FUNCTIONS AND GRAPHS

TECH-FREE TEST 1

Writing time: 30 minutes

Structure of test

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	20

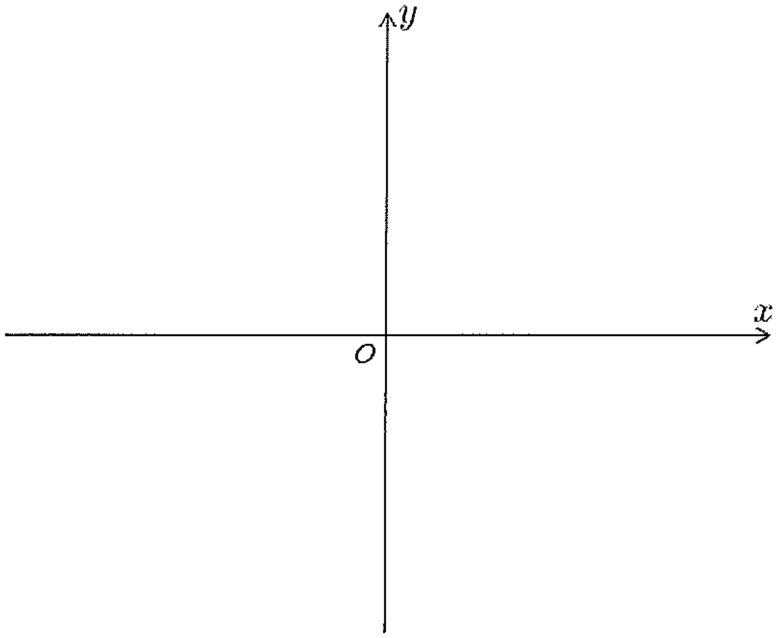
Question 1

Let f be a function defined by $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 6x^2 - 7x + 1$.

a. Find the values of x that satisfy $f(x) \geq 2$.

3 marks

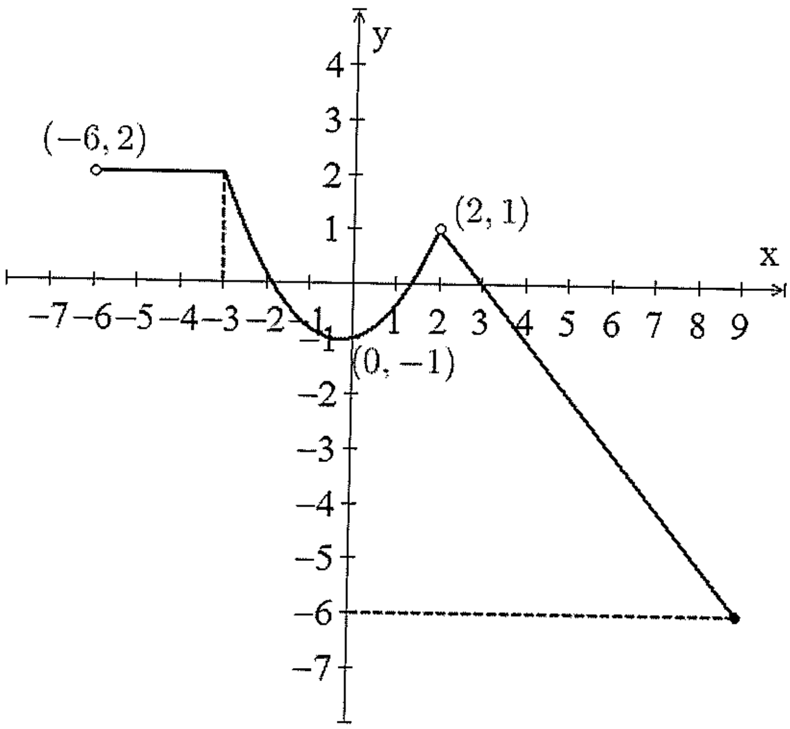
b. Using the result obtained in **part a**, sketch the graph of $y = f(x)$ and hence indicate when $2 \leq y \leq f(x)$, on the axes below. Label all relevant points with their coordinates.



4 marks

Question 2

Define the relation below as a hybrid function, clearly stating domains as appropriate. Note: All relations are in the form $y = mx + d$ or $y = ax^2 + bx + c$



7 marks

Question 3

Find the period of the function, $f(x) = 2\cos\left(\frac{x}{2} + \pi\right) + \sin\left(\frac{2x}{3}\right)$.

3 marks

Question 4

The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$.

The image of the curve $y = 3x^2 + 1$ under the transformation, T, has equation $y = ax^2 + bx + c$. Find the values of a, b, and c.

3 marks

END OF TEST

FUNCTIONS AND GRAPHS

TECH-FREE TEST 2

Writing time: 30 minutes

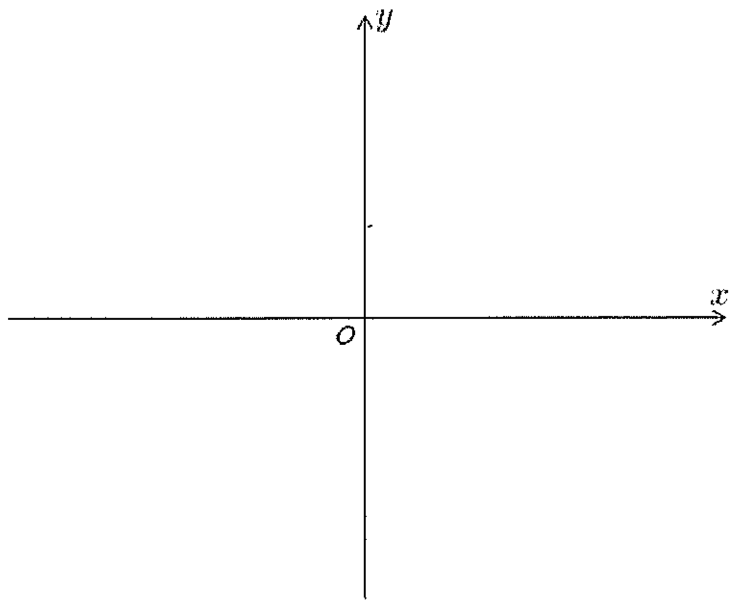
Structure of test

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	20

Question 1

a. On the axes below, sketch the graph of $f : [-\pi, \pi) \to \mathbb{R}$, $f(x) = 2\cos\left(\frac{3}{2}x - \frac{\pi}{4}\right)$.

Label all axes intercepts and endpoints.



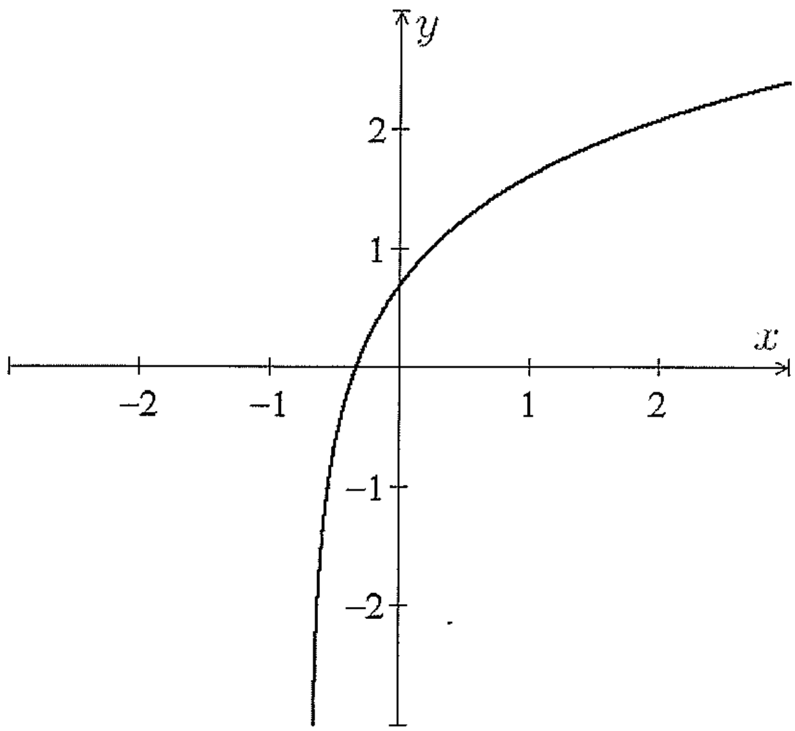
3 marks

b. State the transformation that must occcur for $f(x)$ to have a period of 24.

1 mark

Question 2

Consider the function, f, drawn below where $f(x) = \log_e(3x + 2)$.
Write the rule for $f(|x|)$ as a hybrid function and sketch it on the axes given.



3 marks

Question 3

The image of a curve, $y = \frac{3}{x^2} + 1$, is mapped to another curve, $y = \frac{1}{\sqrt{x+1}} + 2$.

Note: The inverse of $y = \frac{1}{x^2}$ is $y = \frac{1}{\sqrt{x}}$.

a. Specify a series of transformations that describes this mapping.

4 marks

b. State the largest possible domain of the function $f(x) = \frac{3}{x^2} + 1$ such that the inverse function exists.

1 marks

Question 4

A base jumper steps off a building and falls with a speed of $v = ae^{-\frac{3t}{4}} + b$ m/s, where t is the time in seconds after stepping off the building. After 10 seconds, his speed is 108 km/hr. In order to survive the fall, he must land with a speed of less than 40 m/s.

a. Find the exact values of a and b.

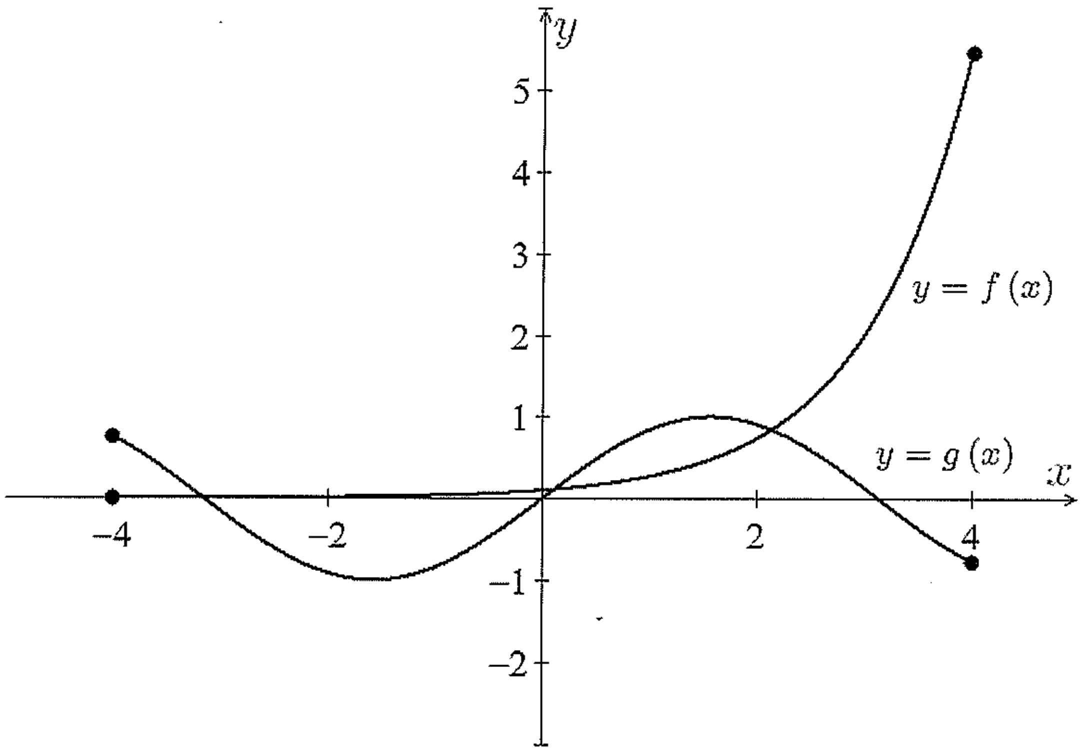
3 marks

b. Calculate the time by which the jumper must land in order to survive.

2 marks

Question 5

a. The graphs of $f(x)$ and $g(x)$ are depicted in the diagram below. Sketch the function of $h(x) = f(x) + g(x)$ for the given domain.



2 marks

b. If $g(x)$ is translated 2 units to the left, state the domain of $h(x)$.

1 marks

END OF TEST

FUNCTIONS AND GRAPHS
TECH-ACTIVE TEST 1

Writing time: 60 minutes

Structure of test

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	11	11	11
2	2	2	29
			40

SECTION 1 - Multiple-Choice Questions

Question 1

The function with rule $f(x) = 7\tan\left(2\left(\frac{2x}{3} - \frac{\pi}{5}\right)\right) + 1$ has period

A. $\frac{2\pi}{3}$

B. $\frac{3\pi}{4}$

C. $\frac{4\pi}{3}$

D. $\frac{3\pi}{2}$

E. 3π

Question 2

Consider the function $w(x) = 3\log_e(2x)$. What is the pré-image of $5a$?

A. $3\log_e(10a)$

B. $3\log_e(5a)$

C. $\frac{1}{2}e^{\frac{5a}{3}}$

D. $e^{\frac{5a}{6}}$

E. $\frac{5a}{6}$

Question 3

Which of the following properties is always true for the function $f(x) = \log_e(x)$?

A. $2f(x) = f(x^2)$

B. $f(x+y) = f(x)f(y)$

C. $f(xy) = f(x) + f(y)$

D. $f(xy) = f(x)f(y)$

E. $f(3x) = f(3) + f(x)$

Question 4

The range of the function $f : [0, 6) \setminus \{2\} \to \mathbb{R}$, $f(x) = \dfrac{8}{(x-2)^2} + 3$ is

A. $(3, \infty)$

B. $[5, \infty)$

C. $(3.5, 5]$

D. $(3.5, \infty)$

E. $[3.5, \infty)$

Question 5

Consider the following functions:

- $f(x) = 2x^6 + 5$
- $g(x) = \cos\left(\dfrac{1}{2}x\right)$
- $h(x) = (x-1)^2$
- $j(x) = |x|^3$
- $k(x) = 3\tan(x)$

Which of these are **even** functions?

A. $k(x)$ only

B. $f(x)$ and $h(x)$ only

C. $f(x)$ and $g(x)$ only

D. $f(x)$, $g(x)$ and $j(x)$ only

E. $f(x)$, $g(x)$, $h(x)$ and $j(x)$ only

Question 6

The following transformations are applied to the graph of $y = \sin(x)$ in the given order:

- Dilation by factor of $\dfrac{1}{2}$ from the x-axis
- Translation of 5 units upwards
- Translation of 1 unit to the left
- Dilation by factor of 3 parallel to the x-axis
- The graph is reflected in the y-axis

Which of the following is the equation of the image?

A. $y = \dfrac{1}{2}\sin\left(\dfrac{1}{3}(1-x)\right) + 5$

B. $y = -\dfrac{1}{2}\sin\left(\dfrac{1}{3}(x-3)\right) + 5$

C. $y = -\dfrac{1}{2}\sin\left(\dfrac{1}{3}(x-3)\right) + 10$

D. $y = \dfrac{1}{2}\sin\left(\dfrac{1}{3}(1-x)\right) + 5$

E. $y = -\dfrac{1}{2}\sin\left(\dfrac{1}{3}(x-1)\right) + 5$

Question 7

Consider the following function

$$h : (-2, 5] \to \mathbb{R},\ h(x) = -2\sqrt{4 + x} + 3$$

Which of the following gives the domain and range, respectively, of $y = h^{-1}(x)$?

A. $[-4, \infty)$ and $(-\infty, 3]$

B. $(-\infty, 3]$ and $[-4, \infty)$

C. $(-3, 3 - 2\sqrt{2}]$ and $(-2, 5]$

D. $(-2, 5]$ and $[-3, 3 - 2\sqrt{2})$

E. $[-3, 3 - 2\sqrt{2})$ and $(-2, 5]$

Question 8

The curve of $y = \dfrac{3}{x}$ is subject to a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with rule $T(\mathbf{X}) = \mathbf{T}(\mathbf{X} + \mathbf{B})$,

where $\mathbf{T} = \begin{bmatrix} 0 & -2 \\ -\frac{1}{3} & 0 \end{bmatrix}$, $\mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $\mathbf{B} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

What is the equation of image of the curve after this transformation?

A. $y = \dfrac{2}{x+4} - \dfrac{1}{3}$

B. $y = \dfrac{3}{x+2} - 2$

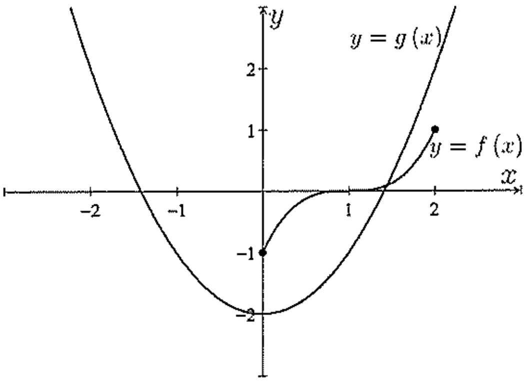
C. $y = \dfrac{9}{2(x+1)} - 2$

D. $y = \dfrac{2}{x-1} + 2$

E. $y = \dfrac{2}{x+4} - \dfrac{1}{2}$

Question 9

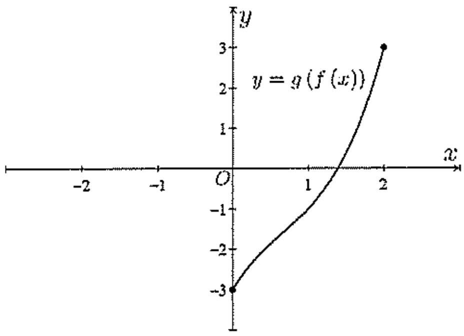
The graphs of $y = f(x)$ and $y = g(x)$ are shown below:



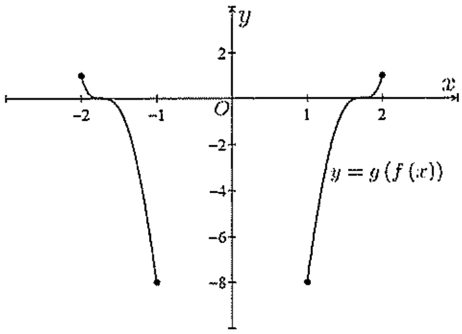
All of the axes below have the same scale as the axes in the diagram above.

Which of the following best represents the graph of $y = g \circ f(x)$?

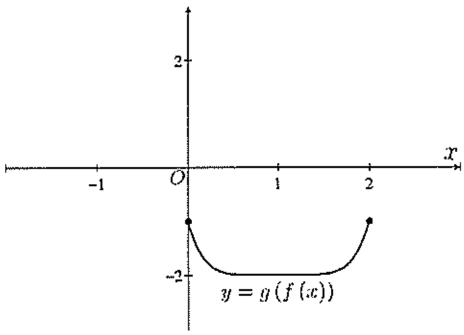
A.



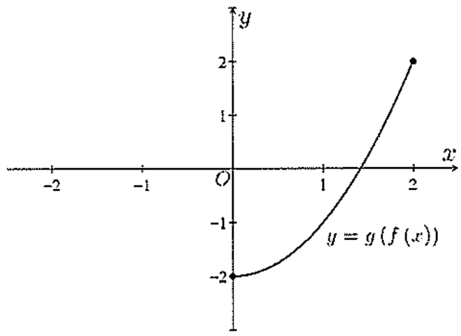
B.



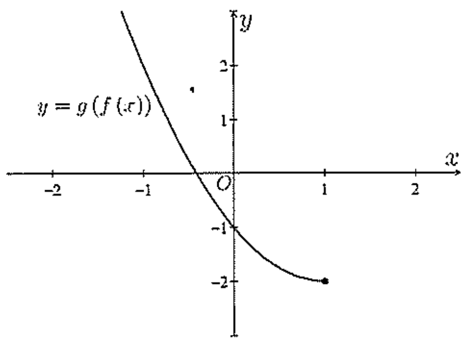
C.



D.

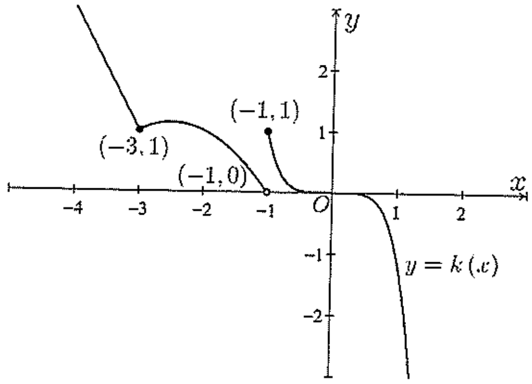


E.



Question 10

The following graph is of the hybrid function $y = k(x)$.



Which of the following is a possible equation for $y = k(x)$?

A. $y = \begin{cases} -x-2, & x < -3 \\ -(x+2)^2+1, & -3 \le x < -1 \\ -x^3, & x \ge -1 \end{cases}$

B. $y = \begin{cases} -2x-5, & x < -3 \\ -\frac{1}{2}\left(x+\frac{5}{2}\right)^2+\frac{9}{8}, & -3 \le x < -1 \\ -x^5, & x \ge -1 \end{cases}$

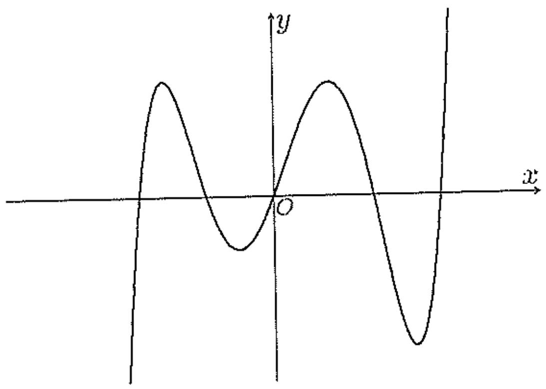
C. $y = \begin{cases} -x-2, & x \le -3 \\ -(x+2)^2+1, & -3 < x < -1 \\ x^3, & x \ge -1 \end{cases}$

D. $y = \begin{cases} -2x-5, & x < -3 \\ -\frac{1}{2}\left(x+\frac{5}{2}\right)^2+\frac{9}{8}, & -3 < x < -1 \\ -x^3, & x \ge -1 \end{cases}$

E. $y = \begin{cases} -x-2, & x \le -3 \\ -\frac{1}{2}\left(x+\frac{5}{2}\right)^2+\frac{9}{8}, & -3 < x \le -1 \\ -x^3, & x > -1 \end{cases}$

Question 11

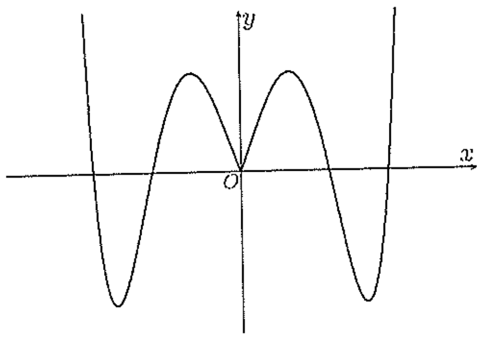
The graph of $y = h(x)$ is shown below:



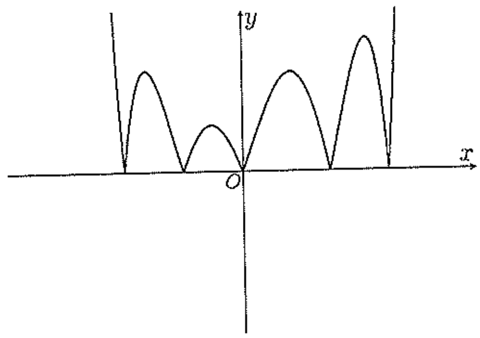
All of the axes below have the same scale as the axes in the diagram above.

Which of the following best represents the graph of $y = h(|x|)$?

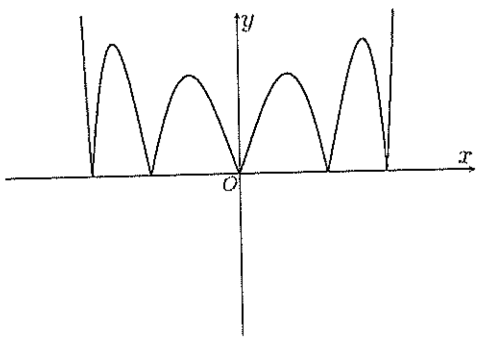
A.



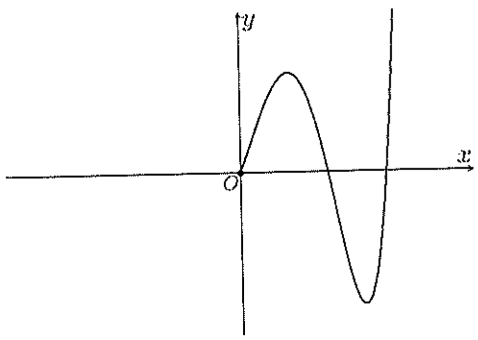
B.



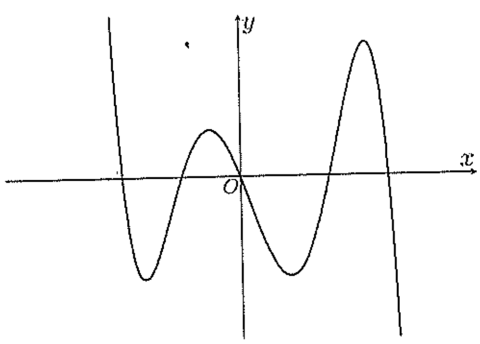
C.



D.



E.



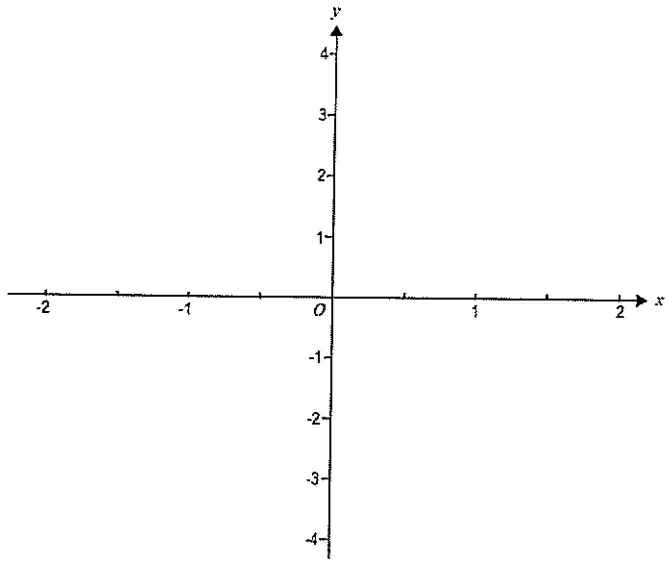
SECTION 2 - Extended-Response Questions

Question 1

Consider the following function

$$f : \left[-\frac{3}{2}, \frac{3}{2}\right) \to \mathbb{R},\ f(x) = -2\cos\left(\frac{\pi}{3}(2x-1)\right) + 1$$

a. **i.** On the axes below, sketch the graph of $y = f(x)$. Label all interecepts and endpoints with their coordinates, and state the period and amplitude in the lines provided below.



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ii. Now consider the function defined by

$$g : (-1, \infty) \to \mathbb{R},\ g(x) = -\frac{4}{x+1} + 2$$

On the set of axes given for **part a.** above, sketch the graph of $y = g(x)$, labelling any intercepts with their coördinates and asymptotes with their equations.

iii. State the point(s) of intersection of $y = f(x)$ and $y = g(x)$, giving your answer(s) correct to **three** decimal places where appropriate.

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iv. Consider now the function defined by $y = (f+g)(x)$.
State the implied domain of $y = (f+g)(x)$.

v. Sketch the graph of $y = (f+g)(x)$ on the axes used above in **part a.** over its implied domain.

$4+3+1+1+2 = 11$ marks

b. A series of transformations are now applied to the graph of $y = f(x)$, and the function $y = h(x)$ is obtained, where

$$h : \left[\frac{2\pi}{3}, \frac{8\pi}{3}\right) \to \mathbb{R},\ h(x) = \cos(x)$$

i. Find a possible series of transformations to obtain $h(x)$ from $f(x)$, and describe it in the correct order.

ii. This series of transformations can also be described in matrix form by the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, which has its rule given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \mathbf{T}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \mathbf{B}\right)$$

Write down the matrices $\mathbf{T}$ and $\mathbf{B}$.

$4+1 = 5$ marks

Question 2

Corey is going on a rollercoaster ride that lasts a total of 20 seconds. His height above ground level, h metres, t seconds after the ride starts, is given by

$$h(t) = \begin{cases} t\sin(t) + 5, & 0 \le t \le a \\ f(t-11), & a < t < b \\ |g(t)| - \frac{1}{2}, & b \le t \le 20 \end{cases}$$

Where $f(t)$ and $g(t)$ are functions of t, and a and b are real constants, with $a < 10$.

Note that when $h < 0$, the riders are submerged underwater - that is, the ride contains a plunge into a body of water.

a. **i.** The function $y = f(t)$ obeys the identity

$$[f(t)]^2 = f(2t) + 2$$

Show that $f(t) = e^t + e^{-t}$ is a possible equation for the function $y = f(t)$.

ii. The function $y = g(t)$ is determined to be of the form

$$g(t) = ke^{mt} + n$$

The graph of $y = g(t)$ has an asymptote given by $y = -1$, and it passes through the points $(16, 0)$ and $(10, e^6 - 1)$. Find the values of k, m and n.

$1 + 3 = 4$ marks

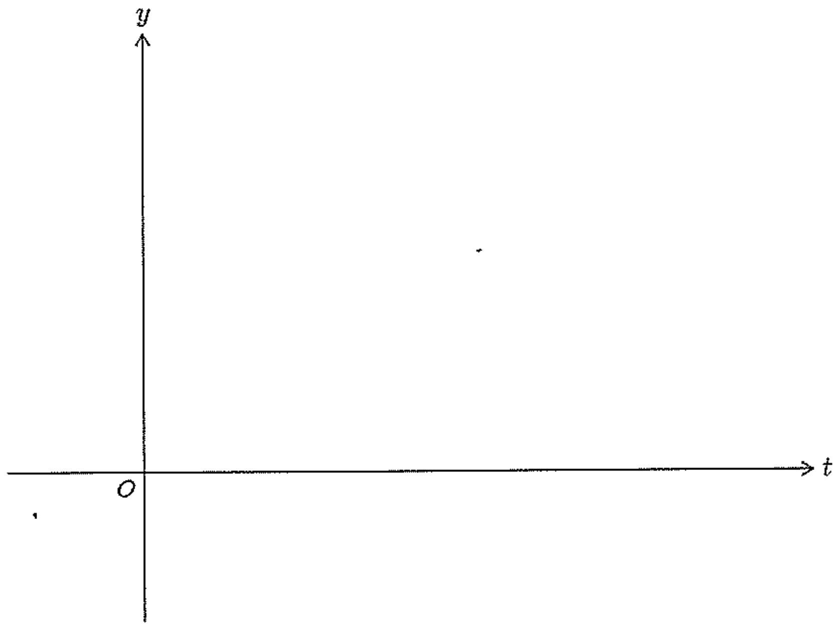
For the remainder of this question, use the possible equation for $f(t)$ suggested in **a.i.** and the equation for $g(t)$ determined in **a.ii.**

b. **i.** Determine the values of a and b, correct to **two** decimal places.

ii. Hence state the rule of the hybrid $y = h(t)$, in simplified form, and in terms of t **only**.

$2 + 1 = 3$ marks

c. On the axes provided below, sketch the graph of $y = h(t)$. Label all intercepts and endpoints, as well as the points where $t = a$ and $t = b$, with coordinates correct to two decimal places. Do **not** attempt to find the turning points.



3 marks

d. For how long will Corey be underwater?

3 marks

END OF TEST

FUNCTIONS AND GRAPHS

TECH-ACTIVE TEST 2

Writing time: 60 minutes

Structure of test

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	11	11	11
2	2	2	29
			40

SECTION 1 - Multiple-Choice Questions

Question 1

Which of the following functions would have a defined inverse function?

- $f : [0, \infty) \to \mathbb{R},\ f(x) = (x-1)^2$
- $g : \mathbb{R} \setminus \{5\} \to \mathbb{R},\ g(x) = \dfrac{1}{(x-5)^2} + 2$
- $h : \mathbb{R}^+ \to \mathbb{R},\ h(x) = 3x^4 - x^3 + 2x^5 + 3x - 7$
- $j : \mathbb{R} \setminus \{0\} \to \mathbb{R},\ j(x) = \log_e(|x|)$
- $k : (1, 5) \to \mathbb{R},\ k(x) = 2\tan\left(\frac{\pi}{4}(x-3)\right) + 1$

A. $k(x)$ only

B. $f(x)$ and $k(x)$ only

C. $f(x)$ and $j(x)$ only

D. $h(x)$ and $k(x)$ only

E. $g(x)$ and $h(x)$ only

Question 2

Consider the function $y = g(x)$, where

$$g : \mathbb{R} \to \mathbb{R},\ g(x) = \frac{1}{\sqrt{1+x^2}} + \sqrt{1+x^2}$$

If the function $y = f(x)$ is defined such that $g(x)$ obeys the identity $[g(x)]^2 = g(f(x)) + 2$, which of the following could be the function $y = f(x)$?

A. $f(x) = \sqrt{x}$

B. $f(x) = \sqrt{x^2 + 2x + 1}$

C. $f(x) = x\sqrt{x^2 + 2}$

D. $f(x) = x^4 + 2x^2$

E. $f(x) = \left(x^2 + 1\right)^2$

Question 3

The following is a list of possible transformations:

I. Dilation by factor of 4 from the x-axis

II. Dilation by factor of 3 from the y-axis

III. Dilation by factor of $\frac{1}{3}$ from the y-axis

IV. Translation of 5 units in the positive direction of the y-axis

V. Reflection in the y-axis

VI. Reflection in the x-axis

VII. Translation of 2 units in the positive direction of the x-axis

VIII. Translation of 2 units in the negative direction of the x-axis

IX. Translation of 6 units in the positive direction of the x-axis

Which of the following sequences of the above transformations could be applied (in the given order) to the graph of $y = \sqrt{x}$ to obtain $y = 4\sqrt{3(2-x)} + 5$?

A. V - VIII - III - I - IV

B. I - V - IV - IX - III

C. V - VII - II - I - IV

D. III - V - VII - IV - I

E. I - IV - VI - IX - III

Question 4

A circle is drawn so that it has a radius of 5 and goes through the points $(5, 7)$ and $(6, 10)$. Which of the following could be the equation of this relation?

A. $(x-4)^2 + (y-9)^2 = 5$

B. $(x-1)^2 + (y-10)^2 = 5$

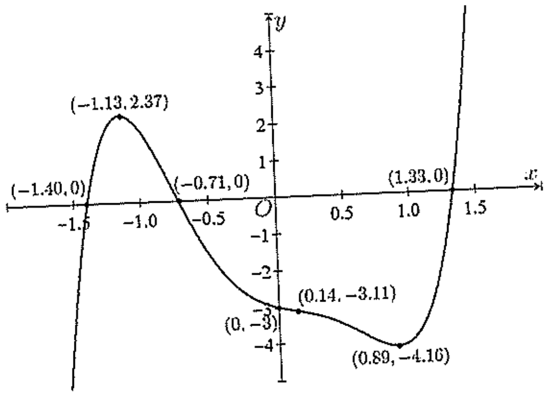
C. $(x-10)^2 + (y-7)^2 = 25$

D. $(x-7)^2 + (y-8)^2 = 5$

E. $(x-1)^2 + (y-7)^2 = 25$

Question 5

The following graph has its rule given by $y = f(x)$.



For what values of a will the equation $f(x) + a = 0$ have three solutions?

A. $-4.16 < a < 2.37$

B. $-1.40 < a < 1.33$

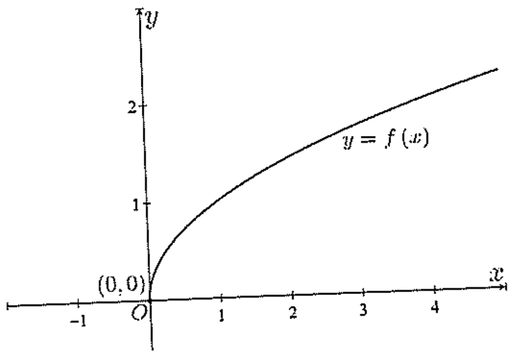
C. $-1.13 < a < 0.89$

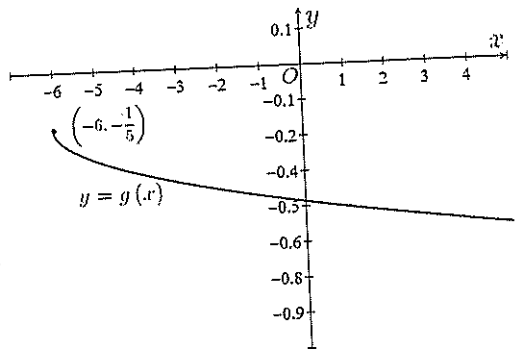
D. $-1.33 < a < 1.40$

E. $-2.37 < a < 4.16$

Question 6

Consider the following graphs of $y = f(x)$ and $y = g(x)$.





A series of transformations, T, is applied to the graph of $y = f(x)$ to obtain the graph of $y = g(x)$. Which of the following is a possible matrix equation for T?

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -\frac{1}{5} \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right)$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -6 \\ -\frac{1}{5} \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -\frac{1}{5} \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -6 \\ -\frac{1}{5} \end{bmatrix}\right)$

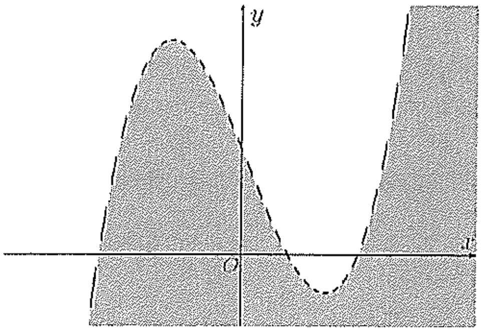
D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -\frac{1}{5} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -6 \\ -\frac{1}{5} \end{bmatrix}$

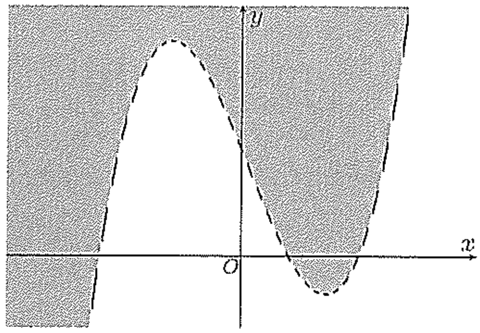
Question 7

Which of the following graphs could be that of $y < -x^2 + 2x^3 - 16x + 15$?

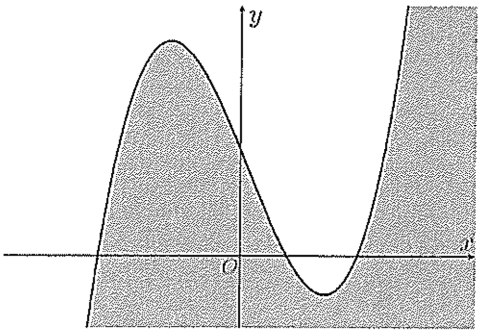
A.



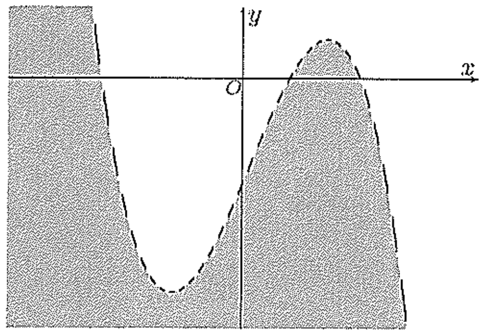
B.



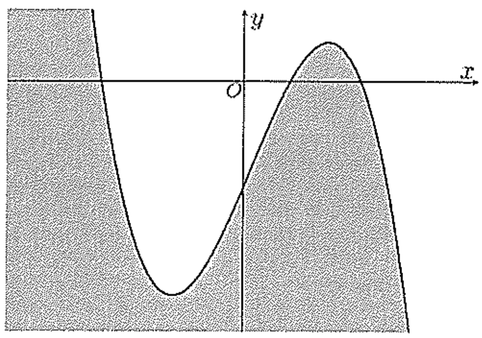
C.



D.



E.



Question 8

The graph of a certain circular function has amplitude of 3 and period of $\frac{\pi}{5}$. There is also an x-intercept at $x = -\frac{\pi}{6}$. Which of the following could be its equation?

A. $y = 3\tan\left(5\left(x + \frac{\pi}{6}\right)\right)$

B. $y = 3\sin\left(\frac{\pi}{5}\left(x + \frac{\pi}{6}\right)\right)$

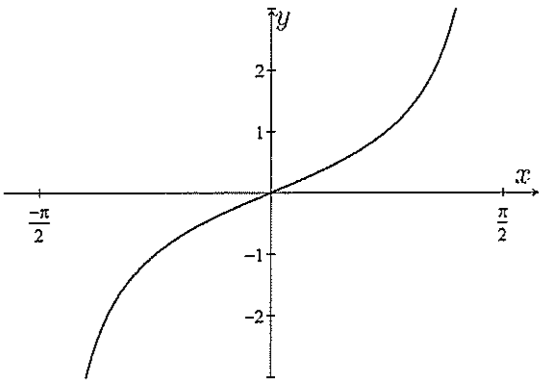
C. $y = \cos\left(10\left(x - \frac{\pi}{6}\right)\right) + 3$

D. $y = 3\sin\left(10x + \frac{5\pi}{3}\right) + 1$

E. $y = -3\cos\left(10\left(x + \frac{\pi}{5}\right)\right) + \frac{3}{2}$

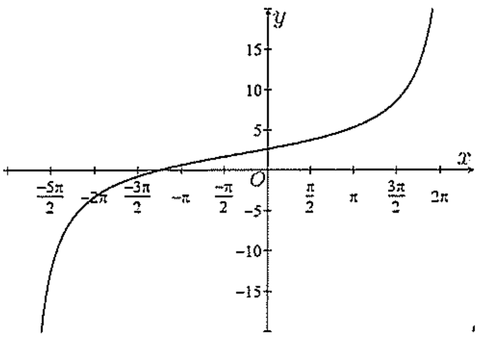
Question 9

The following graph is of the function $y = f(x)$.

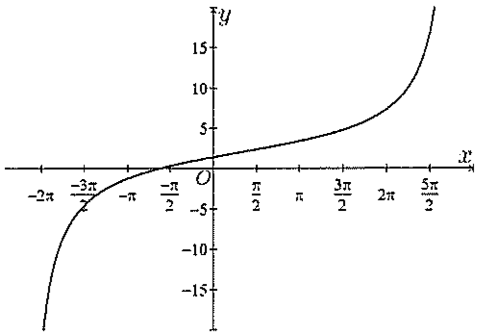


Which of the following graphs could be that of the function $y = 3f\left(\dfrac{x-1}{5}\right) + 2$?

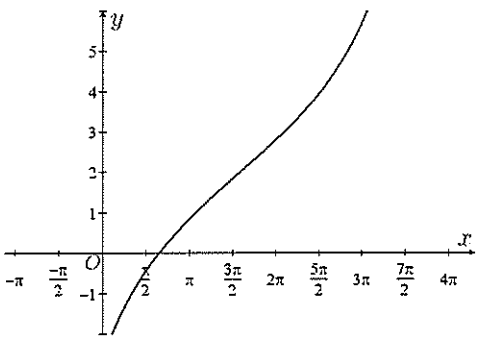
A.



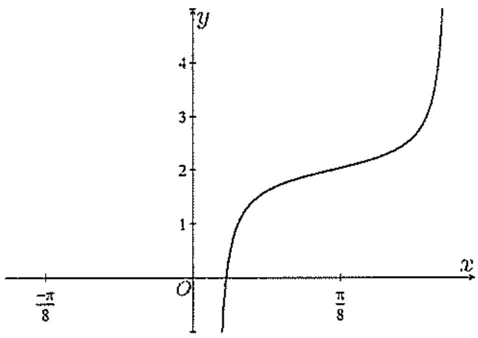
B.



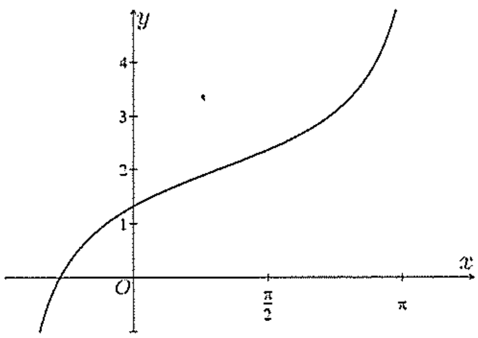
C.



D.



E.



Question 10

Consider the two functions, $y = f(x)$ and $y = f^{-1}(x)$.

Which of the following statements is **not** true regarding these two functions?

A. They are reflections of each other along the line $y = x$

B. The solutions to $f(x) = x$ will be points of intersection between the graphs of $y = f(x)$ and $y = f^{-1}(x)$

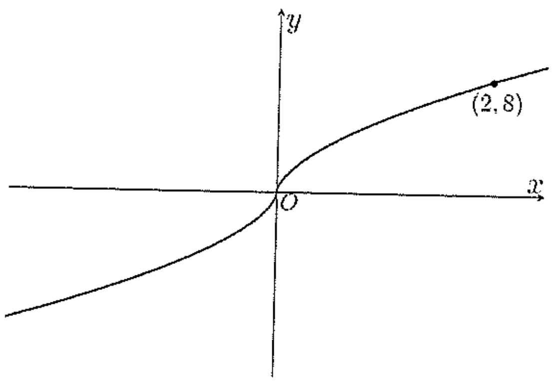
C. The transformation that allows you to obtain $y = f^{-1}(x)$ from $y = f(x)$ cannot be described in matrix notation

D. The range of $y = f(x)$ is equal to the domain of $y = f^{-1}(x)$

E. $y = f^{-1}(x)$ will be defined if and only if $y = f(x)$ is a one-to-one function

Question 11

The following is a graph of the function $y = g(x)$.



Which of the following could be its equation?

A. $g(x) = 2(4x)^{\frac{2}{3}}$

B. $g(x) = x^{\frac{1}{3}}$

C. $g(x) = (16x)^{\frac{3}{5}}$

D. $g(x) = (2x)^{\frac{3}{2}}$

E. $g(x) = \dfrac{(4x)^{\frac{4}{3}}}{2}$

SECTION 2 - Extended-Response Questions

Question 1

a. The following is known about the function $y = u(x)$:

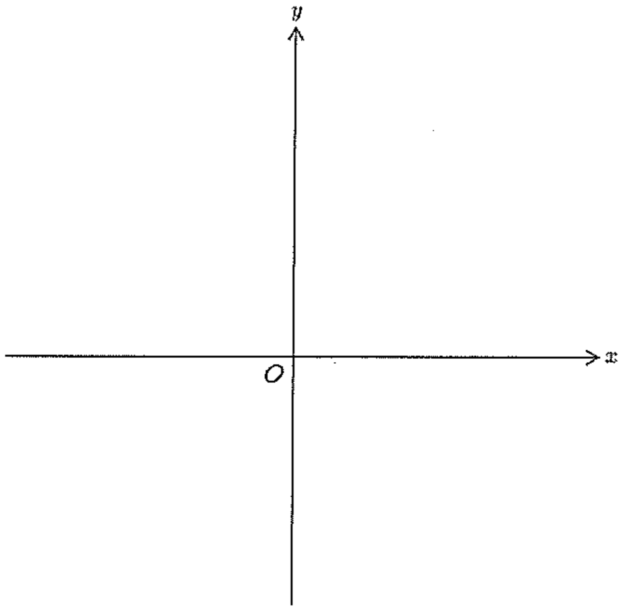
- It is a polynomial of degree 4.
- It goes through the points $(1, 8)$, $(-2, 2)$, $(0, 18)$, $(3, 72)$ and $(-4, 338)$.

i. Write down a system of simultaneous equations in five variables, **in matrix form**, that allows you to deduce the equation of $y = u(x)$ when solved.

ii. Using a CAS calculator or otherwise, state the equation of the function $y = u(x)$.

iii. The function $y = u(x)$ can actually be expressed as a composite function, namely as $y = f \circ g(x)$. If $f(x) = 2x^2$, find the rule of $g(x)$, given that it is a polynomial function in x.

iv. On the axes below, sketch the graph of $y = f \circ g(x)$, labelling any intercepts and turning points with coordinates.

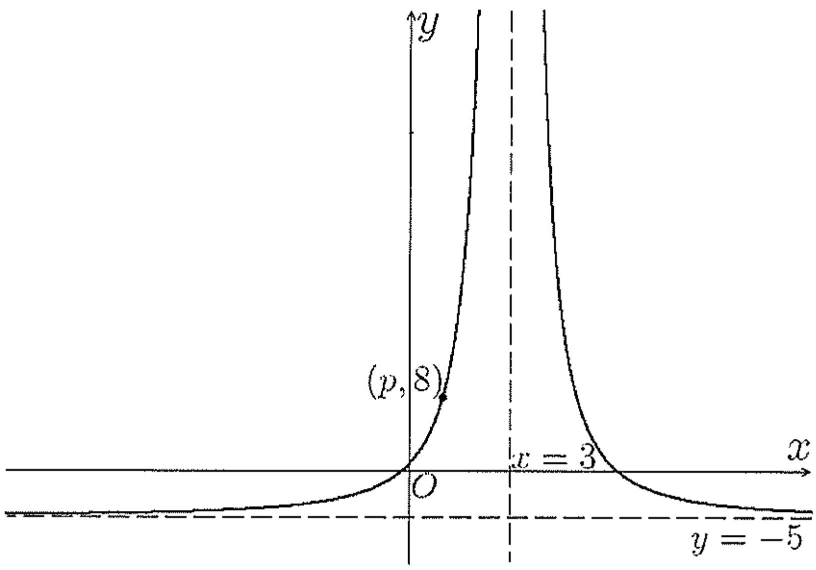


$1 + 1 + 2 + 2 = 6$ marks

b. The graph below is of the function $y = w(x)$, which has its rule given by

$$w(x) = \frac{a}{(x+b)^2} + c$$

where a, b and c are real constants.



i. On the set of axes provided above containing the graph of $y = w(x)$, sketch the graph of $y = |w(x)|$, labelling all asymptotes with their equations. Include the coordinates of any intercepts.

ii. State the values of b and c.

iii. Find a in terms of p.

iv. If the point $(p, 8)$, where $p < 2$, is the intersection of the graph of $y = w(x)$ with $y = f \circ g(x)$, find the value of a.

$2 + 1 + 1 + 3 = 7$ marks

Question 2

Noa is a manager of a recently opened hotel. The hotel opened 12 weeks ago, and she decides to look at the number of rooms that housed guests each week, to try get an idea of how the hotel was faring. Let $R(w)$ be the number of rooms that were being used w weeks after the hotel opened. Assume that rooms are always rented for exactly one week at this hotel.

The following table shows the values of R for some of the first 12 weeks.

w	1	2	3	4	6	8	10	12
$R(w)$	5	17	27	35	41	49	58	67

This means that, during the first week, 5 rooms were used by guests, during the second week, 17 rooms were used, and so on.

a. **i.** To try and determine the most accurate relationship, Noa first draws a line between the first and last point. What would be the equation of this line?

ii. Noa then decides it would be wiser to use a CAS calculator to determine the line of best fit for **all** of the points given above. State the equation of this line, giving coefficients correct to **two** decimal places.

$1 + 1 = 2$ marks

b. The hotel's accountant, Josh, suggests that a cubic function may be more adequate.
Using a CAS calculator, determine the cubic equation that would best fit all the points above, giving coefficients correct to **three** decimal places.

1 mark

c. Several months later, Noa begins collecting data to establish a long term relationship for $R(w)$ that is accurate from week 12 onwards. Noa works out that this new relationship for $R(w)$ is given by:

$$R(w) = -53e^{\left(1 - \frac{w}{12}\right)} + 120$$

i. State the domain and range of $y = R(w)$, given that they are continuous intervals.

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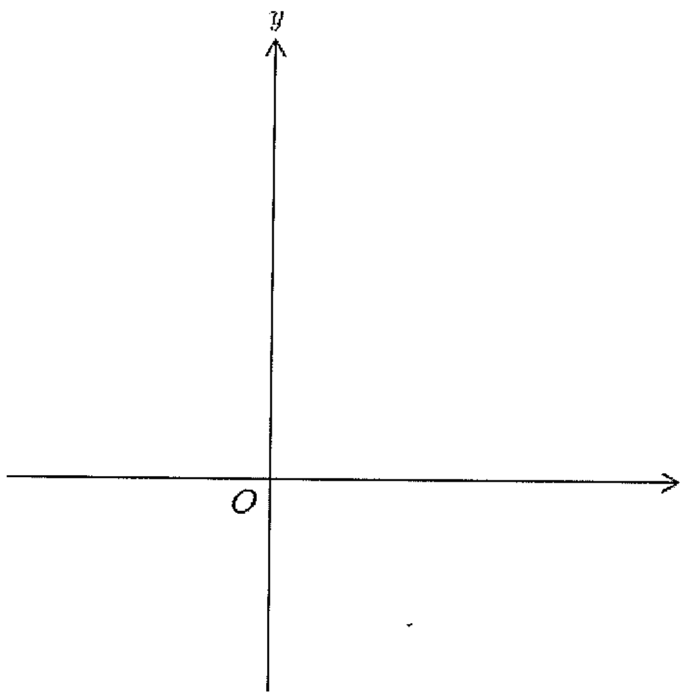
ii. On the axes provided below, sketch and label the graph of $y = R(w)$. Label all asymptotes with their equations and any intercepts and endpoints with their coordinates. Give all values correct to **three** decimal places where appropriate.



iii. Noa notices that, after several months, the hotel is eventually booked out during at least one week. If this model is accurate for a very long period of time, how many rooms must the hotel have?

---

iv. On the set of axes provided above, sketch and label the graph of $y = R^{-1}(w)$, labelling all asymptotes with their equations and including the coordinates of any intercepts/endpoints. Give all values correct to **three** decimal places where appropriate.

v. State the coordinates of any points of intersection between the graphs of $y = R(w)$ and $y = R^{-1}(w)$. Give all values correct to **three** decimal places where appropriate.

$1 + 2 + 1 + 2 + 1 = 7$ marks

d. Ben works at a competing hotel, and manages to hack onto Noa's computer and change her data. He applies the following sequence of transformations to the graph of $y = R(w)$ (given in **part c.**) to make the hotel appear to be losing business:

- The graph is translated 15 units in the negative direction of the w-axis
- The graph is then translated 7 units in the negative direction of the y-axis
- The graph is then dilated by a factor of $\frac{1}{2}$ from the w-axis
- The graph is then dilated by a factor of 3 from the y-axis
- The graph is then reflected in the y-axis

i. After all these transformations are applied, what is the new rule for $R(w)$?

ii. Noa notices the changes in the graph, and decides to correct them by applying a transformation T, where

$$T\left(\begin{bmatrix} w \\ y \end{bmatrix}\right) = \mathbf{T}\left(\begin{bmatrix} w \\ y \end{bmatrix} + \mathbf{B}\right)$$

If the transformation T indeed reverts the graph of $y = R(w)$ back to its original rule (found in **part c.**), find the matrices $\mathbf{T}$ and $\mathbf{B}$.

$3 + 3 = 6$ marks

END OF TEST