SET 2 EXAM 2

Writing time: 120 minutes

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			80

Note: Formula Sheet is NOT supplied. You will need to use your own!

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, **one** bound reference, **one** approved cas calculator and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape

Materials supplied

- Question and answer book

Instructions

- Complete all multiple-choice questions by circling your choice on the book.
- Complete all extended-response questions in the spaces provided.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1 - Multiple-choice questions

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question

Question 1

The function $f(x) = \cos(3x)$ undergoes a transformation in the order of a translation of 1 unit in the negative direction of the y-axis, a reflection in the x-axis, and a translation of $\frac{\pi}{2}$ units in the positive direction of the x-axis. Which of the following is the correct result of this transformation?

A. $f(x) = \sin(-3x) + 1$

B. $f(x) = -\cos\left(3x - \frac{\pi}{2}\right) - 1$

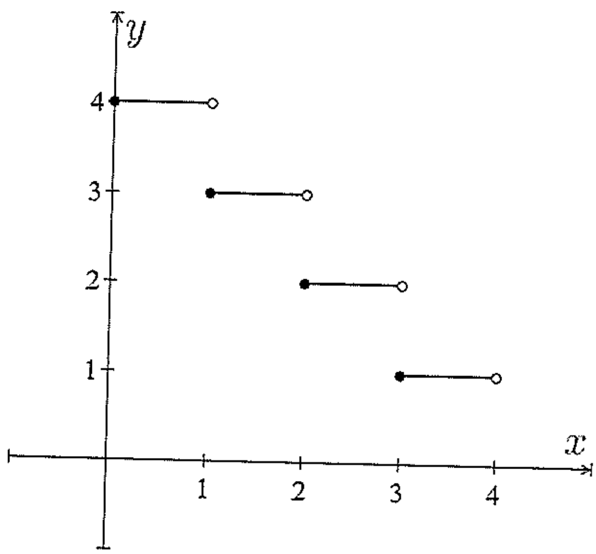
C. $f(x) = \cos\left(-3x + \frac{\pi}{2}\right) + 1$

D. $f(x) = \sin(3x) + 1$

E. $f(x) = \cos(3x) + 1$

Question 2

Consider the graph of a relation below:



Which of the following sets describes when the relation is decreasing?

A. This relation is never decreasing

B. $[0, 4) \setminus \{1, 2, 3\}$

C. $\{1, 2, 3\}$

D. $\{0, 1, 2, 3\}$

E. $[0, 4)$

Question 3

If $f(x) > 0$ for $x \in \mathbb{R}$ and $\int_0^3 f(x)dx = 5$ then $\int_{-2}^{-5} -2f(-(x+2))dx$ is equal to

A. 5

B. −7

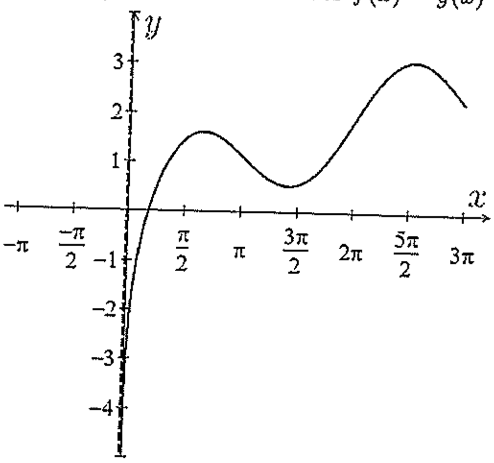
C. 10

D. 7

E. 3

Question 4

The diagram below describes $f(x) = g(x) + h(x)$.



The best fitting functions of $g(x)$ and $h(x)$ are

A. e^x and $\frac{1}{x}$

B. $\log_e(x)$ and $\cos(x)$

C. e^x and x^2

D. $\log_e(x)$ and $\sin(x)$

E. $\log_e(x)$ and x^2

Question 5

The gradient of the function $f(x) = \log_e(kx^2)$ at $x = a$ is

A. $\lim_{h \to 0} \frac{\log_e((ka)^2 + h) - \log_e((ka)^2)}{h}$

B. $\lim_{h \to 0} \frac{\log_e(ka^2 + h^2) - \log_e(ka^2)}{h}$

C. $\lim_{h \to 0} \frac{\log_e(k(a-h)^2) - \log_e(ka^2)}{h}$

D. $\lim_{h \to 0} \frac{\log_e(k(a+h)^2) - \log_e(ka^2)}{h}$

E. $\lim_{h \to 0} \frac{\log_e(k(a+h)^2) + \log_e(ka^2)}{h}$

Question 6

Two curves given by $y = f(x)$ and $y = g(x)$ have tangents $y = 2x - 1$ and $y = x + 1$ at $x = 2$. The largest angle (in degrees) made by the intersection of these curves is closest to

A. 18.43

B. 161.57

C. 122.08

D. 0.33

E. 160.9

Question 7

Consider the function $y = \dfrac{1}{x+3} - 4$. Which of the following statements regarding its inverse is false?

A. It has a horizontal asymptote $y = -3$

B. The point at which it intersects with the function is $(-2.38, -2.38)$

C. When $x = 2$, $y = -\dfrac{7}{2}$

D. It has a vertical asymptote $x = -3$

E. It is a one-to-one function

Question 8

The temperature of coffee just after it has been made is modelled by the function $f(t) = 80e^{-\frac{t}{5}} + 20$ where t is the time in minutes after the coffee is made. What is the average temperature of the coffee over the first 10 minutes?

A. $\dfrac{1}{2}\displaystyle\int_0^{600} f(t)\,dt$

B. $\dfrac{1}{0-10}\displaystyle\int_0^{10} f(t)\,dt$

C. $\dfrac{1}{2}\displaystyle\int_0^{10} f(t)\,dt$

D. $\dfrac{1}{600-0}\displaystyle\int_0^{600} f(t)\,dt$

E. $\dfrac{1}{10-0}\displaystyle\int_0^{10} f(t)\,dt$

Question 9

The discrete random variable X has the following probability distribution,

x	-1	1
$Pr(X = x)$	a	b

Which of the following is correct?

A. $\mu = -a - b$

B. $-a + b = 1$

C. $\text{Var}(x) = a + b - b^2 - 2ab - a^2$

D. $\sigma = \sqrt{a + b - (b - a)^2}$

E. $\text{Var}(x) = a + b - (-a - b)^2$

Question 10

The period, amplitude and range of $f(x) = 3\sin\left(\frac{1}{2}(x-2)\right) - 1$ respectively are

A. $4\pi,\ 3,\ [-2, 4]$

B. $\frac{1}{2},\ 3,\ [-3, 3]$

C. $4\pi + 2,\ 3,\ [-4, 2]$

D. $4\pi,\ 3,\ [-4, 2]$

E. $\frac{1}{2} - 1,\ 2,\ [-4, 2]$

Question 11

Given the following matrix equation, which of the following equations is correct given that there is not a unique solution?

$$\begin{bmatrix} a & 2 \\ 3 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

A. $2a - 3b = 0$

B. $b^2 - 24a = 0$

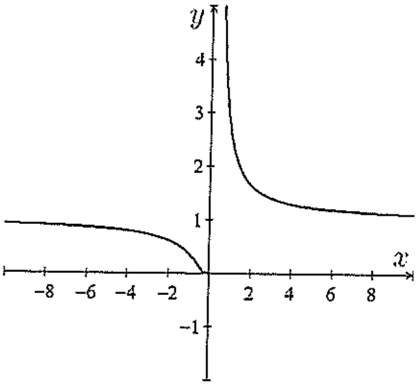
C. $ab - 6 = 0$

D. $2a - 3b = 1$

E. $ab - 6 = 2$

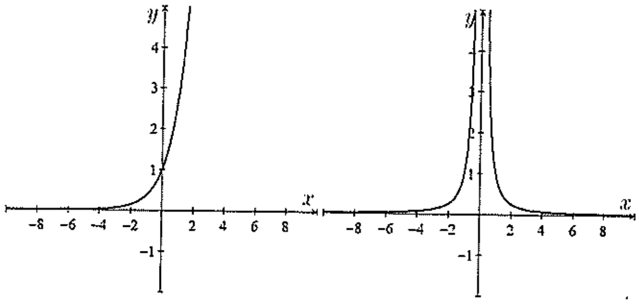
Question 12

The graph of $y = f(g(x))$ is drawn below.

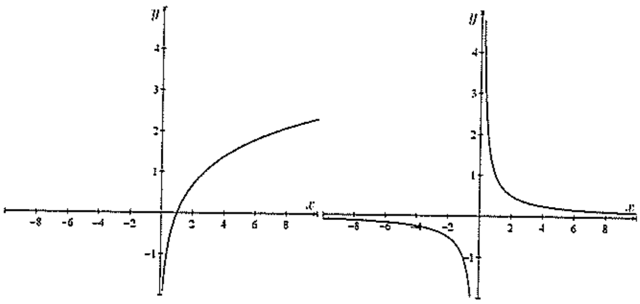


Which of the following pairs of graphs best represent $f(x)$ and $g(x)$ respectively?

A.



B.



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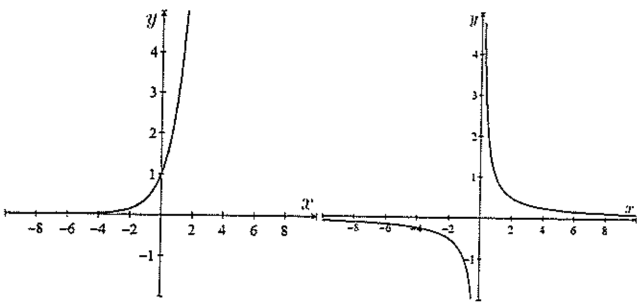
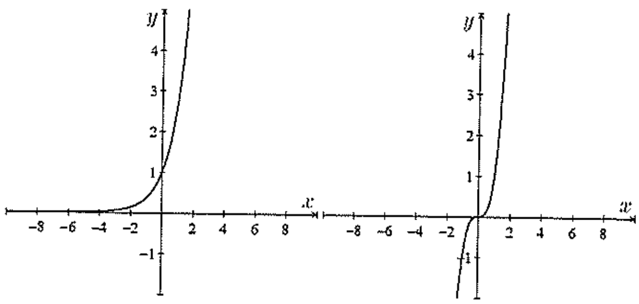
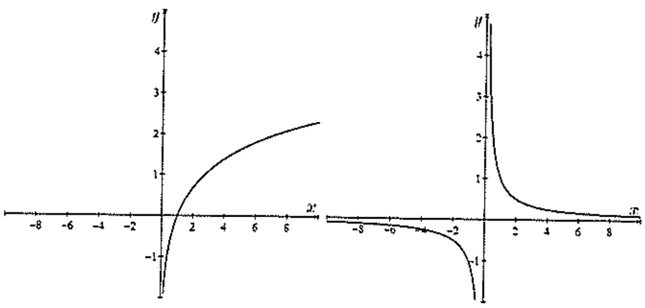


D.



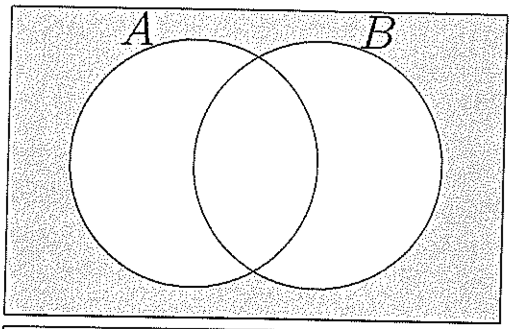
E.



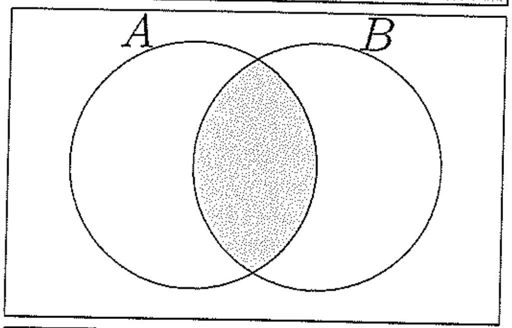
Question 13

Which of the following Venn diagrams best describes the region of $(A' \cap B')'$

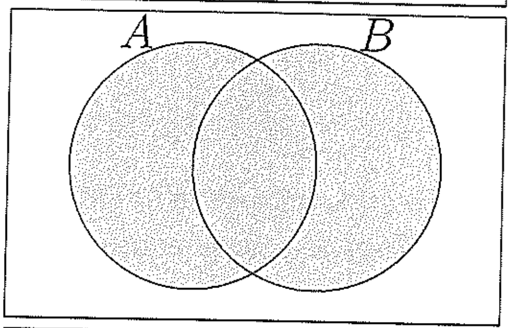
A.



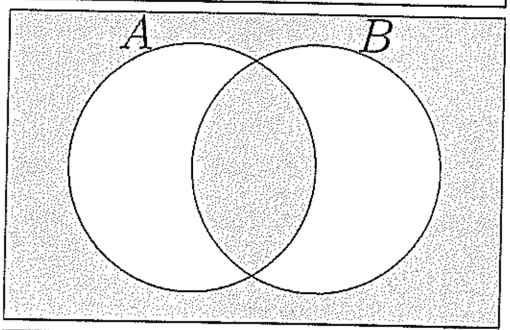
B.



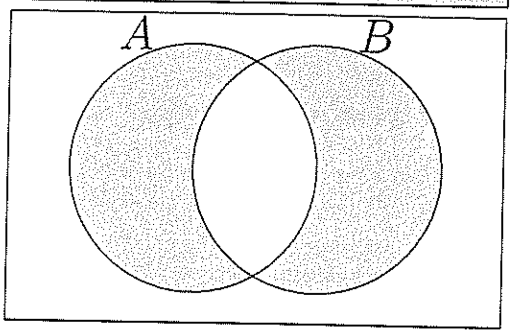
C.



D.



E.



Question 14

Which of the following gives the best left-endpoint approximate area under the graph of $y = x^3$ over the interval $[0, 8]$?

A. $2 \times 0 + 2 \times 8 + 2 \times 64 + 2 \times 216$

B. $0 + 1 + 8 + 27 + 64 + 125 + 216 + 343$

C. $4 \times 0 + 4 \times 64$

D. $1 + 8 + 27 + 64 + 125 + 216 + 343 + 512$

E. $2 \times 0 + 2 \times 1 + 2 \times 8 + 2 \times 27 + 2 \times 64 + 2 \times 125 + 2 \times 216 + 2 \times 343$

Question 15

The following is known about $f(x)$.

- $f'(x) < 0$ for $x < -1$
- $f'(x) = 0$ for $x = -1$
- $f'(x) > 0$ for $-1 < x < 2$
- $f'(x) < 0$ for $x > 2$

Then which of the following must be true

A. There is a local maximum at $x = 2$

B. There is a stationary point of inflection at $x = 2$

C. There is a local minimum at $x = -1$ and a local maximum at $x = 2$

D. There is a stationary point of inflection at $x = -1$ and $x = 2$

E. There is a local minimum at $x = -1$

Question 16

For the function $f(x) = g(x) + h(x)$ where $g(x) = \sqrt{x-3} + 1$ and $h(x) = -\log_e(-x+4)$, the maximum domain of $f(x)$ is

A. (3,4)

B. $(-\infty, \infty)$

C. [3,4)

D. $(-3, 4)$

E. $(-4, 3]$

Question 17

For $y = \sqrt[3]{(x-3)^2}$, the correct expression for the **approximate** change in y, when x changes from a to $a + h$ is

A. $h\dfrac{2}{3(x-3)^{\frac{1}{3}}}$

B. $\sqrt[3]{(a+h-3)^2} - \sqrt[3]{(a-3)^2}$

C. $h\sqrt[3]{(a-3)^2}$

D. $\dfrac{2h}{3(a-3)^{\frac{1}{3}}}$

E. $(a+h)\dfrac{2}{3(a-3)^{\frac{1}{3}}}$

Question 18

A random variable X has probability density function:

$$f(x) = \begin{cases} ke^x, & -20 \le x \le 10 \\ -kx + 10, & 10 < x < 50 \\ 0 & \text{elsewhere} \end{cases}$$

The mean in terms of k is closest to

A. $-41333k + 12000$

B. $2.54 \times 10^{23}k$

C. $-44333k + 10500$

D. $156905k + 12000$

E. $198238k$

Question 19

Thushan is a worker for the science student centre at the University of Competence - whilst looking at a survey of university students' preferences between biomedicine or science, he learns that each year, 30% of students who preferred science continued to prefer science whereas 80% of students who preferred biomedicine continued to prefer biomedicine. In the long term, what percentage of students prefer biomedicine over science?

A. 22%

B. 78%

C. 80%

D. 77%

E. 53%

Question 20

A binomial probability distribution, given by $X \sim \text{Bi}(n, p)$, has a mean of μ and a variance of $\frac{\mu^2}{3}$.

The values of n and p are

A. $n = -\frac{3\mu}{\mu - 3}, p = \frac{-\mu - 3}{3}$

B. $n = -3 + \frac{9}{3 - \mu}, p = 1 - \frac{\mu}{3}$

C. $n = \frac{\mu(3\sqrt{3} + 3)}{2}, p = -\frac{(\sqrt{3} - 3)}{3}$

D. $n = \frac{3\mu}{\mu - 3}, p = -\frac{(\mu - 3)}{3}$

E. $n = \frac{9}{\mu - 3} - 3, p = -\frac{(\mu - 3)}{3}$

Question 21

$f(x)$ exists such that it follows the rule $f(xy) = f(x) + f(y)$. Which of the following functions is a possible rule for $f(x)$?

A. $f(x) = \log_e\left(\frac{1}{e^x}\right)$

B. $f(x) = \log_e(x^2)$

C. $f(x) = \frac{1}{e^x}$

D. $f(x) = \sin(2x)$

E. $f(x) = x^2$

Question 22

A continuous variable X is normally distributed with a mean of μ and variance σ^2. $\Pr\left(X > \mu + \frac{3\sigma}{2}\right)$ is equal to

A. $\Pr(Z > 2.5)$

B. $1 - \Pr(Z > -1.5)$

C. $1 - \Pr(Z < 2.5)$

D. $\Pr(Z < 1.5)$

E. $\Pr(Z > -1.5)$

SECTION 2 - Extended response questions

Instructions for Section 2
Answer **all** questions in the spaces provided
In all questions where a numerical answer is required an exact value must be given unless otherwise specified. is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Tales have indicated the existence of a mystical ancient volcano that has not erupted in thousands of years somewhere in the great midst of Austria. Mr. Williams has travelled very far East in search of this great volcano and after many years of running, he has at last come across the volcano that fits the tales!

He wishes to describe this volcano to his friend Marty, but Marty will only understand if it is described in the form of a function. The volcano is perfectly symmetrical along a vertical axis with the shape of a truncus in the form $y = \frac{a}{x^2} + c$. He measures the top of the volcano (the crater) to have a diameter of 700 m and to be 1.3 km above the ground. The volcano meets the ground 1.5 km away from its centre. x and y have the units of kilometres.

a. What is the function that Mr. Williams must give to Marty in order to describe the volcano correctly?

Specify the domain between the points that the volcano meets the ground. Write all numbers to 3 decimal places.

4 marks

Mr. Williams decides to inspect the inside (in a heat proof suit) of the supposedly dormant volcano but whilst inside, he starts to feel a rumble. He is perched on a ledge he made 5 m from the top of the volcano. He notices that lava is beginning to rise from the floor of the volcano, $y = 0$. The rate at which the radius of the lava decreases is 1 m/s.

b. **i.** What is the rate of change in depth of the lava with respect to time, in terms of x?

ii. Find the average rate of change in lava depth between the floor and his ledge.

iii. Mr. Williams climbs out of the volcano at a vertical rate of 5 cm/s. If he moves after immediately seeing the lava rise from the floor, will he make it to the top of the volcano to be airlifted by a helicopter before the lava engulfs him?

$4 + 3 + 5 = 12$ marks

Question 2

Consider the function $f(x) = (x-1)(x^2-k)$.

a. Find the rule of $g(x)$ if $g(x) = -f(2x-1)+3$. Give your answer in the form ax^3+bx^2+cx+d.

2 marks

b. i. Find the tangent to $f(x)$ when $x=1$ in terms of k.

ii. Find the normal to $g(x)$ when $x=3$ in terms of k.

$3+3=6$ marks

c. Hence or otherwise, find the values for k the two lines found in **part b.** have no intersection. Justify the fact that your values of k do not result in infinite intersections.

4 marks

Question 3

Clare is the owner of Zangolooba, a store that sells high quality watches and wallets. This store is in the town of Blarghsburg which consists of 210 residents and at the start of every month, they throw away either their watch or wallet and purchase a new one at Zangolooba.

Clare carefully monitors the sales every day to determine how much stock she needs to buy in the future. She will always maintain 210 items of either watches or wallets. She finds that in a given month, 90% of the people who bought a watch will buy a watch the next month, and 80% of those who a bought wallet will buy a wallet the next month. In January of 2013, Clare sells 60 wallets and 150 watches.

a. i. How many watches and wallets will Clare need to stock for in April?

ii. If all the wallets cost \$40 and all the watches cost \$80, how much money will Clare make in April from the sales?

$4 + 2 = 6$ marks

b. Calculate the amount of money Clare will make per month in the long run.

3 marks

Clare prides herself on the quality of the watches she sells, and thus keeps track of the error in seconds accumulated over a month for her watches. She will have to fix watches that are more than 15 seconds too fast or too slow. Clare finds that the monthly error is normally distributed and that 2% of watches must be fixed for being too fast and another 3% of watches must be fixed for being too slow.

c. What is the average error accumulated in her watches over a month? Give your answer to two decimal places.

4 marks

Question 4

William has large precious marbles that he would like to hide. He decides to dig out a hole with his own shovel and once he has completed his digging he will drop his marbles into the hole.

He sets up his digging site by placing three sticks in the ground at points, A, B, and C with coordinates $(1,3)$, $(3,5)$, $(5,2)$ respectively. This forms a triangle in the ground that he will dig beneath.

a. i. Find the distance AC.

ii. Find the normal to AC which goes through the point B.

iii. Hence, or otherwise, find the area of the triangle, $\triangle ABC$.

$2+3+4=9$ marks

William digs the hole to be 3 m deep. He now begins to drop marbles from the very top of the hole. He drops an initial marble to test the speed at which it reaches the bottom.

b. **i.** The velocity of the drop is given by $v = -2e^t + b$ where t is the time in seconds after the marble has been dropped. Find the value of b.

ii. Define a function $x(t)$ for $t > 0$, that describes the displacement of the marble after t seconds.

iii. At what velocity does this initial marble hit the bottom of the hole?

$2 + 3 + 3 = 8$ marks

END OF EXAMINATION