SET 1 EXAM 1

Reading time: 15 minutes

Writing time: 60 minutes

Structure of examination

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

Note: Formula Sheet is NOT supplied. You will need to use your own!

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book
- Working space is provided throughout the book

Instructions

- Complete all responses in the spaces provided
- All responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $2\cos(2x+3)$ with respect to x.

1 mark

b. If $f(x) = 3x^3(\log_e(2x) - 1)$, find $f'(2)$.

2 marks

c. Using the result from **part b**, find an antiderivative of $x^2 \log_e(2x)$.

2 marks

Question 2

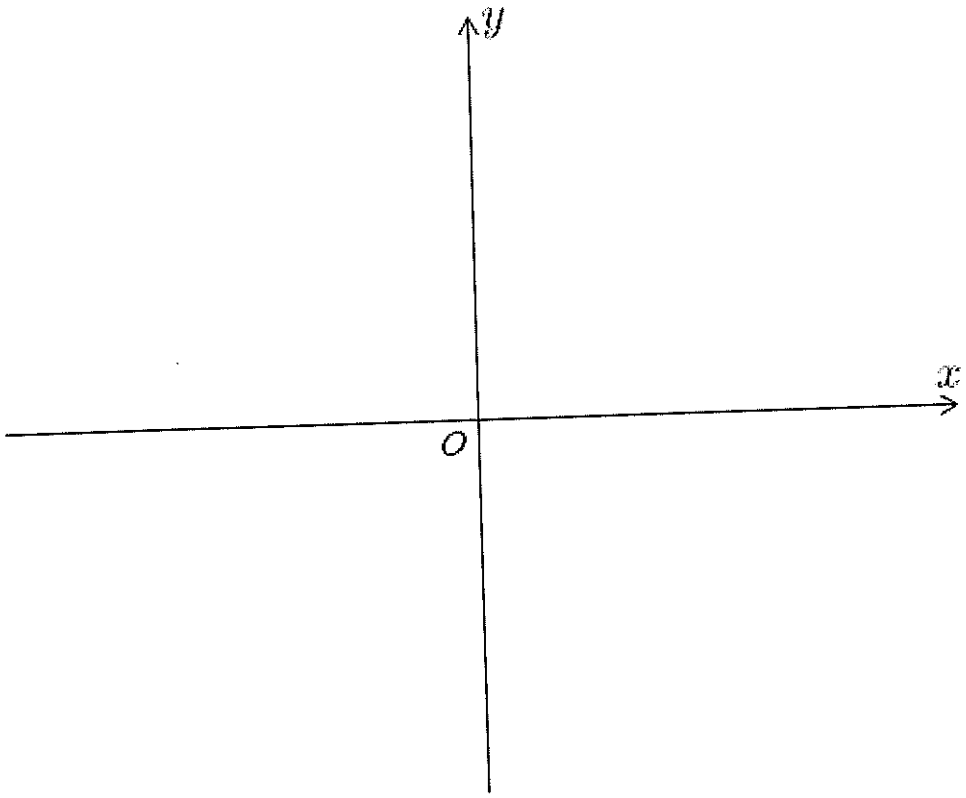
a. Find an antiderivative of $\frac{1}{2x-1}+4$ with respect to x.

1 mark

b. Find the values of a and b if $\int_1^3 \left(\frac{1}{2x-1}+4\right) dx = \log_e(a)+b$ where $a, b \in \mathbb{R}$.

2 marks

c. Sketch the graph of $y = \dfrac{1}{2x-1} + 4$, showing all axial intercepts and asymptotes.



3 marks

Question 3

Solve the equation $2 \times 3^{2x} - 48 \times 3^{x} = 162$ for x.

3 marks

Question 4

Find the inverse of $f : \left(-1, \frac{4}{3}\log_e(2)\right] \to \mathbb{R}, f(x) = 2e^{-3x} - 2$, stating the domain and rule for the inverse function.

3 marks

Question 5

a. State the range and the period of the function $f : \mathbb{R} \to \mathbb{R}, f(x) = 5\cos\left(\frac{\pi}{2}(x-4)\right) + 2$.

2 marks

b. Solve the equation $2\sin\left(2x - \frac{\pi}{2}\right) + 3 = 2$ for $x \in [0, 2\pi]$.

2 marks

Question 6

A probability density function for the continuous random variable X is given by the following function

$$f(x) = \begin{cases} k|x(x-1)(x-3)| & 0 \leq x \leq 3 \\ 0 & x \in \mathbb{R} \setminus [0,3] \end{cases}$$

a. Find the value of k.

2 marks

b. Find $\Pr(X \leq 2)$.

2 marks

c. Find $\Pr(X \geq 2 | X \geq 1)$.

3 marks

Question 7

Consider the set of simultaneous linear equations

$$\begin{aligned} mx + y &= m - 2 \\ 6x + (m-1)y &= 12 \end{aligned}$$

Find the value(s) of m for which the set of linear equations has no solution.

3 marks

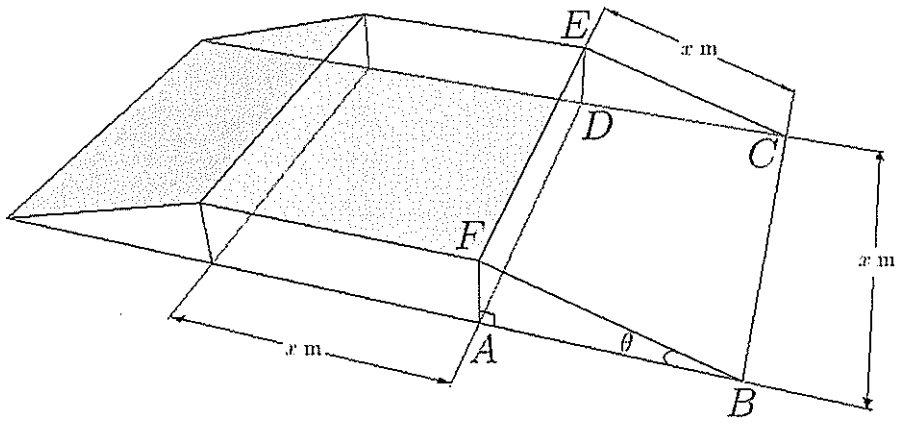
Question 8

Tim's car is leaking oil while he has it parked in his driveway. The oil takes the shape of a thin cylinder, that is, a circle of thickness 3 mm that expands outwards in all directions. If the rate that the oil is leaking out of the car is 5π mm^3/s, find the rate that the radius is increasing when the radius is 10 mm.

2 marks

Question 9

The local council is planning on building a small skate ramp, which will be made from three concrete blocks as shown below. One is a rectangular prism, and the other two are identical triangular prisms. x is a constant length.



a. Find AB and AF in terms of x and θ.

2 marks

b. Find an expression for the volume of the whole structure, V, in terms of x and θ.

1 mark

c. Find $\frac{dV}{d\theta}$ and the value of θ (in radians) for which $\frac{dV}{d\theta} = 0$.

3 marks

d. Find the maximum volume of the structure if $x = 5$ m.

1 mark

END OF EXAMINATION