal rule with three equal intervals is used to approximate the shaded area, its value, correct to 2 decimal places, would be

A. 19.69

B. 14.50

C. 13.13

D. 6.56

E. 3.70

Question 24

A particle of mass 2 kg is acted on by a resultant force of $(4\underset{\sim}{i} - 3\underset{\sim}{j})$ newton. The magnitude of the particle's acceleration in m/s^2 is

A. 1.3

B. 2

C. 2.5

D. 3.5

E. 10

Question 25

Using the substitution $u = x + 4$, $\int_0^1 2x(x+4)^5 dx$ becomes

A. $\int_4^5 (2u^6 - 4u^5)\text{du}$

B. $\int_0^1 (2u^6 - 4u^5)\text{du}$

C. $\int_0^1 (2u^6 - 8u^5)\text{du}$

D. $\int_4^5 (2u^6 - 8u^5)\text{du}$

E. $\int_4^5 (2u^6 + 8u^5)\text{du}$

Question 26

The region bounded by the coordinate axes and the graph of $y = \sin x$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the y-axis to form a solid of revolution. The volume of the solid is given by

A. $\int_0^1 \pi (Sin^{-1} y)^2 dy$

B. $\int_0^{\frac{\pi}{2}} \pi (Sin^{-1} y)^2 dy$

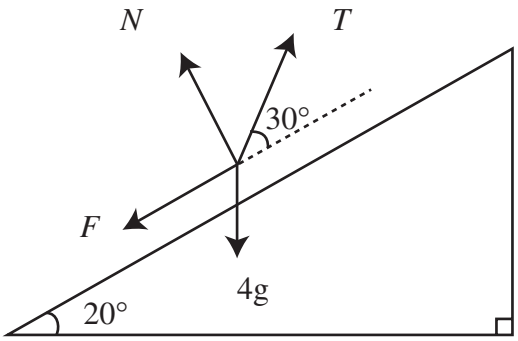
C. $\int_0^1 \pi Sin^{-1} y \text{ dy}$

D. $\int_0^1 \pi \sin^2 x \text{ dx}$

E. $\int_0^{\frac{\pi}{2}} \pi \sin^2 x \text{ dx}$

Question 27

The diagram shows all the forces acting on a block of mass 4 kg, which is being pulled up a rough inclined plane at constant speed. The plane is inclined at 20° to the horizontal and the coefficient of friction between the block and the plane is 0.3. The block is being pulled by a rope which makes an angle of 30° with the plane. T newtons is the tension in the rope, N newtons is the normal reaction of the plane on the block and F newtons is the frictional force which is assumed to be limiting.



Resolving forces parallel to the plane gives

A. $T \cos 30° = 4g \cos 20° - N$

B. $T \cos 30° = 4g \sin 20°$

C. $T \cos 30° = 4g \sin 20° + 2g$

D. $T \cos 30° = 0.3N + 4g \cos 20°$

E. $T \cos 30° = 0.3N + 4g \sin 20°$

Question 28

A body is in equilibrium under the action of three forces $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$, and $\underset{\sim}{F}_3$. If $\underset{\sim}{F}_1 = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\underset{\sim}{F}_2 = 4\underset{\sim}{i} + 2\underset{\sim}{j}$, the magnitude of $\underset{\sim}{F}_3$ to one decimal place is given by

A. 9.5

B. 9.0

C. 7.3

D. 5.0

E. 4.5

Question 29

A particle moves in a straight line, such that its velocity is given by $v = (2x - 3)^2$ cm/s. When x = 3 cm, its acceleration in cm/s^2 is

A. 6

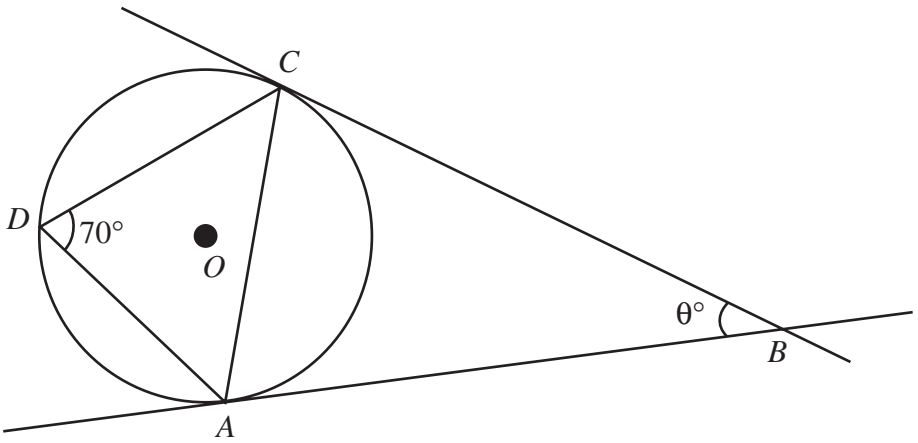
B. 9

C. 12

D. 54

E. 108

Question 30



This diagram shows a circle with centre O. Two tangents of the circle intersect at B and intersect the circle at A and C. The angle $<CDA$ is 70°. The angle $<CBA$ is

A. 110

B. 65

C. 20

D. 40

E. 10

Part II (Short answer questions)

Question 1

An object is dropped from rest at time $t = 0$. A body falling from rest under gravity is subject to air resistance proportional to its speed. Its acceleration is given by the differential equation

$$\frac{dv}{dt} = g - kv,$$

where g is the acceleration due to gravity and k is a constant.

Express v in terms of t.

[3 marks]

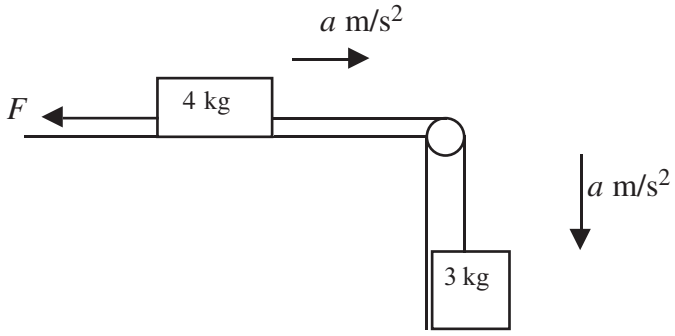
Question 2

The position vector $\underset{\sim}{r}(t)$ of a particle at time t is given by $\underset{\sim}{r}(t) = (e^t \sin t)\underset{\sim}{i} - (e^t \cos t)\underset{\sim}{j}$.

Show that the speed of the particle at any time t is given by $\sqrt{2}e^t$.

[3 marks]

Question 3



A 4 kg mass on a rough horizontal table is connected by a light string that passes over a smooth pulley to a 3 kg mass. The direction of the acceleration is as shown on the diagram above, and the coefficient of friction between the rough table and the 4 kg mass is 0.3.

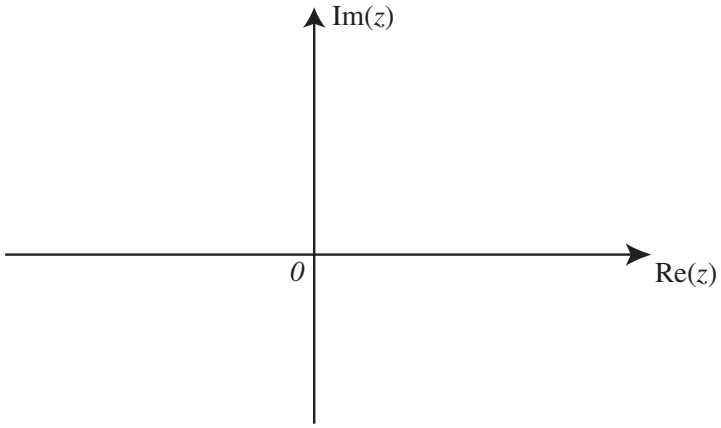
a. Find, to one decimal place, the magnitude of the acceleration of the two masses.

b. Find, to one decimal place, the tension force, T, in the string.

[4 marks]

Question 4

a. On the set of axes below sketch the graphs of $\{z : \text{Re}(z) - \text{Im}(z) = -3\}$ and $\{z : |z + 2 - i| = 3\sqrt{2}\}$



b. Evaluate the points of intersection.

[3 + 2 = 5 marks]

Question 5

Use calculus to find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2(2x)dx$.

[3 marks]

Question 6

Given $f'(x) = 2x\sqrt{x-2}$ and $f(2) = 3$, find $f(x)$.

[3 marks]

END OF PAPER – TOTAL MARKS 51